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Productive public expenditure in a new economic geography model

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Abstract. This paper elaborates on Baldwin's (1999) New Economic Geography model allowing for capital accumulation and capital mobility between a "rich" and a "poor" region. A central government decides upon the level and the regional and sectoral allocation of productivity enhancing public investments. We derive results on how such policies affect the overall private capital stock and its regional allocation under alternative financing schemes. We show that the regional and sectoral distribution of public capital matters in determining the final impact of an increase in public capital on the level of private capital. Furthermore, we find that increasing public capital in the "poor" region does not always increase the share of manufacturing in that region as the final result depends on the relative strength of two effects which have been studied separately in the literature so far: the "productivity" and the "demand" effects. Finally, we show that in order to be effective regional policy must not confine itself to the expenditure side but has to take into account the financing side at the same time.

JEL Classification: F20; H5; R12.
Keywords: New Economic Geography; Public Expenditure; Footloose Capital.

Résumé. Cet article part du modèle de nouvelle économie géographique développé par Baldwin (1999) qui combine accumulation et mobilité du capital entre une région riche et une région pauvre. Un gouvernement central décide du niveau et de l'allocation régionale et sectorielle de la productivité en augmentant les investissements publics. Comment de telles politiques affectent le stock total de capital privé et son allocation régionale ? En retenant différents schémas financiers, nous montrons que la répartition régionale et sectorielle du capital public n'est pas neutre quant à l'impact final d'une hausse du capital public sur le niveau du capital privé. Bien plus, accroître le capital public dans une région « pauvre » n'y entraîne pas toujours une hausse de la part de l'industrie : le résultat dépend du poids relatif de deux effets que la littérature a jusqu'à présent étudiés séparément, l'effet « productivité » et l'effet « demande ». En définitive, pour être efficace, la politique régionale ne doit pas se cantonner à gérer la dépense mais, en même temps, tenir compte du mode de financement.

Classification *JEL* : F20 ; H5 ; R12.
Mots-clefs : Nouvelle économie géographique ; dépense publique ; mobilité du capital.

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1. Introduction

The European Union is characterized by quite substantial regional inequalities – deepened with the recent enlargement. In order to reduce those disparities, European regional policy heavily relies on public investment, viewed as "contributing directly to economic growth and strengthening the productive potential of the economy" (European Commission, 2007, p. 135). However, in a comprehensive survey on empirical studies, Romp and de Haan (2007) summarize that the growth effect of public capital is not always positive, that it appears to be rather small and that it differs across regions (see Romp and de Haan, 2007, p. 33). In addition, the authors note that only few studies are based on solid theoretical models, and that especially the channels through which public capital should enhance growth are not well understood.

Our paper intends to contribute to closing this gap by presenting a variant of the New Economic Geography (NEG) model developed by Baldwin (1999). Our analysis involves a "rich" northern region and a "poor" southern region and one central government that decides upon the level of (productivity enhancing) public investment and its regional and sectoral distribution and at the same time upon the financing scheme. The model allows for private capital accumulation and for capital mobility across the regions. The framework, therefore, seems to be particularly suited because it allows to study in a unified framework how the provision and the financing of (productivity enhancing) public investments affect the overall private capital stock and at the same time its regional distribution. We are particularly interested in the role of different financing schemes (i.e. whether or not the regional tax burden corresponds to the expenditures for regional public goods); therefore we assume one central government (instead of two regional ones) and deliberately abstract from issues of tax competition or competition in public inputs.

We show that an increase in (productivity enhancing) public capital does not always increase the overall private capital. The regional and sectoral distribution of the public investments matters. The "productivity" effect tends to increase private capital, while a "crowding-out" effect between public and private investments works in the opposite direction.

Turning to the regional consequences, we show that an increase in (productivity enhancing) public capital in the "poor" region does not always increase the share of manufacturing in that region. The "productivity" effect, which works through productivity of labour in the local manufacturing sector, tends to increase the share of manufacturing, whereas a "demand" effect – decisively depending on the tax scheme – influences location *via* the relative size of local market and can work in the opposite direction. Only if the "rich" region also contributes to the financing of the public capital in the "poor" region, public investments unambiguously increase the southern share of manufacturing. The paper's most important contribution consists in accounting for the interaction of the so-called "demand" and "productivity" effects of government expenditure on agglomeration, which have been studied separately in the literature so far. In addition, we show that regional policy – in order to be effective – must not confine itself to the expenditure side but has to take into account the financing side at the same time.

Our results, therefore, provide a theoretical basis for the mixed empirical evidence and reveal a possible trade-off between regional equity and overall efficiency.

