Pension Risk, Saving and Retirement Decisions

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Abstract: We analyse the impact of changes in social security provisions on the joint saving and retirement decisions. We argue that both decisions cannot be regarded as independent ones. Our main finding is that policy changes may impact the combined saving and retirement decisions in a markedly different way than they would do if either of the decisions were analysed independently. A major focus of our analysis is the reaction of saving and retirement decisions upon changes in pension risk. This is relevant as many current pension reforms involve a higher (perceived) riskiness or unreliability of social security benefits. Again, the combined saving/retirement choice may look quite different than the isolated saving and retirement responses.

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1 Introduction

The decreasing participation of older workers in the workforce is a major concern in many OECD countries. According to Gruber and Wise (1999) the participation rates of 60 to 64 year old men in OECD countries have fallen from 70-80% in the 1960s to typically less than 50%, in some countries to well below 20% in the mid 1990s. Starting with Boskin (1977) many economists argued that developments in social security schemes bear the main responsibility for this trend. This view is very convincingly supported by recent empirical studies, reported in Gruber and Wise (1999), which argue that the generosity of, and the easy eligibility for, public pensions create strong incentives for departing from the labour force.[1] Early retirement is problematic for at least two reasons: First, it magnifies adverse population trends by increasing the dependency ratio (i.e., the number of persons that are not working relative to the number of those working), putting additional pressure on social security schemes. Second, withdrawals from the labour force mean foregone productive capacities and thus output losses. Increasing the age of retirement therefore has become one of the major concerns of pension politics.

Another major issue in the arena of pension policy is a partial shift from PAYG pension schemes (which are held to be unsustainable due to demographic changes) towards funded schemes. While pension funding itself will certainly not alleviate the demographic crisis itself, it is hoped to increase the capital stock of the economy, to lead to higher outputs, and to thus contribute to render the strains of the demographic crisis less severe. Saving, as the prerequisite for capital accumulation, is therefore one of the crucial variables in averting the old-age crisis.

Notwithstanding many differences in the details, recent reforms of PAYG pension schemes in OECD countries typically involve or imply two elements (see McHale, 1999, for ample evidence): lowering the generosity of the scheme and increasing its volatility (ie., reducing its reliability from an *ex ante* perspective). E.g., the actual German PAYG reform not only involves a (poorly disguised) cut in pensions but, by moving from a policy of keeping the replacement ratio fixed towards a policy of keeping the contribution rate stable, also exposes pensioners to a higher riskiness of their old-age incomes (see Section 7 for details). Apart from explicit changes in pension policies, the permanent discussion about the unsustainability and the poor prospects of PAYG schemes by itself generates the impression that state-run pension schemes are getting less and less reliable. Hence, the public trust in PAYG schemes has been rapidly deteriorating recently. In fact, Diamond (1997) and McHale (1999) argue that the political risk – defined as the volatility of the benefit rules of the social security scheme – is the main source of uncertainty associated with PAYG schemes.

As van der Klaauw and Wolpin (2001) stress, little is known about the interactions between saving and retirement decisions since most studies on retirement assume that individuals can

[1] See, however, Ruhm (1996) for evidence in the opposite direction.

neither borrow nor lend. However, saving and retirement decisions cannot be analysed independently since there are obvious relationships between these choices: A shorter working period *ceteris paribus* leaves a shorter time-span to accumulate funds and implies that the funds available at the age of retirement have to be stretched over a longer time-span. In so far, a move towards more funding may alleviate the early-retirement problem. Similarly, reducing the generosity of PAYG schemes may at first sight have two beneficial effects: inducing higher saving and postponing retirement.

In this paper we propose a simple life-cycle model that allows to analyse the joint decisions on saving *and* retirement in the presence of a public pension scheme (see Section 2). Using this model, we set out to discuss the impact of different changes in pension policies on joint saving/retirement choices. We focus on the two major trends in pension policies alluded to above: reducing the generosity of the scheme and increasing its riskiness. Stochastics thus make a major ingredient of our analysis, and we use results and techniques that recently have been proposed in the analysis of decision-making under uncertainty.

Our results suggest that less generous PAYG schemes tend to postpone retirement and to boost saving. While sounding quite natural, this result does not hold unless restrictions on preferences are imposed (precisely, the coefficients of relative risk aversion should by below unity). Otherwise, the interdependencies between saving and retirement may overcompensate the "first-order" effects. The main – and, to our knowlegde, new – focus of our paper is, however, on the effects of the second stylized fact of recent changes in pension policies, namely on the effects of changes in the pension risks on joint retirement and saving decisions. The following simple *gedankenexperiment* may exhibit the underlying trade-off:

Suppose that, due to a change in pension policies, old-age income becomes riskier (ie., a zero-mean noise is added). Ignoring the retirement decision, such a policy change increases the incentives to save – due to the precautionary motive. Ignoring, on the other hand, the saving decision, a riskier old-age income depresses the incentive to early retirement – due to risk aversion; Friedberg and Webb (2000) provide empirical evidence for this prediction. However, viewing saving and retirement decisions as dependent choices, the effects are far less clear-cut: Higher saving *ceteris paribus* also makes longer periods of retirement affordable, while later retirement *ceteris paribus* reduces the need for old-age saving. The sum of these effects is by no means clear. In fact, we show that a greater riskiness of old-age income may well induce earlier retirement or lower saving (although not both at the same time).

One of the (fairly obvious) lessons we learn from our analysis is that clear-cut results on the impact of changes in pension policies for single decision variables loose their validity if several decisions are analysed jointly. Recent research has made similar findings made in various other areas related to old-age provision. E.g., Cigno (1993) shows that, if one sees child-raising and saving as interdependent choices, the impact of social security schemes on the combined choice is far less clear than the impact on the isolated saving or fertility decision. In particular, it may well happen that social security schemes boost saving – contrary to the traditional Feldstein-hypothesis that social security depresses saving. Lau and Poutvaara (2001) analyse the effect of social security schemes on the simultaneous decisions of retirement and human capital investment. They also find a rather complex interaction of the two variables in reaction to changes in the social security scheme. Elmendorf and Kimball (2000) analyse the impact of labour income risk on the joint saving-portfolio composition problem. This paper is relevant here because it is the only one (we are aware of) that analyses the impact of changes in the structure of a multiple-risk setting on a two-dimensional decision problem with interdependent choice variables. Elmendorf and Kimball (2000) identify quite complex interactions between consumption and portfolio choices. Moreover, they use new concepts from the theory of decision-making under uncertainty to disentangle these complexities.

After presenting the simple model in Section 2, the paper goes through various scenarios each of which compares the optimal saving and retirement choices in two different situations: Prior and after changes in the generosity of the pension scheme (Section 3), with and without pension risks (Section 4), with and without financial risks (Section 5), and with and without background risks (Section 6). Each of these comparisons comes in a double form: We first analyse how saving and retirement decisions would be affected were they independent choices. Second, we discuss the eefects in the joint decision problem. Section 7 discusses our findings against the background of the recent German pension reform and concludes.

2 The Model

We consider a representative individual whose economically active lifespan extends over $D > 1$ periods. He can spend this time either working or in retirement. Retirement is an irreversible decision: Having once left the workforce, there is no return. The individual derives utility out of consumption and leisure. We assume that the individual does not have any discretion over their working hours in their active period. Without loss of generality we normalize leisure to be zero then. As individuals do not work at all during retirement, we can identify total lifetime leisure with the time spent in retirement.

Preferences are assumed to be additively separable between consumption at different dates and leisure:

$$U = \sum_{t=1}^{D} u(c_t) + \tilde{v}(D - R) \tag{1}$$

where $u, \tilde{v} : \mathbb{R}_{++} \rightarrow \mathbb{R}$ are increasing and strictly concave sub-utility functions for, respectively, consumption and leisure. To exclude corner solutions, we assume that u satisfies the Inada limit conditions. D means the certain date of death, while R with $0 \leq R \leq D$ denotes retirement age; this is endogenous. $\tilde{v}$ reflects the pleasures of retirement or, alternatively, the disutility of labour.

It will be helpful to define right here the Arrow-Pratt-index of relative risk aversion for the utility function u:

$$\mathcal{R} := -c \cdot \frac{u''(c)}{u'(c)} \tag{2}$$

Note that there is no time preference in (1). Correspondingly, we shall assume that the rate of interest is zero. We assume this for simplicity; none of the results to come depends on these assumptions (yet, the exposition would be much more complicated).