The remainder of the paper is organized as follows. Section 2 reviews the scarce literature on public expenditure and NEG. Section 3 presents the basic framework of the model. In section 4 we characterize the short-run equilibrium. Section 5 is devoted to the long-run equilibrium and to the study of the impact of public expenditure on industrial location. Section 6 analyses the welfare impact of public policy and section 7 concludes.

2. Related literature

NEG scholars have studied the impact of public expenditure on industrial location within the standard Core Periphery (CP) model (Trionfetti, 1997), as well as three variants of the original Krugman (1991) framework: the Vertical-Linkages (VL) version of the CP model (Trionfetti, 2001), the Footloose Capital (FC) model (Martin and Rogers, 1995; and Martin, 1999) and the Footloose Entrepreneur (FE) model (Brakman *et al.*, 2008).[2]

The standard result of the CP model is that a sufficient reduction in trade barriers will destabilize the symmetric equilibrium and will result in complete agglomeration of the industrial activity in one region. Trionfetti (1997) adds public expenditures to a standard CP model providing an additional channel to the "market-access" effect, leading to a stable equilibrium with partial agglomeration. The government is assumed to spend tax revenues on consumption goods which are destroyed after the purchase.[3] Intuitively, additional public demand for manufactured goods increases local demand. This creates a new demand-linked effect in the model which can, under certain conditions, dominate all the others, leading to stable equilibria with no catastrophic agglomeration.

In the VL version of the CP model proposed by Venables (1996) and Krugman and Venables (1995), it is shown that self-reinforcing agglomeration can be driven by input-output relationships between industrial firms, even in the absence of inter-regional labour migration. Trionfetti (2001) accommodates for government procurement, by extending the demand side of the model by Krugman and Venables (1995). The author assumes that the differentiated good produced by the manufacturing sector is used as an intermediate input by the manufacturing sector itself as well as by national governments for the provision of public services. In the presence of home-biased public procurement, an exogenous increase in the number of firms in one country will not result in the complete industrial specialization of that same country. Indeed, public expenditure could counter agglomeration and contribute to the stability of the initial equilibrium. In addition to Trionfetti (1997), it is also shown that home-biased public procurement may be welfare improving. The stabilization results discovered in Trionfetti (1997 and 2001) find empirical confirmation in Brülhart and Trionfetti (2004).

Martin and Rogers (1995) develop a FC model in which public policy plays a role in facilitating trade both within and across regions. The FC model departs from the CP model for two assumptions which

2. There exists also a related strand of literature on tax competition and public input competition in models with agglomeration forces – see e.g. Andersson and Forslid (2003), Baldwin and Krugman (2004), Bucovetsky (2005), Borck and Pflüger (2006), Commendatore and Kubin (2006), Commendatore, Currie and Kubin (2008) and Ihara (2008) – which, as noted above, we do not pursue here.

3. This assumption is made in order to isolate the "demand" effect of public expenditure, at the cost of neglecting further effects which could arise from alternative uses of public resources.

rule out the CP outcome of catastrophic agglomeration: a fixed capital requirement for each variety of the differentiated good and international immobility of workers.[4] However, agglomeration still occurs due to the working of the "market-access" effect. Compared to the CP model, the FC model has the advantage of obtaining closed form solutions for the spatial distribution of industry, while it does not feature the circular causality that is so much of the source of the CP model's richness and intractability (Baldwin *et al.*, 2003, p. 68).

An improvement in domestic (international) infrastructures is modeled by Martin and Rogers (1995) as a reduction in transaction costs within (across) regions.[5] Public infrastructure and services affect industrial location through demand, while neither domestic nor international public infrastructures are assumed to affect the production function, that is, any impact on factors' productivity is neglected. Better domestic infrastructures imply higher demand and lower prices for industrial goods produced in the region, thus attracting mobile capital. When regions are integrated, differentials in terms of domestic infrastructures drive the relocation of capital, with firms tending to locate in regions with better infrastructures. On the other hand, improvements in international infrastructures lead to less concentration in poorer regions.

Brakman *et al.* (2008) introduce the government sector in the FE model developed by Forslid (1999), Ottaviano (2001) and Forslid and Ottaviano (2003), assessing the effectiveness of public spending as an instrument for locational competition across regions. The crucial assumption in the FE model is that producing a new manufactured variety requires one unit of mobile human capital (one entrepreneur). Contrary to the FC model, the mobile factor and its owner move together. This implies demand linked-circular causality. Furthermore, the attractiveness of one region as compared to the other is judged on the basis of real rewards paid to the mobile factor, thus leading to cost-linked circular causality. In the model by Brakman *et al.* (2008), the total amount of capital in each region is defined by the sum of "public" and "market" capital, with the government assumed to produce public goods by means of the former, while manufacturers use the latter. Public goods – financed by a uniform income tax rate – are productive in that they reduce both fixed and variable costs of manufacturing firms. As a main conclusion, it is shown that local governments can perturbate the equilibrium between agglomeration and dispersion forces, as the introduction of public goods fosters agglomeration. Since the attractiveness of locations is influenced by their endowments of public goods, depending on the share of total regional capital used to produce public goods, a periphery can turn into a core and *vice versa*. In Brakman *et al.* (2008), however, the provision of public goods has impact neither on the size nor on the composition of the expenditure in manufactured goods, so that public policy does not affect industrial location through the demand channel.