Consumption during working period equals the net wage minus saving s_t. The net wage differs from the gross wage w_t by a social security tax τ. Consumption during retirement has to be financed out of a pension P_t and capital income, i.e., the (zero) returns to saving plus the principal. Hence,

$$c_t = \begin{cases} w_t(1-\tau) - s_t & \text{if } t \text{ belongs to working period} \\ \text{cap. income} + P_t & \text{when retired} \end{cases} \tag{3}$$

To keep the analysis as simple as possible we assume that $w_t = w$ and for all t and $P_t = P$ for all $t \geq R$. Define

$$\lambda := \frac{D-R}{D} \tag{4}$$

as the fraction of the lifespan spent in retirement. We will henceforth treat λ as a continuous variable. Further set $v(\lambda) \equiv D^{-1} \cdot \tilde{v}(D\lambda)$; this implies $v' = \mathrm{d}\tilde{v}/\mathrm{d}\lambda$. As is shown in the Appendix, the problem of maximizing (1) subject to (3) can be equivalently represented by the following program:

$$\max_{s>0,\ 0\leq\lambda\leq 1} \left\{ (1-\lambda)u\left(w(1-\tau)-s\right) + \lambda u\left(\frac{1-\lambda}{\lambda}s + P\right) + v(\lambda) \right\}. \tag{5}$$

It will be helpful to introduce the notation c_1 and c_2 for annual consumption in the working period and during retirement, respectively.

3 Choices and Comparative Statics under Certainty

3.1 General results

The FOCs for problem (5) are:

$$u'(c_1) - u'(c_2) = 0 \tag{6a}$$

$$-u(c_1) + u(c_2) - \frac{s}{\lambda} \cdot u'(c_2) + v'(\lambda) = 0. \tag{6b}$$

Condition (6a) contains intertemporal consumption smoothing over working and retirement period. It clearly implies $c_1 = c_2 = c$. Therefore saving amounts to

$$s = \lambda \cdot [(1-t)w - P], \tag{7}$$

which is positive if and only if net labor income exceeds pensions. Saving thus makes up for a share of λ of the income differential between working period and retirement. The optimal level of consumption as a function of the point of retirement is

$$c = (1-\lambda)(1-\tau)w + \lambda P. \tag{8}$$

The RHS of (8), which is the sum of working and retirement incomes, equals lifetime income. Given that the lifespan is normalized to one, condition (8) is just a different way to state that consumption is distributed equally across periods.

Condition (6b) comprises three marginal effects of extending the time spent in retirement: First, an additional year of retirement yields utility $u(c_2)$ from retirement consumption, but costs utility $u(c_1)$ from consumption in the working period; due to consumption smoothing (6a) these effects cancel out. Second, an additional year of retirement means one period less for wealth accumulation but one period more of dissaving. The effect on old-age consumption therefore is

$$\frac{\partial c_2}{\partial \lambda} = -s/\lambda^2$$

which is negative for positive saving (which we shall assume below). Hence, there is consumption disutility from earlier retirement. Third, the pleasures of longer retirement show up in the utility gain $v' > 0$.

Note that without any utility from retirement (i.e., for $v \equiv 0$) individuals would either never retire (if $w(1-\tau) > P$) or never work (if $P > w(1-\tau)$). Using $c_1 = c_2$, we can rewrite the condition (6b) as

$$-((1-\tau)w - P) \cdot u'[(1-\tau)(1-\lambda)w + \lambda P] + v'(\lambda) = 0. \tag{9}$$

We will henceforth assume that net labour income exceeds pensions:

$$\Delta := (1-\tau)w - P > 0. \tag{10}$$

This is in harmony with reality where replacement rates are typically below unity. As a consequence, saving will be positive and, given that v' shows enough variation on $[0,1]$, there is an interior value $0 < \lambda < 1$ for optimal retirement. Condition (9) then simply states that one should retire when the margianl utility of another year's higher income from working ($\Delta u'(c)$) is equal to the disutility of work ($v'(\lambda)$); see Kingston (2000) for this familiar condition.

Note that condition (9) depends on λ only. It can thus be solved to obtain the optimal value of $\lambda = \lambda(w, \tau, P)$. Plugging this into (7), we get optimal saving $s = s(w, \tau, P)$. Hence, the problem is technically very simple, and its comparative statics are rather straightforward:

Fact 1 • *Retirement age decreases if the pension increases:* $\dfrac{\partial\lambda}{\partial P} > 0$.

- *Suppose that* $\mathcal{R} \leq 1$. *Then the retirement age decreases if the tax rate is increased or if wages are reduced:* $\dfrac{\partial\lambda}{\partial\tau} > 0$ *and* $\dfrac{\partial\lambda}{\partial w} < 0$.
- *Suppose that the pension is initially small:* $P = 0$. *Then:*

$$\mathcal{R} \leq 1 \quad \Longleftrightarrow \quad \frac{\partial\lambda}{\partial\tau} > 0 \quad \Longleftrightarrow \quad \frac{\partial\lambda}{\partial w} < 0. \tag{11}$$

Proof: Totally differentiating (9), we get:

$$\begin{aligned} -\mathrm{d}\lambda \cdot [\Delta^2 u''(c) + v''] &= [\mathrm{d}\tau \cdot w - \mathrm{d}w \cdot (1-\tau)] \cdot \big(u'(c) + \Delta(1-\lambda)u''(c)\big) \\ &\quad + \mathrm{d}P \cdot [u'(c) - \lambda\Delta u''(c)]. \end{aligned} \tag{12}$$

Consider the signs of the square-bracketed expressions in this equation. The one on the LHS is negative while the last one on the RHS is positive. To determine the sign of the first and second one on the LHS check that

$$0 < \Delta(1-\lambda) = (1-\lambda)(1-\tau)w - (1-\lambda)P = c - P \leq c.$$

Hence,

$$u'(c) + (1-\lambda)\Delta u''(c) = u'(c)\left[1 - \frac{c-P}{c} \cdot \mathcal{R}\right] \geq u'(c)[1-\mathcal{R}],$$

where the equality sign holds if $P = 0$. Combining these observations leads to Fact 1. ∎

As expected, a higher pension makes retirement unambiguously more attractive. An increase in net wages has, however, an ambiguous effect, reflected in $u' + (1-\lambda)\Delta u''$: Increases in net wages make retirement less attractive since they mean higher amounts of income foregone. (This income effect is reflected in $u'(c) > 0$.) Yet, an increase in net wages also means an increase in lifetime income which, by saving, is spread equally across working age and retirement. This increase in lifetime income (and lifetime consumption) reduces the marginal utility of consumption and thus makes consumption less costly in terms of leisure. This raises the incentives to retire. (This substitution effect is reflected in $(1-\lambda)\Delta u'' < 0$.) The magnitude of $\mathcal{R}$ then determines which effect dominates the other. The third item in Fact 1 shows, however, that this unabatedly only holds when $P = 0$. The existence of a positive pension weakens the substitution effect (such that the second item in Fact 1 is not an equivalence result.)

Fact 2 • *Suppose that* $v'' \equiv 0$, *i.e.,* $v(\lambda) = \bar{v} \cdot \lambda$.

(i) Savings increase upon an increase in pensions.

(ii) If the replacement rate in the pension scheme is above 50%, then saving is reduced in reaction to an increase in wages or to a reduction in wage taxes if $\mathcal{R} \leq 1$. The same happens for $\mathcal{R} > 1$ if the income discrepancy Δ is small.

(iii) If the initial pension is small ($P = 0$), then saving increases in reaction to a wage increase or a tax cut whenever $\mathcal{R} \geq 1$.

- *Otherwise, the effects on saving are ambiguous.*

Proof: From (7), we get

$$\mathrm{d}s = \Delta \cdot \mathrm{d}\lambda + \lambda \cdot \mathrm{d}\Delta, \tag{13}$$

or

$$\frac{\partial s}{\partial P} = \Delta \cdot \frac{\partial \lambda}{\partial P} - \lambda \tag{14a}$$

$$\frac{\partial s}{\partial w} = -\frac{1-t}{w} \cdot \frac{\partial s}{\partial \tau} = \Delta \cdot \frac{\partial \lambda}{\partial w} + (1-\tau)\lambda \tag{14b}$$

Combining (14a) and (12), we obtain

$$\frac{\partial s}{\partial P} \gtreqless 0 \iff -\Delta(u' - \lambda \Delta u'') \lesseqgtr \lambda(v'' + \Delta^2 u'')$$
$$\iff -\Delta u' \lesseqgtr \lambda v''. \tag{15}$$

For $v'' = 0$, we thus have $\partial S/\partial P > 0$. Otherwise, the effect is unclear. Analogously to (15) calculate from (14b) and (12) that

$$\frac{\partial s}{\partial w} \gtreqless 0 \iff \Delta(u' + (1-\lambda)\Delta u'') \lesseqgtr -\lambda(v'' + \Delta^2 u'')$$
$$\iff \Delta u' \left[1 - \frac{\Delta}{c} \cdot \mathcal{R}\right] \lesseqgtr -\lambda v''. \tag{16}$$

For $v'' = 0$ we thus have that $\partial S/\partial w < 0$ iff $\mathcal{R} < c/\Delta$. Without any pension ($P = 0$) we have $\Delta/c = (1-\lambda)^{-1} > 1$. However, for replacement rates $P/[(1-\tau)w]$ above 50%, one gets $\Delta/c < 1$. Namely:

$$\frac{\Delta}{c} \lesseqgtr 1 \iff P \gtreqless \frac{\lambda}{1+\lambda} \cdot (1-\tau)w \tag{17}$$

where $\lambda/(1+\lambda) \in (0, 1/2)$ for all $0 < \lambda < 1$.
Since $-\lambda v'' \geq 0$ in (16), we get a "normal reaction" $\partial S/\partial w \geq 0$ whenever $\mathcal{R} \geq c/\Delta$. ■

As Fact 2 shows, the effects of pension provision (i.e., contributions and pensions) are complex whence retirement choices are endogenized. Consider, e.g., an increase in the pension P. Such an increase reduces the incentive to save since shifting income from working age to retirement

becomes less necessary (captured in $-\lambda$ in (14a). If the retirement age were exogenous, this would be the only effect. It has since long been identified as one of the negative effects of social security schemes that they reduce capital formation. However, if retirement choices are endogenous an increase in pensions also positively impacts saving since, due to earlier retirement, there is higher necessity for saving (captured by $\Delta\partial\lambda/\partial P$ in (14a)). Similar observations apply to increases in net wages which – if considered under isolation – typically increase saving via the consumption-smoothing argument (cf. the term $(1-\tau)\lambda > 0$ in (14b)). If higher wages induce earlier retirement, the incentives to save are further increased. If, however, individuals with higher wages retire later (i.e., if $\partial\lambda/\partial w < 0$ in (14b)), then there is an offsetting effect.
Generally, the effects of social security provisions on saving are ambiguous. In its items (i) and (ii), Fact 2 assembles sufficient conditions such that saving shows reactions that are opposite to those one would observe when retirement were exogenous. This is to be understood as a possibility result; "odd" effects can also emerge under circumstances other than the special case constructed in Fact 2.

3.2 A pension reform

We now consider the effects of a pension reform that is neutral with respect to the social security budget. I.e., we consider a change in the contribution rate τ, accompanied by a change in the pension P such that the size of the social security scheme remains unchanged, taking into account, of course, that retirement decisions affect either side of the scheme's budget equation. Recalling that the interest rate is zero, the requirement that the present value of the social security deficit or budget is constant can be written as

$$\mathrm{d}[(1-\lambda)\tau w - \lambda P] = -\mathrm{d}\lambda(\tau w + P) + w(1-\lambda)\mathrm{d}\tau - \lambda \mathrm{d}P = 0. \tag{18}$$

Note from (18) that it is not *a priori* clear whether an increase in the contribution rate $\mathrm{d}\tau < 0$ indeed allows for an increase in pensions $\mathrm{d}P > 0$: If the retirement age decreases considerably upon the pension reform ($\mathrm{d}\lambda \gg 0$), then a cut in pensions is required to keep the social security budget balanced. As the following result states, this can, however, not happen:

Fact 3 *A pension reform (18) with* $\mathrm{d}\tau > 0$ *(i) decreases retirement age, (ii) has an ambiguous effect on saving, (iii) always involves an increase in pensions, and (iv) reduces utility.*

Proof: Plugging (18) into (12), setting $\mathrm{d}w = 0$, one obtains after some algebraic manipulations:

$$\left.\frac{\mathrm{d}\lambda}{\mathrm{d}\tau}\right|_{(18)} = -\frac{wu'}{\lambda\left(\Delta w u'' + v'' - \frac{\tau w + P}{\lambda} u'\right)} > 0. \tag{19}$$

which proves (i).Verify that

$$\mathrm{d}\Delta|_{(18)} = -w\mathrm{d}\tau - \mathrm{d}P = \frac{\mathrm{d}\lambda}{\lambda}(\tau w + P) - \frac{1}{\lambda} w \mathrm{d}\tau. \tag{20}$$

Invoking this in (13), one gets

$$\left.\frac{\mathrm{d}s}{\mathrm{d}\tau}\right|_{(18)} = w\left(\left.\frac{\mathrm{d}\lambda}{\mathrm{d}\tau}\right|_{(18)} - 1\right). \tag{21}$$

Check from (19) that

$$\left.\frac{\mathrm{d}\lambda}{\mathrm{d}\tau}\right|_{(18)} - 1 \gtreqless 0 \quad \Longleftrightarrow \quad \Delta(u' + \lambda w u'') + \lambda v'' \gtreqless 0. \tag{22}$$

The sign of this expression is ambiguous (even for $v'' = 0$). Hence, the effect of (18) on saving is generally unclear, which proves (ii). If $\Delta = 0$ or positive, but small, then (22) is negative and, thus, the pension reform (18) depresses saving.
To see (iii), check from (18) and (19) that

$$\begin{aligned} \lambda\left.\frac{\mathrm{d}P}{\mathrm{d}\tau}\right|_{(18),(19)} > 0 \quad &\Longleftrightarrow \quad w(1+\lambda) > -\frac{wu'(\tau w + P)}{\lambda\left(\Delta w u'' + v'' - \frac{\tau w + P}{\lambda} u'\right)} \\ &\Longleftrightarrow \quad -(1+\lambda)(\Delta w u'' + v'') > -u'(\tau w + P), \end{aligned}$$

which always holds.
To see (iv) use the envelope theorem to get that $\frac{\partial u^*}{\partial P} = \lambda u'(c_2)$ and $\frac{\partial u^*}{\partial \tau} = -(1-\lambda) w u'(c_1)$, where u^* denotes indirect utility. Then,

$$\begin{aligned} \mathrm{d}u^*|_{(18)} &= \left[\frac{\partial u^*}{\partial P}\left(\frac{\mathrm{d}P}{\mathrm{d}\tau}\right)_{(18),(19)} + \frac{\partial u^*}{\partial \tau}\right]\mathrm{d}\tau \\ &= u'(c) \cdot \frac{w}{\lambda\left(\Delta w u'' + v'' - \frac{\tau w + P}{\lambda} u'\right)} \cdot \mathrm{d}\tau < 0 \end{aligned}$$

where we used (19) and (18). ■

3.3 Penalties to early retirement

So far we assumed that the pension P is the same, regardless of the age of retirement. In reality, this is typically not the case for two reasons: First, in Bismarckian pension schemes the pension one receives typically depends on the amount and/or the duration of contributions one has made to the pension scheme. Second, the social security provisions in many countries (try to) penalize early retirement by cutting the pension if the pensioner-to-be retires before some "general" retirement age. I.e., we typically observe pension formula $P = P(\lambda)$ with $P'(\lambda) < 0$. Without further information on the pension formula nothing of interest can be said. Therefore we use a linear specification which has a straightforward interpretation:

$$P(\lambda) = \alpha - \beta(\lambda - \bar{\lambda}) = \bar{P} - \beta\lambda \tag{23}$$

Here, $(1-\bar{\lambda})$ can be interpreted as the general retirement age. For each year that he retires before this age, the pensioner faces a cut in his pension by β. The second formulation in (23) is particularly convenient to work with. Using it in the framework of the previous section, the modified FOCs can be rearranged to read:

$$s = \lambda \cdot \bar{\Delta} \tag{24a}$$

$$v'(\lambda) = -\bar{\Delta} \cdot u'\left((1-\tau)(1-\lambda)w + \bar{P} - \beta\lambda\right) \tag{24b}$$

where $\bar{\Delta} := (1-\tau)w - \bar{P}$ does neither depend on λ nor on β.[2] The comparative statics of λ and s with respect to $\bar{P}$ and τ are identical to those given in Facts 1 and 2 when P is replaced by $\bar{P}$. It is an easy exercise to calculate that an increase in the tax on early retirement increases retirement age and reduces saving:[3]

$$\frac{\partial\lambda}{\partial\beta} = -\frac{\lambda^2\bar{\Delta}u''}{\bar{\Delta}^2u'' + 2\lambda\beta + v''} < 0$$

$$\frac{\partial s}{\partial\beta} = \bar{\Delta}\frac{\partial\lambda}{\partial\beta} < 0.$$

An increase in β is in general beneficial to the social security budget. Namely,

$$\frac{\partial}{\partial\beta}\left((1-\lambda)\tau w - \lambda P(\lambda)\right) = -(\bar{\Delta} - 2\lambda\beta)\cdot\frac{\partial\lambda}{\partial\beta} + \lambda\beta.$$

Here, the second term $\lambda\beta > 0$ is the direct revenue effect of increasing the "retirement tax", while the first term covers the tax base effect. If $\bar{\lambda}\beta$ is small, relative to $\bar{\Delta}$, both effects are positive. This will, in particular happen, if $\beta = 0$ initially, i.e., if a "retirement tax" is newly introduced.

4 Choices with Old-Age Income Risks

In this section, we introduce uncertainty into the economy. In particular, we assume that retirement income contains a risky component $\tilde{b}$. We will interpret the risk associated with $\tilde{b}$ as a pension risk.[4] We wish to capture the effect that, due to changes in pension policy, social security schemes become, or are perceived to become, more volatile and less reliable.

In this section we shall assume that the pension consists of a constant part, P, as before, plus a stochastic part $\tilde{b}$. Without loss of generality we assume that the expected value of $\tilde{b}$ is zero, while it has a non-degenerate distribution over a closed interval of the real line. He further assume that the lower bound of the support of $\tilde{b}$ is strictly greater than $-P$ such that non-positive total pensions will never emerge. By $\mathbf{E}$ we denote the expectation operator, and all expectations in

[2] Since we are only interested in the retirement tax we ignore that, in the first specification in (23) also $\bar{P}$ depends on β.