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4. The mobile factor (capital) earnings are repatriated and spent where the capital owner resides. Therefore, the typical CP feature of demand-linked circular causality – production changes brought about by factor movements yield expenditure switching that in turn generate further production changes – does not occur. Furthermore, the cost-linked circular causality of the CP model – shifts in production alter prices inducing workers migration with further production shifting – is eliminated.

5. Public infrastructures are viewed as any public "good, facility or institution which facilitate the juncture between production and consumption" (Martin and Roger, 1995, p. 336), affecting the actual amount of output that reaches the consumer. The authors distinguish infrastructures that can facilitate domestic trade – such as law and contract enforcement of telecommunication networks, public administration and, in general, intra-regional transport infrastructure – and those that facilitate trade among regions, such as the international communication and transportation systems.

3. The basic framework

We base our analysis on the Footloose *cum* Constructed Capital (CC) model developed by Baldwin (1999).[6] In a FC model, capital is mobile across regions, while their owners are immobile and capital earnings are repatriated. In the CC variant of the FC model, total capital stock is not constant as the construction of new capital goods and depreciation are explicitly allowed for. We further extend this model bringing in a government sector that – aiming to reduce regional inequalities – provides public goods which enhance local factors productivity. A crucial question is how these goods are financed. We study two simple cases. Considering a two-region economy, the central government finances an additional provision of public goods in the backward region by taxing residents of that region; alternatively, the residents of the other region also contribute to the financing of the public goods provision.[7]

3.1. The economy

The economy is composed of two trading regions – South (S) and North (N) – and four sectors: the agricultural sector (A), the consumer goods manufacturing sector (M), the investment goods sector (I) and the government sector (G). There are two factors of production, capital (K) and labour (L). K is mobile across regions, whereas L is inter-regionally immobile but freely mobile across sectors.[8]

3.2. Agriculture

The market for the homogeneous agricultural good is perfectly competitive. The technology available to each agricultural firm is such that one unit of L yields one unit of output. Under the assumption of perfect competition, the price of the agricultural good is equal to the wage rate in agriculture, $p_A = w_A$. Since neither region has enough workers to satisfy the total demand of both regions for the agricultural product, both regions always engage in agricultural production – the so-called "non-full specialization condition" (Baldwin *et al.*, 2003, p. 72).

3.3. Manufacturing sector

Manufacturing involves Dixit-Stiglitz monopolistic competition. The assumption of increasing returns requires two factors of production. Each manufacturer requires a fixed input of one unit of capital to operate and has a constant labour requirement a_M for each unit of the product. Total cost of producing x_i units of a specific variety i is:

$$TC(x_i) = F + w_M a_M x_i \tag{1}$$

6. In our exposition we use the reduced form as found in Baldwin et *al.*, 2003, Ch. 6; it is better accessible intuitively without loosing the compatibility with the fully fledged intertemporal structure that is made explicit in Baldwin (1999).

7. Our analysis, taking into account the perspective of a central government aiming to resolve regional disparities, does not contemplate the possibility of local governments competition. Such an analysis has been recently presented in Ihara (2008) who considers the case of two symmetric regions whose governments compete over the provision of productive public goods in order to attract capital from the other region. Note that Ihara (2008) does not allow for regionally differentiated financing schemes.

8. In this section, we do not specify the region we are referring to when the same description holds for both regions.

where F corresponds to the fixed cost necessary to activate the production of i (that is, F is the cost of a unit of capital), w_M is the nominal wage rate in manufacturing and x_i is the total output of industry i.

Given consumers' preference for variety and increasing returns, a firm will always produce a variety different from those produced by other firms. Furthermore, since one unit of capital is required for each manufacturing firm, the total number of firms / varieties, denoted by n, always equates total private capital stock:

$$n = K_P \tag{2}$$

If λ is the share of private capital located and used in region S, the number of varieties produced in region r ($r = S, N$) is defined by:

$$n_S = \lambda n = \lambda K_P \quad \text{and} \quad n_N = (1 - \lambda)n = (1 - \lambda)K_P \tag{3}$$

3.4. Investment sector

In the investment sector perfect competition prevails. The technology available to produce investment goods requires only the use of labour. The construction of one unit of capital requires a_I units of L. Therefore, if L_I units of labour are employed in the sector, the supply of *new* capital goods corresponds to $\frac{L_I}{a_I}$. Following Baldwin (1999) we assume that capital depreciates according to a "Blanchard" process: i.e., each unit of capital faces a constant probability δ of "dying" at every instant of time; with a large number of firms (capital units), δ also denotes the depreciation rate of aggregate capital.