[3] The denominator in $\partial\lambda/\partial\beta$ is negative due to the second-order condition.

[4] Risks to saving will be discussed below, in Section 5.

this section will be is taken with respect to the distribution of $\tilde{b}$. We assume that all decisions are to be made before the stochastic shock $\tilde{b}$ is realized.

In order to obtain informative results we have to make further restrictions on preferences. To capture changes of risk perceptions with wealth we impose the standard assumption that individuals have *decreasing absolute risk aversion* (DARA):

$$\left(-\frac{u''(c)}{u'(c)}\right)' = -\frac{u'(c)u'''(c) - u''(c)^2}{u'(c)^2} < 0. \tag{25}$$

Recall that DARA requires prudence, i.e., that the marginal utility of consumption is convex: $u'''(c) > 0$.

Our analysis will proceed as follows: We first discuss the effects of a higher riskiness in pensions on retirement and saving decisions under the premise that either decision can be made independently of the other; we call this "isolated choices". We then discuss what happens if both choices have to be made simultaneously ("joint decisions").

4.1 Isolated Choices

Consider the isolated decision problems that only deal with one choice variable:

- If the household only has to make a saving decision (while retirement age is fixed), his optimization problem reads

$$\max_s \left\{(1-\lambda)\cdot u((1-\tau)w - s) + \lambda \cdot \mathbf{E}u\left(\frac{1-\lambda}{\lambda}s + P + \tilde{b}\right)\right\}. \tag{26}$$

  Denote the solution to (34) by s^a. Similarly, denote by s^b the solution for the saving problem under certainty (i.e., when ϵ and the expected value are removed from (26)).

- If savings are predetermined at some level $\bar{s}$ and the single choice variable of the individual is his retirement age, the decision problem reads

$$\max_\lambda \left\{(1-\lambda)u(w(1-\tau) - \bar{s}) + \lambda\mathbf{E}u\left(\frac{1-\lambda}{\lambda}\bar{s} + P + \tilde{b}\right) + v(\lambda)\right\}.$$

  while it reduces under certainty to

$$\max_\lambda \left\{(1-\lambda)\cdot u((1-\tau)w - \bar{s}) + \lambda \cdot u\left(\frac{1-\lambda}{\lambda}\bar{s} + P\right) + v(\lambda)\right\}. \tag{27}$$

  Denote the maximizers to these problems by λ^a and λ^b, respectively.

Comparing s^a to s^b and λ^a to λ^b we obtain:

Fact 4 *Given DARA, $s^a > s^b$ and $\lambda^a < \lambda^b$.*

Proof: Convexity of marginal consumption implies $\mathbf{E}u'(c_2) > u'(\mathbf{E}c_2)$ by Jensen's inequality. Hence, the result for saving is obvious. Retirement levels λ^a and λ^b satisfy:

$$\begin{aligned} u(w(1-\tau)-\bar{s}) &= v'(\lambda^b) + u\left(\frac{1-\lambda^b}{\lambda^b}\bar{s}+P\right) - \frac{\bar{s}}{\lambda^b}\cdot u'\left(\frac{1-\lambda^b}{\lambda^b}\bar{s}+P\right) \\ &= v'(\lambda^a) + \mathbf{E}u\left(\frac{1-\lambda^a}{\lambda^a}\bar{s}+P+\tilde{b}\right) - \frac{\bar{s}}{\lambda^a}\cdot \mathbf{E}u'\left(\frac{1-\lambda^a}{\lambda^a}\bar{s}+P+\tilde{b}\right) \\ &< v'(\lambda^a) + u\left(\frac{1-\lambda^a}{\lambda^a}\bar{s}+P\right) - \frac{\bar{s}}{\lambda^a}\cdot u'\left(\frac{1-\lambda^a}{\lambda^a}\bar{s}+P\right). \end{aligned}$$

The inequality follows from risk aversion ($\mathbf{E}u(c_2) < u(\mathbf{E}c_2)$) and prudence ($\mathbf{E}u'(c_2) > u'(\mathbf{E}c_2)$). As the objective function for the "riskless" problem is globally concave in λ,

$$\frac{\partial}{\partial\lambda}\left(v'(\lambda) + u\left(\frac{1-\lambda}{\lambda}\bar{s}+P\right) - \frac{\bar{s}}{\lambda}\cdot u'\left(\frac{1-\lambda}{\lambda}\bar{s}+P\right)\right) = v''(\lambda) + \frac{s^2}{\lambda^3}u'' < 0,$$

we obtain $\lambda^a < \lambda^b$. ■

As is well-known (see Kimball, 1990), DARA implies a precautionary motive of saving: Since its returns saving are riskless, saving serves as a buffer against the pension shocks to old-age consumption. Incentives to save are thus the higher the greater is the old-age income risk. A similar prudence argument (paired with a risk aversion argument) also implies that retirement age rises when pensions get riskier. First, a higher pension risk makes living in retirement *per se* less attractive for risk averse individuals ($\mathbf{E}u(c_2) < u(\mathbf{E}c_2)$). Second, prudent individuals will wish to build up an additional buffer of safe consumption against risky income. One way to do this is to increase saving, another one is to postpone retirement (Note that, for given saving, second-period consumption increases with retirement age: $\partial c_2/\partial\lambda = -\bar{s}/\lambda^2 < 0$). Risk aversion and prudence thus work into the same direction here: If pensions get riskier, individuals retire later.

Friedberg and Webb (2000) have recently provided empirical evidence that higher pension risks lead to later retirement. They compare the retirement behaviour of individuals who have defined-benefit (DB) pension plans to that of people with defined-contribution (DC) plans. DC plans leave the return risk with the pensioner-to-be while DB plans promise a certain pension stream from the outset. Hence, DC plans are *ceteris paribus* riskier than DB plans. Friedberg and Webb (2000) find that DB plans induce individuals to retire almost two years earlier in average, compared to people with DC plans. Our Fact 4 provides a theoretical underpinning for this observation.

4.2 Joint Decisions

If saving and retirement decisions are jointly made, the optimization problem reads:

$$\max_{s,\lambda}\left\{(1-\lambda)u(w(1-\tau)-s) + \lambda\mathbf{E}u\left(\frac{1-\lambda}{\lambda}\cdot s+P+\tilde{b}\right)+v(\lambda)\right\} \tag{28}$$

and its FOCs are:

$$-u'(c_1) + \mathbf{E}u'(c_2) = 0 \tag{29a}$$

$$-u(c_1) + \mathbf{E}u(c_2) - s/\lambda \cdot \mathbf{E}u'(c_2) + v'(\lambda) = 0. \tag{29b}$$

First note that conditions (29a) and (29b), unlike their counterparts (6a) and (6b) in the certainty case, cannot be analysed independently. In particular, (29a) cannot be explicitly solved to yield a function $s = s(\lambda)$ which could be plugged into (29b).

The precautionary motive for saving, which is implied by DARA, is also at work in (29a): Convexity of marginal consumption implies $\mathbf{E}u'(c_2) > u'(\mathbf{E}c_2)$. Using the concavity of u, we thus get that consumption during working age falls below the expected value of consumption during retirement: $c_1 < \mathbf{E}c_2$ or, in terms of saving:

$$s > \lambda \cdot (w(1-\tau) - P) = \lambda \cdot \Delta. \tag{30}$$

Hence, if the retirement age λ were the same as in the certainty setting (compare (7)), saving would be higher. This claim naturally only holds when the retirement age were the same with and without a pension risk. Since (6b) and (29b) do not coincide, this is obviously not the case. In what follows we try to compare the optimal decisions under certainty (i.e., the solution to (6a) and (6b)) with those under uncertainty (i.e., the solution to (29a) and (6b)). Denote the latter by (λ_1, s_1) and the former by (λ_0, s_0). Also, in what follows, variables with subscript 0 (subscript 1) denote values of that variable for the riskless (risky, respectively) scenario.

Define the difference in (expected) utilities of consumption between (any) retirement and (any) working period by

$$\delta_u := \mathbf{E}u(c_{2,1}) - u(c_{1,1}) \tag{31}$$

As a first observation we get that expected utility from consumption in old age is higher than in working age:

Lemma 1 *Under DARA, $\delta_u > 0$.*

Proof: Define ψ and π as, respectively, the (equivalent) precautionary premium and the (equivalent) risk premium for the pension risk (see Kimball (1990) for these concepts). I.e., $u'(\mathbf{E}c_{2_1} - \psi) = \mathbf{E}u'(c_{2,1})$ and $u(\mathbf{E}c_{2,1} - \pi) = \mathbf{E}u(c_{2,1})$. From Kimball (1990, p. 65) we know that $\psi > \pi$. Now verify from (29a) that $c_{1,1} = \mathbf{E}c_{2,1} - \psi$. Hence,

$$u(c_{1,1}) = u(\mathbf{E}c_{2,1} - \psi) < u(\mathbf{E}c_{2,1} - \pi) = \mathbf{E}u(c_{2,1}),$$

which proves the Lemma. ■

Hence, individuals expect to be better off in retirement: They enjoy the pleasures of leisure and they enjoy a higher expected utility from consumption. This second effect certainly increases the incentives to retire early as compared to the certainty case where utility from consumption is split equally across all periods of life.

However, there is an offsetting effect, captured in $-s_1/\lambda_1 u'(c_{1,1})$ in (29b). Namely, upon using (30) we get that

$$-s_1/\lambda_1 \cdot u'(c_{1,1}) < -\Delta \cdot u'((1-\tau)w - s_1) < -\Delta \cdot u'((1-\tau)w - \lambda_1 \cdot \Delta).$$

The final expression denotes the marginal disutility of consumption foregone due to earlier retirement if s_1 and λ_1 were the optimal choices under certainty. We thus find that the marginal disutility of consumption foregone due to retirement increases when uncertaimty enters the play. This depresses the incentices to retirement.

Hence, introducing uncertainty exerts two opposing effects on thge marginal utility of retiring: a positive consumption effect (consumption is, in expected terms, more pleasureful during retirement than during working age) and a negative income effect (the loss of lifetime income becomes more serious). We sum this up in

Fact 5 *If pensions get riskier individuals retire earlier (i.e., $\lambda_1 > \lambda_0$) if the positive income effect more than outweighs the negative consumption effect. If that happens, saving will be unambiguously higher: $s_1 > s_0$. Otherwise (i.e., if $\lambda_0 > \lambda_1$) savings can also be decreased, as compared to their level with a riskless pension. Lower savings always go along with later retirement.*

Proof: Conditions (29a) and (29b) imply

$$\delta_u = -v'(\lambda_1) + s_1/\lambda_1 u'(c_{1,1}) > v'(\lambda_1) - \Delta \cdot u'((1-\tau)w - \lambda_1 \cdot \Delta) =: f(\lambda_1) \tag{32}$$

where f defines the function $v' - \Delta u'(\cdot)$. Note that $f(\lambda_0) = 0$ from (6b). Further, $f'(\lambda) < 0$. Hence, if δ_u were zero, we would have $\lambda_1 < \lambda_0$. However, if δ_u is large enough, we get $\lambda_1 > \lambda_0$. If $\lambda_0 < \lambda_1$ we clearly have $s_0 < s_1$. Namely:

$$s_0 = \Delta \cdot \lambda_0 < \Delta \cdot \lambda_1 < s_1.$$

However, for $\lambda_0 > \lambda_1$, no clearcut result for saving can be obtained. We know that $s_0 = \Delta \cdot \lambda_0 > \Delta \cdot \lambda_1$ and that $s_1 > \Delta \cdot \lambda_1$, which does not enables us to rank s_1 and s_0. If we know, however, that $s_0 \geq s_1$, we also have that individuals retire later: $\lambda_1 < \lambda_0$. ■

There is a different way to exhibit the opposed incentives for saving and retirement under uncertainty. Denote the objective function under uncertainty (i.e., the maximand in (28)) by

$Z = Z(s, \lambda)$. Then calculate:

$$\begin{aligned}
\frac{\partial Z(s_0, \lambda_0)}{\partial s} &= (1 - \lambda_0) \cdot \big(-u'(c_{1,0}) + \mathbf{E}u'(c_2(s_0, \lambda_0))\big) \\
&> (1 - \lambda_0) \cdot \big(-u'(c_{1,0}) + u'(\mathbf{E}c_2(s_0, \lambda_0))\big) \\
&= (1 - \lambda_0) \cdot \big(-u'(c_{1,0}) + u'(c_{2,0})\big) = 0 \qquad (33a) \\
\frac{\partial Z(s_0, \lambda_0)}{\partial \lambda} &= -u(c_{1,0}) + \mathbf{E}u(c_2(s_0, \lambda_0)) - \frac{s_0}{\lambda_0}\mathbf{E}u'(c_2(s_0, \lambda_0)) + v'(\lambda_0) \\
&< -u(c_{1,0}) + u(\mathbf{E}c_2(s_0, \lambda_0)) - \frac{s_0}{\lambda_0}u'(\mathbf{E}c_2(s_0, \lambda_0)) + v'(\lambda_0) \\
&= -u(c_{1,0}) + u(c_{2,0}) - \frac{s_0}{\lambda_0}u'(c_{2,0}) + v'(\lambda_0) = 0. \qquad (33b)
\end{aligned}$$

Moreover,

$$\frac{\partial^2 Z}{\partial s \partial \lambda} = u'(c_1) - \mathbf{E}u'(c_2) - \frac{s(1-\lambda)}{\lambda^2}\mathbf{E}u''(c_2). \qquad (33c)$$

This is positive at and close to (s_1, λ_1). We assume that it is positive throughout:[5] $Z_{s\lambda} > 0$. Starting from (s_0, λ_0), the introduction of uncertainty requires, in isolation, an increase in saving (as (33a) is still positive) and an increase in retirement age (as (33b) is already negative). From (33c) it can be seen, however, that either of such reactions also works to bring the other choice closer to an optimum. Hence, the overall direction of necessary moves towards $Z_s = Z_\lambda = 0$ is unclear.

The following graphical argument may be helpful to understand the situation. The FOC $\partial Z/\partial s = 0$ implicitly defines a function $s = \sigma_1(\lambda)$ that maps given λ into a corresponding value of s such that $\partial Z/\partial s = 0$ holds. This function is increasing as $Z_{s\lambda} > 0$:

$$\sigma_1'(\lambda) = -Z_{s\lambda}/Z_{ss} > 0.$$

The second FOC $\partial Z/\partial \lambda = 0$ similarly defines a locus $s = \sigma_2(\lambda)$ which is also increasing in λ:

$$\sigma_2' = -Z_{\lambda\lambda}/Z_{s\lambda} > 0.$$

The function σ_1 and σ_2 cross at (s_1, λ_1): $\sigma_1(\lambda_1) = \sigma_2(\lambda_1) = s_1$. Note that

$$\sigma_1'(\lambda) < \sigma_2'(\lambda) \quad \Longleftrightarrow \quad \frac{\partial^2 Z}{\partial s^2} \cdot \frac{\partial^2 Z}{\partial \lambda^2} - \left(\frac{\partial^2 Z}{\partial s \partial \lambda}\right)^2 > 0.$$

I.e., σ_2 is steeper than σ_1 if Z is (locally) concave (as it is the case at (s_1, λ_1) due to the maximum property). Figure 1 shows this.

Figure 1 goes here

[5] This is without any loss to generality as long as we assume that changes are small.

Points (such as (s_0, λ_0)) with $Z_s > 0 > Z_\lambda$ lie below the σ_1- and σ_2-lines in the shaded area of Figure 1 which we subdivided into three regions A, B, and C. Compared to the joint decisions on saving and retirement under certainty, it may under uncertainty happen that

- individuals retire earlier and save more (region A), or
- individuals save less and retire later (region B), or
- individuals retire later and save more (region C).

These findings should be contrasted with the optimal reactions of retirement and saving towards risk that would be observed when either decision were taken in isolation. In Fact 4 we observed that riskier old-age wealth then leads to additional saving and later retirement. I.e., only region C is relevant.

One interesting question of course is: How likely are situations where the relation between (s_0, λ_0) and (s_1, λ_1) is such as in A or B? As a preliminary measure for that likelihood we look at the sizes of regions A and B. Figure 1 suggests that Region A is the larger, the flatter is σ_2, and region B is the larger the steeper is σ_1. I.e., with joint decisions phenomena that are not in concordance with the predictions made for isolated choices are the more likely to occur the more elastically the optimal choices in the isolated problems react to (exogenous) changes in the other, ignored variable.[6]

The bottom line of this is: Upon an increase in the pension risk, anything can happen – except for a combination of lower saving *and* earlier retirement.

4.3 Comparative Statics

We now turn to the comparative statics. We are interested in the effects of changing P and τ under the joint saving/retirement-decisions under uncertainty. As a benchmark, we verify that the *isolated* decisions under uncertainty (i.e., the choices s^a and s^b) have the expected and highly plausible comparative statics. Implicit differentiation of the FOCs yields:

$$\frac{\partial s^a}{\partial P} < 0 \quad \text{and} \quad \frac{\partial s^a}{\partial [w(1-\tau)]} > 0$$
$$\frac{\partial \lambda^a}{\partial P} > 0 \quad \text{and} \quad \frac{\partial \lambda^a}{\partial [w(1-\tau)]} < 0.$$

[6] Restricting our attention to small risks, we can focus on the slopes of σ_1 and σ_2 at (s_1, λ_1). Using our previous results, we calculate:

$$\sigma_1' = \frac{s/\lambda}{\lambda + (1-\lambda)\frac{\mathbf{E}u''(c_2)}{u''(c_1)}} \quad \text{and} \quad \sigma_2' = \frac{s}{\lambda(1-\lambda)} + \frac{\lambda^2}{s(1-\lambda)}\frac{v''(\lambda)}{\mathbf{E}u''(c_2)}.$$

Hence, σ_2 gets flatter when $|v''|$ gets smaller. Further, σ_1 is the steeper and σ_2 is the flatter the larger is $|\mathbf{E}u''(c_2)|$, relative to $|u''(c_1)|$. The latter observation is particularly interesting since it means that situations A or B are more likely to be observed with agents who have a greater degree of *temperance* $-u^{(4)}/u'''$.