Finally, under the assumption of perfect competition, the cost of one unit of capital is given by $F = a_I w_I$.

3.5. Government

Public policy can affect production in the manufacturing sector, *via* its impact on factor productivity. The government uses tax revenues to purchase capital goods required for the production of freely available public goods. The government technology involves a requirement of one unit of capital for each unit of public goods;[9] the corresponding production function is:

$$H = K_G \tag{4}$$

where H represents the provision of public goods and K_G the public capital stock.

This way of modelling public policy adds two forces potentially driving industrial agglomeration. The government competes with manufacturers on the demand side of the capital market: the presence of the government reduces the amount of capital available to private agents. As for the use that the government does of purchased goods, we assume that they positively impact on workers productivity.

In particular, we think of public expenditures for research, innovation and education. Such freely available public goods potentially impact upon all sectors. However, the possibilities of transforming the public goods into specific productivity gains will differ among the sectors, R&D intensive sectors

9. We could assume that production of public goods requires also a labour input but we leave the study of the consequences of this complication for future work.

being more capable of benefiting from them. We therefore assume that labour productivity in the agricultural sector – where we expect the R&D intensity to be comparatively low – is left unchanged by the provision of public goods and that it increases in the investment goods and manufacturing sectors.[10] Since the focus of our paper is on regional policy, we deliberately assume *local* public goods, i.e. we assume that the provision of a public good in one region *does not* affect productivity of workers in the other region.

More formally, we assume that the provision of public goods affects respectively labour input requirements a_I and a_M according to the relationships:

$$a_I = f(H) \quad \text{and} \quad a_M = g(H) \tag{5}$$

both being monotonically decreasing, positive and definite in the relevant range $H \geq 0$. Very simple functions which satisfy these properties are:

$$f(H) = A(1+BH)^{-1} \quad \text{and} \quad g(H) = C(1+DH)^{-1} \tag{6}$$

where A, B, C and D are positive constants.

3.6. Households

There is, in total, a fixed number L of identical – infinitely lived – households. Each household owns capital and supplies a unit of labour. Henceforth we denote s_L the (fixed) share of households located in the South; s_K the (fixed) share of capital owned by capitalists living in the South; and s_I the share of investment undertaken by capital owners in the South. From the assumption of invariant shares of capital owned by households in the South, it follows $s_I = s_K$.[11] In order to simplify the analysis, from now on we also assume $s_L = s_K$, that is, capital is uniformly distributed among the L households. Households in both regions have the following instantaneous utility function:[12]

$$U = C_A^{1-\mu} C_M^{\mu} \tag{7}$$

From (7), the utility of the representative household depends in the usual form on quantity consumed of the homogeneous agricultural good, C_A, and on a quantity index C_M that is a CES function of the varieties of manufactured goods:

$$C_M = \sum_{i=1}^{n} c_i^{\frac{\sigma-1}{\sigma}} \tag{8}$$

10. Thus, as far as the manufacturing sector is concerned, it is possible to identify a direct favourable effect on variable input costs (*via* a_M) and an indirect favourable effect on fixed costs (*via* a_I).

11. The assumption of fixed shares of capital ownership between regions is typical of the FC model. It reduces notably the complexity of the analysis. In the absence of capital mobility, the share of capital owned by southern residents, s_K, is necessarily equal to the capital located and used in that region, that is, $s_K = \lambda$. It follows that a change in the share of capital located in a region impacts on residents income and therefore on the size of the market, activating the demand-linked cumulative process. Moreover, since s_K is allowed to change through time, the investment shares can differ from the capital shares. For an extensive treatment of a CC model with immobile capital, see Baldwin (1999).

12. Note that the nature of the productive public goods is such that they do not have any direct effect on households' utility.

where c_i is consumption of variety i, n is the total number of varieties produced in the two regions and $\sigma > 1$ denotes the constant elasticity of substitution between the manufactured varieties; the lower σ, the greater the consumers' taste for variety. The exponents of C_A and C_M in the utility function – $(1 - \mu)$ and μ, respectively – indicate the invariant shares of disposable income devoted to the agricultural good and to manufactures.

3.7. Transport costs

Both agricultural and investment goods are traded costless. On the other hand, transport costs for manufacturers take an "iceberg" form (Samuelson, 1954): when 1 unit is shipped only a fraction $1/T$ arrives at destination whereas the rest, i.e., $1 - (1/T)$, "melts" along the way, where $T \geq 1$. Following Baldwin *et al.* (2003), to compact the notation, we introduce the parameter $\phi \equiv T^{1-\sigma}$ which is conventionally labelled "Trade Freeness", where $0 < \phi \leq 1$, with $\phi = 1$ corresponding to no trade cost ($T = 1$) and with $\phi \to 0$ corresponding to trade cost becoming prohibitive ($T \to \infty$).