For the joint decisions, things are far less clear-cut:

Fact 6 • *An increase in pensions P always leads to earlier retirement: $\frac{\partial \lambda}{\partial P} > 0$. It also leads to higher saving ($\frac{\partial s}{\partial P} > 0$) if $v'' = 0$ or if $v'' < 0$, but small in absolute value. Otherwise, the impact of P on saving is unclear.*

- *The effects of a decrease in net wages on retirement and on saving is generally unclear. If relative risk aversion $\mathcal{R}$ is below unity, then a decrease in $(1-\tau)w$ will, under "realistic conditions" (see below), lead to later retirement ($\frac{\partial \lambda}{\partial \tau} > 0$) and, for $v'' = 0$, to an increase in saving ($\frac{\partial s}{\partial \tau} > 0$).*

The **proof** is in the Appendix. The "realistic conditions" alluded to in the second item of Fact 6 are as follows: The individual spends one quarter or more of his life in retirement, and his average saving rate (i.e., saving/income) does not exceed 20%. These conditions imply that $s/(\lambda c_1) < 1$ – which is the precise condition that is required for the second item in Fact 6 to hold (also see (46) in the proof).

5 Choices with Financial Risks

We now consider a different type of risk, namely that saving yields a stochastic (rather than a riskless) return. To keep our analysis simple, we assume that the magnitude of that risk is independent of the length of the period of wealth accumulation. I.e., we assume that after $(1-\lambda)$ periods of saving s the accumulated wealth amounts to $(1-\lambda)(1+\epsilon)s$, where ϵ is a random variable with expecetd value zero, but positive variance. In particular note that ϵ is independent on λ.

5.1 Isolated choices

Consider the isolated decision problems that only deal with one choice variable:

- If the household only has to make a saving decision (while retirement age is fixed), his optimization problem reads

$$\max_s \left\{ (1-\lambda) \cdot u((1-\tau)w - s) + \lambda \cdot \mathbf{E}u\left(\frac{1-\lambda}{\lambda}(1+\epsilon)s + P\right) \right\}. \tag{34}$$

  Denote the solution to (34) by s^a. Similarly, denote by s^b the solution for the saving problem under certainty (i.e., when ϵ and the expected value are removed from (34)).

- If savings are predetermined at some level $\bar{s}$, but nevertheless yield a random return, and the only choice variable of the individual is his retirement age, the decision problem reads

$$\max_\lambda \left\{ (1-\lambda) \cdot u((1-\tau)w - \bar{s}) + \lambda \cdot \mathbf{E}u\left(\frac{1-\lambda}{\lambda}(1+\epsilon)\bar{s} + P\right) \right\}. \tag{35}$$

Denote the solution to this problem by λ^a. Similarly, denote the solution for the retirement problem under certainty (again, remove ϵ and the expected value) by λ^b.

Comparing s^a to s^b and λ^a to λ^b we obtain:

Fact 7
- *If $s^a \geq s^b$, then $\lambda^a < \lambda^b$. The converse is not true.*
- *(Rothschild and Stiglitz, 1971) A sufficient condition for $s^a \geq s^b$ is that relative risk aversion $\mathcal{R}$ is increasing and greater than one. A sufficient condition for $s^a \leq s^b$ is that relative risk aversion is decreasing and smaller than one.*

Proof: Define

$$g(\epsilon) := (1+\epsilon)u'(\cdot) \quad \text{and} \quad h(\epsilon) := u(\cdot) - \frac{(1+\epsilon)s}{\lambda} \cdot u'(\cdot) \tag{36}$$

where the omitted argument in u and u' is always $c_2(\epsilon) = \frac{1-\lambda}{\lambda}(1+\epsilon)s + P$. The FOCs for problem (34) *with* and *without* the return risk then read

$$u'((1-\tau)w - s) = \mathbf{E}g(\epsilon) \quad \text{and} \quad u'((1-\tau)w - s) = g(0),$$

respectively (recall that $\mathbf{E}\epsilon = 0$). Similarly, we get the FOCs for the risky and the riskless version of (35) as

$$u((1-\tau)w - s) = v'(\lambda) + \mathbf{E}h(\epsilon) \quad \text{and} \quad u((1-\tau)w - s) = v'(\lambda) + h(0),$$

respectively. By Jensen's Inequality we then have that (i) $s^a \geq [\leq] s^b$ whenever $g(\epsilon)$ is convex [concave] and (ii) $\lambda^a \geq [\leq] \lambda^b$ whenever $h(\epsilon)$ is convex [concave]. Calculate:

$$\begin{aligned} g''(\epsilon) &= \frac{1-\lambda}{\lambda} s \left[2u'' + (1+\epsilon)\frac{1-\lambda}{\lambda} s u''' \right], \\ h''(\epsilon) &= -\frac{1-\lambda}{\lambda^2} s^2 \left[(1+\lambda)u'' + (1+\epsilon)\frac{1-\lambda}{\lambda} s u''' \right]. \end{aligned}$$

where $(1+\epsilon)\frac{1-\lambda}{\lambda}s \leq c_2$ and $\lambda < 1$. Hence, convexity of g implies strict concavity of h and, similarly, convexity of h implies strict concavity of g. This proves the first item of the assertion. To see the second item verify that

$$\mathcal{R}' \gtreqless 0 \quad \Longleftrightarrow \quad u''(1+\mathcal{R}) + cu''' \lesseqgtr 0. \tag{37}$$

Use this to evaluate the sign of g'' under the various assumptions. ■

Fact 7 re-states the well-known observation that it is unclear whether risk-averse individuals will extend their risky activities if the risk associated to the return of that activity increases: Any reaction is possible here (Rothschild and Stiglitz, 1971). Fact 7 then further states that if a more risky return on saving induces the individual to *ceteris paribus* increase his saving, then it will also *ceteris paribus* retire earlier. While Fact 7 itself states (and correctly states) that the converse does not hold, a glance at the proof of Fact 7 shows that the converse is indeed quite likely to hold.[7] This correspondence does not come as a surprise: The return risk ϵ hits saving and retirement choices in a roughly identical manner. Hence, if it induces the individual to reduce its risky activity (namely, saving) in problem (34), then one could expect it to induce a reduction of the risky activity (namely, [early] retirement) in problem (35) as well.

5.2 Combined Choices

The FOCs for the joint saving/retirement problem

$$\max_{s,\lambda} Z(s,\lambda) := \left\{(1-\lambda)\cdot u((1-\tau)w-s)+\lambda\mathbf{E}u\left(\frac{1-\lambda}{\lambda}(1+\epsilon)s+P\right)+v(\lambda)\right\} \tag{38}$$

are

$$\frac{\partial Z}{\partial s} = -u'(c_1)+\mathbf{E}\left((1+\epsilon)u'(c_2)\right)=0 \tag{39a}$$

$$\frac{\partial Z}{\partial \lambda} = \mathbf{E}u(c_2)-u(c_1)-\frac{s}{\lambda}\cdot\mathbf{E}\left((1+\epsilon)u'(c_2)\right)+v'(\lambda)=0. \tag{39b}$$

Recall that we denoted the optimal decisions in the joint saving/retirement-problem under certainty (i.e., the solution to (6a) and (6b)) by (s_0,λ_0). Denote the solution to (39a) and (39b) by (s_2,λ_2). We now wish to compare (s_0,λ_0) and (s_2,λ_2). Given that the comparative statics of the isolated saving or retirement decisions are replete with ambiguities when return risks are added, this is quite a hopeless task. We nevertheless obtain

Fact 8 *Suppose that the function g (defined in (36)) is convex. Then $s_2<s_0$ is possible as well as $\lambda_2>\lambda_0$, but never both.*

Proof: We will evaluate the partial derivatives of Z at (s_0,λ_0). Subscripts 0 to variables indicate the values of these variables associated with (s_0,λ_0). In particular, $c_{1,0}=(1-\tau)w-s_0$ and $c_{2,0}(\epsilon)=\frac{1-\lambda_0}{\lambda_0}(1+\epsilon)s_0+P$. First calculate:

$$\frac{\partial Z(s_0,\lambda_0)}{\partial s} = -u'(c_{1,0})+\mathbf{E}\left((1+\epsilon)u'(c_{2,0}(\epsilon))\right) > -u'(c_{1,0})+u'(\mathbf{E}c_{2,0}(\epsilon))=0.$$

The inequality stems from the convexity of g. The final equality is by definition.

$$\begin{aligned}\frac{\partial Z(s_0,\lambda_0)}{\partial \lambda} &= \mathbf{E}u(c_{2,0}(\epsilon))-u(c_{1,0})-\frac{s_0}{\lambda_0}\cdot\mathbf{E}\left((1+\epsilon)u'(c_{2,0}(\epsilon))\right)+v'(\lambda_0)\\ &< -\frac{s_0}{\lambda_0}\cdot\mathbf{E}\left((1+\epsilon)u'(c_{2,0}(\epsilon))\right)+v'(\lambda_0)<\frac{s_0}{\lambda_0}\cdot u'(\mathbf{E}c_{2,0}(\epsilon))+v'(\lambda_0)=0.\end{aligned}$$

[7] Look at g'' and h'' in that proof: The difference that prevents the signs of g'' and h'' to be always opposite is between $2u''$ and $(1+\lambda)u''$.