4. Short-run general equilibrium

We first characterize a short-run equilibrium contingent on the overall given capital stock $K \equiv K_P + K_G$ and on the given regional allocation of private capital λ. In a short-run general equilibrium all markets are in equilibrium.

Beginning with the factor markets, in each region perfect intra-regional labour mobility ensures the equalisation of the wage rate among sectors:

$$w_A^r = w_M^r = w_I^r = w^r \tag{9}$$

where $r = S,N$. Moreover, the definition of unitary capital cost implies $F_r = w^r a_I^r$ in the investment goods market in region r.

As for the commodity markets, we consider that with no agricultural transport costs, the equilibrium agricultural price is the same in both regions, $p_A^S = p_A^N = p_A$. Since competition results in zero agricultural profits, the equilibrium nominal wage of workers is equal to the agricultural product price and is therefore always the same in both regions $w^S = w^N = w$. Setting the agricultural price and, therefore, the wage equal to 1, $p_A = w = 1$, we define the *numéraire* in terms of which the other prices are obtained.

Facing a wage of 1, each firm – depending on the region where it is located – has a fixed cost corresponding to $F_r = a_I^r$ and a marginal cost of a_M^r.

Given the beliefs attributed to a firm in the Dixit-Stiglitz model, each firm maximises profit on the basis of a perceived price elasticity of $-\sigma$. Considering that the indirect demand function is

$$p_{M,i}^r = \frac{\left(c_i^r\right)^{-\frac{1}{\sigma}}}{\sum_{i=1}^{n}\left(c_i^r\right)^{\frac{\sigma-1}{\sigma}}} E \tag{10}$$

where c_i^r is the quantity demanded of good i in region r and E the overall consumption expenditure of the economy, under the assumption of symmetric behaviour, each firm i sets the same local (mill) price p_M^r for its variety as follows

$$p_M^r = \frac{\sigma}{\sigma - 1} a_M^r \tag{11}$$

Note that from (11), public expenditures affect the price level of manufactured goods, *via* its impact on a_M^r. The greater is the consumers' taste for variety (i.e., the lower σ), the lower is the degree of competition and the higher is the excess of price over marginal cost. Given transport costs, the effective price paid by consumers for a variety produced in the other region is $p_M^r T$.

Short-run general equilibrium requires that each firm meets the demand for its variety. For a variety produced in region r, we have:[13]

$$x_r = d_r \tag{12}$$

where d_r is the demand for a variety in region r. From (11), the short-run equilibrium operating profit *per* variety in region r is:

$$\pi_r = p_M^r x_r - a_M^r x_r = \frac{p_M^r x_r}{\sigma} = \frac{a_M^r}{\sigma - 1} x_r \tag{13}$$

This profit per variety defines the regional profit or rental *per* unit of capital.

Adding the government sector to the analysis, total expenditure in consumption corresponds to total income Y minus total investment L_I, the latter being defined by the sum of overall private spending in new capital goods, $L_{IP} = L_{IP}^S + L_{IP}^N$ and overall public expenditures in investment goods, $L_{IG} = L_{IG}^S + L_{IG}^N$:[14]

$$E = Y - L_I = Y - L_{IP} - L_{IG} \tag{14}$$

Given the overall expenditure planned by the government (L_{IG}), the tax burdens of the two regions are:[15]

$$TB_S = s_F L_{IG} \qquad TB_N = (1 - s_F) L_{IG} \tag{15}$$

where s_F is the share of public expenditure financed by residents in the South. When $s_F = 1$ ($s_F = 0$) the provision of public goods is entirely financed by southern (northern) residents.

In the presence of a central government purchasing investment goods, the price of manufactured goods in each region will depend upon local productive public goods H_r. Denoting by $q \equiv p_M^S / p_M^N$ the relative price of manufactured goods produced in the South, from (5) and (11) it follows

$$q = \frac{a_M^S}{a_M^N} = \frac{g(H_S)}{g(H_N)} \tag{16}$$

13. Note that from now on, given the assumption of firms' symmetric behaviour, we disregard the subscript *i*.

14. Note that private and public spending in investment goods correspond to the wages paid to the units of labour necessary to produce these goods given the technology and the location where they are produced.

15. In our analysis, the level of provided public goods is the primary policy tool on the basis of which taxation is fixed.

where $\frac{\partial q}{\partial H_S} \leq 0$. That is, the higher the provision of public goods in the South – as compared to the North – the lower the relative price of manufactured goods produced in the South.