The first inequality is by risk aversion: $\mathbf{E}u(c_{2,0}(\epsilon)) < u(\mathbf{E}c_{2,0}(\epsilon)) = u(c_{1,0})$. The second inequality is due to the convexity of g. The final inequality is by definition.
As $Z_{s\lambda}(s, \lambda) > 0$ these findings are compatible with all relationships between (s_0, λ_0) and (s_2, λ_2) expect for the one excluded in the assertion. ■

Fact 8 should be contrasted with Fact 7. There the condition that g is convex leads to $s^a \geq s^b$ and $\lambda^a < \lambda^b$. I.e., increases in the risk to saving boost saving and postpone retirement if either of these decisions is taken isolatedly. In the joint decision problem, convexity of g does not exclude that increases in financial risks depress saving or induce postponed retirement.

6 Pension Risk as Background Risk

Now consider the following scenario: There is a pre-existing return risk on saving; thus, saving *and* retirement are risky choices. Due to some political decision now the pension risk is increased (or, as we shall discuss below, newly introduced). We assume that the pension risk and the return risk are independently distributed. The question we wish to answer here is: interested is: How will this additional risk affect the risky activities of saving and retirement?
Technically, this is the question for the optimal response towards a background risk which has recently found much attention in the literature on decision making under uncertainty. Gollier and Pratt (1996) introduced the concept of *risk vulnerability* to capture the idea that the presence of an exogenous background risk (here: the pension risk) with zero mean raises the aversion against any other, independent risk (here: the return risk on saving).[8] Unfortunately, it is much easier to verbally describe risk vulnerability than to formally characterize it in terms of properties of the underlying risk preferences. In what follows, it will be helpful to define a *derived* utility function: Suppose pensions have a risky component, $\tilde{b}$ with $\mathbf{E}\tilde{b} = 0$. Then we write

$$\nu(c_2) = \mathbf{E}_{\tilde{b}} u(c_2 + \tilde{b}). \tag{40}$$

The subscript indicates that the expectation is formed for the distribution of $\tilde{b}$. Reactions of optimal choices when risks change crucially depend on the properties of the derived utility function ν – which in turn are determined (in a non-trivial way) by the properties of the original utility index u.
As above, our analysis will proceed as follows: We first discuss the effects of background pension risks on retirement and saving decisions under the premise that either decision is made independently of the other ("isolated choices"). We then discuss what happens if both choices have to be made simultaneously ("joint decisions").

[8]The theory has recently been extended to also cover dependent risks. See, e.g., Eeckhoudt et al. (1996).

6.1 Isolated Choices

Consider the isolated decision problems that only deal with one choice variable:

- If the household only has to make a saving decision (while retirement age is fixed), his optimization problem reads

$$\max_s \left\{ (1-\lambda) \cdot u((1-\tau)w - s) + \lambda \cdot \mathbf{E}_\epsilon \nu \left(\frac{1-\lambda}{\lambda}(1+\epsilon)s + P \right) \right\}. \tag{41}$$

  Note that $\mathbf{E}_\epsilon \nu(\cdot) = \mathbf{E}_\epsilon \mathbf{E}_{\tilde{b}} u(\cdot)$ is a double integral that captures both the return risk ϵ and the pension risk $\tilde{b}$. Denote the solution to (41) by s^c. Recall that we already defined the saving problem without any background risk in (34) and that we labelled its solution s^a.

- If savings are predetermined at some level $\bar{s}$ and the single choice variable of the individual is his retirement age, the optimal decision in the presence of a pension risk in the background is given by

$$\lambda^c := \arg\max_\lambda \left\{ (1-\lambda) u(w(1-\tau) - \bar{s}) + \lambda \mathbf{E}_\epsilon \nu \left(\frac{1-\lambda}{\lambda}(1+\epsilon)\bar{s} + P \right) + v(\lambda) \right\}.$$

  while in the absence of such a risk it is λ^a which we already defined in (35).

Comparing s^a to s^c and λ^a to λ^c we obtain:

Fact 9 *(i) If ν is more risk averse than u, $s^c < s^a$. (ii) It is generally not possible to rank λ^c and λ^a.*

Proof: (i) Comparing the FOCs that characterize s^a and s^c one readily sees that $s^c < s^a$ if and only if[9]

$$\mathbf{E}_\epsilon \left((1+\epsilon) u' \left(\frac{1-\lambda}{\lambda}(1+\epsilon)s^c + P \right) \right) > \mathbf{E}_\epsilon \left((1+\epsilon) \nu' \left(\frac{1-\lambda}{\lambda}(1+\epsilon)s^c + P \right) \right). \tag{42}$$

It is well known from Pratt (1964) that condition (42) holds for arbitrary values of λ and P and for arbitrary distributions of ϵ if and only if ν is more risk averse than u, i.e., if $-\nu''(c)/\nu'(c) > -u''(c)/u'(c)$ for all c.

(ii) Comparing the FOCs that characterize λ^a and λ^b one sees that $\lambda^c > \lambda^a$ if and only if

$$\begin{aligned} &\mathbf{E}_\epsilon u \left(\frac{1-\lambda^c}{\lambda^c}(1+\epsilon)\bar{s} + P \right) - \mathbf{E}_\epsilon \nu \left(\frac{1-\lambda^c}{\lambda^c}(1+\epsilon)\bar{s} + P \right) \\ &> \frac{\bar{s}}{\lambda^c} \cdot \left[\mathbf{E}_\epsilon (1+\epsilon) \cdot u' \left(\frac{1-\lambda^c}{\lambda^c}(1+\epsilon)\bar{s} + P \right) - \mathbf{E}_\epsilon (1+\epsilon) \cdot \nu' \left(\frac{1-\lambda^c}{\lambda^c}(1+\epsilon)\bar{s} + P \right) \right]. \end{aligned} \tag{43}$$

[9] This tacitly, but innocuously assumes that the second-order conditions of all optimization problems are satisfied.

Note that $\mathbf{E}_\epsilon \nu = \mathbf{E}_\epsilon \mathbf{E}_{\tilde{b}} u(\cdot + \tilde{b}) < \mathbf{E}_\epsilon u(\cdot + \mathbf{E}\tilde{b}) = \mathbf{E}_\epsilon u(\cdot)$ due to risk aversion. Hence, the LHS of (43) is positive. If ν is more risk averse than u and thus (42) holds, the RHS is positive, too. Hence, the overall effect on λ is unclear. Only if (42) did not hold would we obtain $\lambda^c > \lambda^a$. ■

Under the assumption that ν is more risk averse than u, a higher pension risk in the background will *ceteris paribus* depress saving and lead to earlier retirement. The effect on saving is the (by now fairly) standard tempering effect of background risks: Additional uncertainty about pensions makes saving less desirable. The same risk-vulnerability argument would speak in favour of an *earlier* retirement: The background risk decreases marginal utility of old-age consumption (this is the essence of (42)). Hence, from that point of view, a lower per-period consumption in old age is desirable – which can be achieved by prolonging the period of retirement. However, there is an offsetting effect due to risk aversion, of course: The additional pension risk makes life in retirement less attractive, demanding for later retirement. The overall effect thus remains ambiguous.

6.2 Joint Decisions

If saving and retirement decisions are jointly made, the optimization problem is identical to (38) except that $\mathbf{E}u''(\cdot)$ has to be replaced by $\mathbf{E}_\epsilon \nu(\cdot) \equiv \mathbf{E}_\epsilon \mathbf{E}_{\tilde{b}} u(\cdot)$. Analogously, the FOCs are identical to (39a) and (39b) where $\nu(c_2)$ and its derivatives are exchanged for $u(c_2)$ and derivatives. Call the optimal solution to this problem (s_3, λ_3).

At first glance, the formal structure of comparing (s_3, λ_3) and (s_1, λ_1) is very similar to the comparison between (s_1, λ_1) and (s_0, λ_0) which we dealt with in Fact 5. However, there are some complicating factors due to the double uncertainty we introduced. This requires further research.