Total expenditure on the agricultural product is $(1-\mu)E$.[16] Expenditure on manufacturers is μE. Since, from (13), profit equals the value of sales times $1/\sigma$, the total profit received by capitalists is defined by:

$$\Pi = \frac{\mu}{\sigma} E \tag{17}$$

Therefore, given a wage of 1, the overall income in the economy, including wages and profits, is

$$Y = L + \frac{\mu}{\sigma} E$$

and total expenditure corresponds to:

$$E = \frac{\sigma}{\sigma - \mu}\left(L - L_{IP} - L_{IG}\right) \tag{18}$$

Since capitalists allocate their capital across regions in the same manner, they all receive the average profit. Given that all profits are repatriated to capital owners, the regional expenditures are:

$$E_S = s_L\left(L + \frac{\mu}{\sigma}E - L_{IP}\right) - TB_S = s_L\left(L + \frac{\mu}{\sigma}E - L_{IP}\right) - s_F L_{IG} \tag{19}$$

$$E_N = (1 - s_L)\left(L + \frac{\mu}{\sigma}E - L_{IP}\right) - TB_N = (1 - s_L)\left(L + \frac{\mu}{\sigma}E - L_{IP}\right) - (1 - s_F)L_{IG} \tag{20}$$

where E_r is the nominal expenditure in region r; and where, under the assumption $s_L = s_K = s_I$, $s_L L$ and $s_L \frac{\mu}{\sigma}E$ are, respectively, the wages and the profits earned by the households living in the South and $s_L L_{IP}$ are the investments undertaken by southern households. Similar meanings apply to the terms $(1 - s_L)L$, $(1 - s_L)\frac{\mu}{\sigma}E$ and $(1 - s_L)L_{IP}$ which refer to northern households.

From equations (18) and (19) region *S*'s share in total expenditure $s_E \equiv E_S/E$ can be expressed as:

$$s_E = s_L + (s_L - s_F)\frac{\sigma - \mu}{\sigma}\frac{L_{IG}}{L - L_{IG} - L_{IP}} \tag{21}$$

With no provision of (new) public goods and no taxation, $L_{IG} = 0$, the southern share in total expenditure is equal to the share of factor endowment of the South, which in turn corresponds to the number of resident households in that region, $s_E = s_L$. When $s_L \neq \frac{1}{2}$, factor endowments (labour and private capital) are unevenly distributed between regions. In particular, when $s_L < \frac{1}{2}$, the South is poorer (has a smaller factor endowment) than the North.

With the provision of public goods, $L_{IG} > 0$:

$$s_E < (>)\ s_L \quad \text{if} \quad s_L < (>)\ s_F$$

16. The non-full specialization condition requires total expenditure on the agricultural good, $(1-\mu)E$, to exceed agricultural production in each region taken separately even in the case of complete agglomeration of the industrial activity in one of them. That is, $(1-\mu)E > \max\{s_L L;\ (1-s_L)L\}$, where, given the assumption on the technology, $s_L L$ is the agricultural output in the South and $(1-s_L)L$ is the agricultural output in the North.

In the South, the expenditure share on manufactured goods after taxation will be smaller (larger) than before taxation, if consumers in the South contribute more (less) than consumers in the North to the financing of public goods taking into account their contributive capacity s_L. If $s_L = s_F$ the expenditure share on manufactured goods will not be affected by taxation.

The regional manufacturing price indices facing consumers are given by:

$$P_S = \left[n_S \left(p_M^S \right)^{1-\sigma} + n_N \left(p_M^N T \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = \Delta_S^{\frac{1}{1-\sigma}} n^{\frac{1}{1-\sigma}} p_M^S \tag{22}$$

$$P_N = \left[n_S \left(p_M^S T \right)^{1-\sigma} + n_N \left(p_M^N \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = \Delta_N^{\frac{1}{1-\sigma}} n^{\frac{1}{1-\sigma}} p_M^N q \tag{23}$$

where $\Delta_s \equiv \lambda + \phi(1 - \lambda)z$, $\Delta_N \equiv \phi\lambda + (1 - \lambda)z$ and $z \equiv q^{\sigma-1}$.

With any barrier to trade and symmetric provision of public goods, $z = 1$ ($H_S = H_N$), the cost-of-living is lower, and the real incomes of both workers and resident capitalists are higher in the region with the larger manufacturing sector (i.e., for $\phi < 1$, $P_N < P_S$ iff $f\lambda < 1/2$). Decreasing z (increasing the provision of public goods in the South compared to the North) could reverse this inequality, since $P_S < P_N$ iff $f\lambda > z/(1 + z)$.

The demand for a variety produced in region S is:

$$d_S = \left(\mu E_S P_S^{\sigma-1} + \mu E_N P_N^{\sigma-1} \phi \right) \left(p_M^S \right)^{-\sigma} = \left[s_E P_S^{\sigma-1} + (1 - s_E) P_N^{\sigma-1} \phi \right] \left(p_M^S \right)^{-\sigma} \mu E \tag{24}$$

and the demand for a variety produced in region N is:

$$d_N = \left(\mu E_S P_S^{\sigma-1} \phi + \mu E_N P_N^{\sigma-1} \right) \left(p_M^N \right)^{-\sigma} = \left[s_E P_S^{\sigma-1} \phi + (1 - s_E) P_N^{\sigma-1} \right] \left(p_M^N \right)^{-\sigma} \mu E \tag{25}$$

From (12), (22), (23), (24) and (25):

$$x_S = d_S = \left(\frac{s_E}{\Delta_S} + \frac{1 - s_E}{\Delta_N} \phi \right) \frac{\mu E}{n p_M^S} \tag{26}$$

$$x_N = d_N = \left(\frac{s_E}{\Delta_S} \phi + \frac{1 - s_E}{\Delta_N} \right) \frac{z \mu E}{n p_M^N} \tag{27}$$

From (12), (13), (16), (26) and (27), the short-run equilibrium profits per variety / unit of capital are:

$$\pi_S = \left(\frac{s_E}{\Delta_S} + \frac{1 - s_E}{\Delta_N} \phi \right) \frac{\mu E}{\sigma K_P} \tag{28}$$

$$\pi_N = \left(\frac{s_E}{\Delta_S} \phi + \frac{1 - s_E}{\Delta_N} \right) \frac{z \mu E}{\sigma K_P} \tag{29}$$

5. Long-run equilibrium

A long-run equilibrium is characterised as follows.[17] Firstly, since capital is perfectly mobile across regions, regional profit rates are equalised, i.e. $\pi_S = \pi_N = \pi$.

Secondly, there is no incentive to further expand the existing private capital stock; i.e. the present value of the profit stream resulting from one additional unit of capital (where the given discount rate ρ and the depreciation rate δ are appropriately accounted for) should be equal to its construction / replacement cost, denoted by $\hat{a}_I$. Using (17), it follows:

$$\frac{1}{\rho+\delta}\pi^* = \frac{1}{\rho+\delta}\frac{\mu E^*}{\sigma K_P^*} = \hat{a}_I \tag{30}$$

or

$$\frac{E^*}{K_P^*} = (\rho+\delta)\frac{\sigma}{\mu}\hat{a}_I \tag{31}$$

where * denotes long-run values.

Finally, since private and public capital stocks are constant, investment is enough to reintegrate capital depreciation $I_P^* = \delta K_P^*$ and $I_G^* = \delta K_G^*$, where $I_P^* = \frac{L_{IP}}{\hat{a}_I}$ and $I_G^* = \frac{L_{IG}}{\hat{a}_I}$ represent, respectively, private and public investment. Observing that the labour input requirements to produce private and public investment goods correspond to $L_{IP} = \hat{a}_I I_P^*$ and $L_{IG} = \hat{a}_I I_G^*$ respectively, from (18) we obtain:

$$\frac{E^*}{K_P^*} = \frac{\sigma}{\sigma-\mu}\left[\frac{L}{K_P^*} - \delta\hat{a}_I\left(\frac{K_P^*+K_G^*}{K_P^*}\right)\right] \tag{32}$$

Using (30) and (32), we get:

$$K_P^* = \frac{\mu\left(L-\delta\hat{a}_I K_G^*\right)}{\left[(\sigma-\mu)(\rho+\delta)+\mu\delta\right]\hat{a}_I} \tag{33}$$

which gives the long-run equilibrium level of private capital stock (corresponding to the total number of manufacturing firms).

In studying the impact of changes in the provision of public goods on the long-run equilibrium, we need to take into account where the investment sector is located. Assuming that the investment goods sector agglomerates where capital construction is less costly allows us to distinguish two cases:

case a) $H_S < H_N$ and $a_I^N < a_I^S$. The investment sector is agglomerated in the North, $\hat{a}_I = a_I^N = f(H_N)$.

case b) $H_S > H_N$ and $a_I^S < a_I^N$. The investment sector is agglomerated in the South, $\hat{a}_I = a_I^S = f(H_S)$.

Proposition 1. *When case a) prevails, an increase in the provision of public goods in the South always involves a reduction in the overall stock of private capital; instead, when case b) prevails, an increase in the provision of public goods in the South enlarges the overall stock of private capital, if*

17. For an explicit derivation see Baldwin (1998), in particular the *Supplemental Guide to Calculations*; there, the stability properties of the long-run equilibrium – for the model without government sector – are studied as well. In Commendatore, Kubin and Petraglia (2009), in which we present a much simpler model without endogenous capital but with a government sector, we investigate explicitly the stability properties of industrial location.

the improvement of labour productivity in the investment goods sector – following such an increase – is sufficiently large to exceed the "crowding-out" effect induced by the increase in public investment.