7 Application and Conclusion

From our analysis we draw at once one obvious lesson: Clearcut results on the impact of changes in pension policies for single decision variables loose their validity if several decisions are analysed jointly. Let us briefly apply this result to the current German pension reform. Apart from newly introducing tax incentives for private (funded) old-age provisions, this reform entails two distinctive features:

- There will be a cut in future pensions. This cut is not made explicit in the reform law, but hidden in changes of the formulae that are used to calculate pensions.
- "Stability of contributions" should be the primary goal for the future development of the German PAYG scheme. This is indeed novel since in the past the guideline of pension

politics was to stabilize the benefits of the scheme.

Several authors interpret the second item as a switch from a fixed-replacement (or defined-benefit) policy to a defined contribution policy within the PAYG scheme (see, e.g., Schmähl, 2000). Such a switch unambiguously makes pensions riskier from an *ex ante* perspective as it forces pensioners-to-be to participate in future economic ups and downs, which they would be protected against under a defined-benefit regime.

If one focusses on the univariate choice problems (i.e., the isolated saving and retirement decisions) the German pension reform can be expected to show the following effects: The cut in benefits leads to higher saving and to later retirement. The switch towards a defined-contribution scheme also induces higher (precautionary) saving and postponed retirement (Fact 4). In so far, the German reform seems to be an unambiguously promising measure with respect to both increasing capital formation and the participation of older workers in the labour force.

If one accounts for the interdependencies of saving and retirement choices the effects of the German pension reform may get somehow more ambiguous. It may well happen that savings are *reduced* as a response to the pension cut (Facts 2 and 6). The response to the increase in the pension risk may be even more striking: It is well conceivable that individuals will retire earlier (Facts 5). Yet, for those looking for a piece of "good news", it will probably not happen that the German pension reform is entirely counterproductive in that it decreases both saving and retirement age.

These tentative conclusions suggest that there is substantial scope for additional research on saving and retirement (see van der Klaauw and Wolpin (2001) for the same view). Several of our findings can probably be further strengthened in a formal sense. Given the importance of the topic, such research certainly ought to be undertaken.

Appendix

Derivation of (5)

Here we demonstrate that the maximization of (1) subject to (3) can indeed be equivalently represented by maximization of (5). Given that the interest rate is zero and denoting capital income in period t by cap_t, the lifetime budget constraint of the individual is

$$\sum_{t=1}^{R}(w(1-\tau)-s_t)=\sum_{t=R+1}^{D}(P+cap_t). \tag{44}$$

Given that preferences for consumption are equal during working age and retirement, the individual will wish to perfectly smooth consumption across dates. As wages are constant, we thus get $s_t = s$ for all $t \leq R$. The amount of wealth accumulated at the age of retirement is therefore $R \cdot s$. With constant pensions during retirement, this will be spent in equal shares during retirement. I.e., capital income during retirement amounts to $s \cdot R/(D-R)$ per year. Hence, the optimization problem boils down to

$$\max_{s,R} \left\{ R \cdot u((1-\tau)w - s) + (D-R) \cdot u\left(\frac{R}{D-R} \cdot s + P\right) + \tilde{v}(D-R) \right\}$$

Using $\lambda = (D-R)/D$ this is equivalent to

$$\max_{s,\lambda} D \left\{ (1-\lambda) \cdot u((1-\tau)w - s) + \lambda \cdot u\left(\frac{1-\lambda}{\lambda} \cdot s + P\right) + D^{-1}\tilde{v}(D\lambda) \right\},$$

which is tantamount to (5). ■

Proof of Fact 5

Total differentiation of (29a) and (29b) yields a linear system

$$\begin{pmatrix} \mathrm{d}\lambda \\ \mathrm{d}s \end{pmatrix} = \mathbf{A}^{-1} \cdot \mathbf{B} \cdot \begin{pmatrix} \mathrm{d}\tau \\ \mathrm{d}P \end{pmatrix}$$

where

$$\mathbf{A} = \begin{pmatrix} \frac{s}{\lambda^2}\mathbf{E}u'' & -u'' - \frac{1-\lambda}{\lambda}\mathbf{E}u'' \\ -\frac{s^2}{\lambda^3}\mathbf{E}u'' - v'' & \frac{s(1-\lambda)}{\lambda^2}\mathbf{E}u'' \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} wu'' & \mathbf{E}u'' \\ wu' & \mathbf{E}u' - \frac{s}{\lambda}\mathbf{E}u'' \end{pmatrix}.$$

Here, the arguments of u, $\mathbf{E}u$, v and their derivatives are, respectively, $c_{1,1}$, $c_{2,1}$, and λ_1. Verify that

$$\det \mathbf{A} = -\left[\frac{s^2}{\lambda^3}u''\mathbf{E}u'' + u''v'' + \frac{1-\lambda}{\lambda}v''\mathbf{E}u''\right] < 0.$$

Solving the linear system gives:

$$\frac{\partial \lambda}{\partial \tau} = \det{}^{-1}\mathbf{A} \cdot \left[\frac{1-\lambda}{\lambda}w\mathbf{E}u''\left(\frac{s}{\lambda}u'' + u'\right) + wv'u''\right] \tag{45a}$$

$$\frac{\partial \lambda}{\partial P} = \det{}^{-1}\mathbf{A} \cdot \left[u''\mathbf{E}u' - \frac{s}{\lambda}u''\mathbf{E}u'' + \frac{1-\lambda}{\lambda}\mathbf{E}u''\mathbf{E}u'\right] \tag{45b}$$

$$\frac{\partial s}{\partial \tau} = \det{}^{-1}\mathbf{A} \cdot \left[\frac{ws}{\lambda^2}\mathbf{E}u''\left(\frac{s}{\lambda}u'' + u'\right) + wv''u''\right] \tag{45c}$$

$$\frac{\partial s}{\partial P} = \det{}^{-1}\mathbf{A} \cdot \mathbf{E}u'' \cdot \left[v'' + \frac{s}{\lambda^2}\mathbf{E}u'\right]. \tag{45d}$$

Only (45b) can be unambiguously signed; it is positive. For $v'' = 0$, (45d) is positive, too. The same holds if $|v''|$ is small.

Next consider the term $(\frac{s}{\lambda}u'' + u')$ which appears in (45a) and (45c). It is equal in sign to $1 - \mathcal{R}s/(\lambda c_1)$, the magnitude of which is generally unclear, though. Check that

$$s/(\lambda c_1) \underset{>}{\overset{<}{=}} 1 \iff s \underset{>}{\overset{<}{=}} \frac{\lambda}{1+\lambda} \cdot (1-\tau)w. \tag{46}$$

For realistic values of retirement behaviour we get $\lambda \geq 0.25$ or $\lambda/(1+\lambda) \geq 0.2$; this should be a conservative estimate. Since we observe saving rates well below 20% in reality, one can, thus, reasonably assume that $s/(\lambda c_1) \leq 1$. Together with $\mathcal{R} \leq 1$ we thus get $1 - \mathcal{R}s/(\lambda c_1) > 1$. This implies that (45a) is positive. For $v'' = 0$, (45c) would be positive, too. The same holds if $|v''|$ is small. ■

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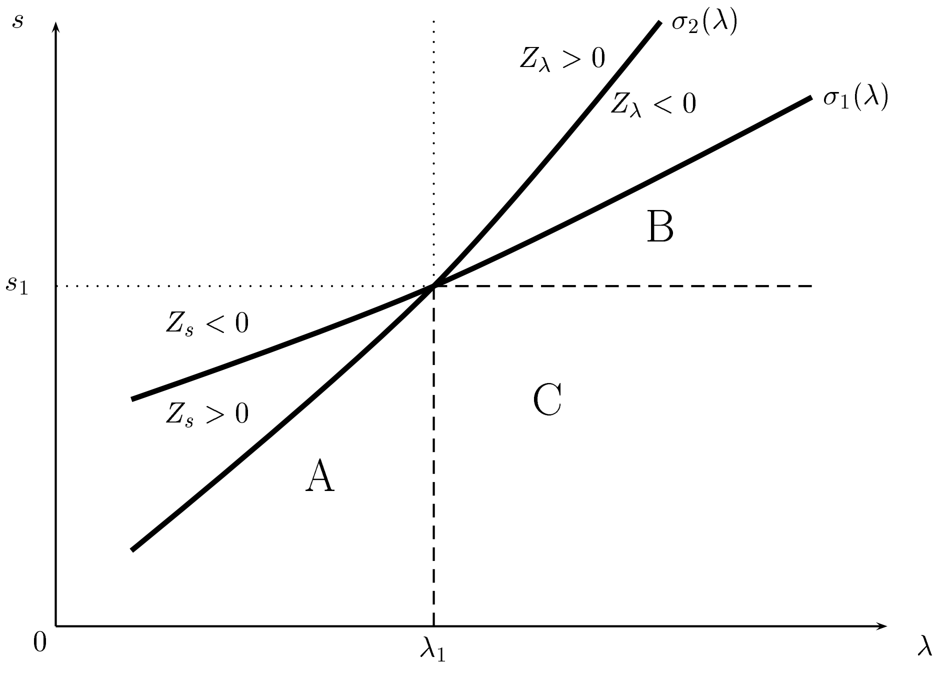

Figure 1: Illustration of Fact 5