When case a) prevails, it can be easily checked that:

$$\frac{\partial K_P^*}{\partial H_S} = -\frac{\mu\delta}{(\sigma-\mu)(\rho+\delta)+\mu\delta} < 0$$

which is always negative. That is, in the absence of any effect on labour productivity in the investment goods sector, an increase in the provision of public goods will unambiguously reduce the stock of private capital.

On the other hand, when case b) prevails the effect on K_P^* of an increase in H_S can be separated into two components:

$$\frac{\partial K_P^*}{\partial H_S} = \frac{\mu}{(\sigma-\mu)(\rho+\delta)+\mu\delta}\left(\frac{f'(H_S)}{f(H_S)^2}L - \delta\right) \tag{34}$$

The derivative (34) is positive, $\frac{\partial K_P^*}{\partial H_S} > 0$ when $|f'(H_S)| > \delta\frac{f(H_S)^2}{L} > 0$ or, taking into account expression (6), when $\frac{B}{A} > \frac{\delta}{L}$. That is, when its effect on labour productivity in the investment goods sector is sufficiently large to overcome the "crowding–out" effect of an increase in public investment, an increase in the provision of public goods enlarges the overall stock of private capital.

Inspecting (32) can provide some economic intuition for the result summarized in proposition 1. If the increase in southern public capital does not increase the productivity in the investment goods sector (i.e., in case a)) increasing the (equilibrium) public capital stock in the South requires more resources for its maintenance (to compensate for the depreciation); therefore, expenditures on the manufactured goods decline, which in turn lowers the profit rate in this sector. In order to satisfy condition (31), the private capital stock has to decline to a new long-run equilibrium value.[18] This result can be overturned if the increase in the southern public capital increases the productivity in the investment goods sector as well, i.e. in case b). In that case an increase in the public capital stock needs not to increase the maintenance costs (see (32)); in addition, the right hand side of the accumulation condition (31) declines as well. Therefore, in that case the provision of public capital can increase the overall accumulation incentive.

Now we turn to the question of how changes in the provision of public goods affect the long-run regional allocation of private capital, λ^*. Our analysis is concerned with the interior equilibrium, that is, an equilibrium in which the manufacturing sector is located in both regions, $0 < \lambda^* < 1$.

At the interior equilibrium $\pi_N^* = \pi_S^* = \pi^*$. Equating (28) and (29), we are able to obtain an explicit solution for λ^*:

$$\lambda^* = \frac{1}{2} + z\frac{(1-\phi)(1+\phi)}{(1-z\phi)(z-\phi)}\left(s_E^* - \frac{1}{2}\frac{(1+z\phi)(z-\phi)}{(1-\phi)(1+\phi)z}\right) \tag{35}$$

18. Note that this is a case of input competition discussed in Bucovetsky (2005). Since the wage rate is used as *numéraire*, a decrease in expenditures is equivalent to an increase in the wage rate (which is the pivotal variable in Bucovetsky, 2005).

and from (21) and (23) an explicit solution for s_E^*:

$$s_E^* = s_L + (s_L - s_F)\frac{\sigma(\rho+\delta)-\mu\rho}{\sigma(\rho+\delta)}\frac{\delta\hat{a}_I K_G^*}{L-\delta\hat{a}_I K_G^*} \qquad (36)$$

From equation (35) it is straightforward to note that $\lambda^* < \frac{1}{2}$ for $s_E^* < \boldsymbol{\Omega}$, where $\boldsymbol{\Omega} \equiv \frac{1}{2}\frac{(1+z\phi)(z-\phi)}{(1-\phi^2)z}$. Moreover, $\boldsymbol{\Omega} \geq (<) \frac{1}{2}$ for $z \geq (<) 1$. That is, in correspondence of an equal provision of public goods in the two regions, $H_S = H_N$ ($z = 1$), if the share in total expenditure in the South is smaller compared to the North, $s_E^* < \frac{1}{2}$, the size of the manufacturing sector is also smaller, $\lambda^* < \frac{1}{2}$.

In order to study the effect of public expenditure on the location of the manufacturing sector in a simple framework, we assume that the investment sector is located in the North (case a) above).[19] Moreover, we assume that the South is the "poor" region in the sense that it has the smaller factor endowment, $s_L < \frac{1}{2}$.

An increase in the provision of public goods in the South could reduce regional inequalities relocating firms to the backward region. As the following proposition 2 states, the effect of an increase in the provision of public goods in the South, H_S, on the long-run equilibrium location of manufacture, λ^*, depends crucially on the difference between s_F, the share of public expenditures financed by residents in the South and s_L, interpreted as a measure of fiscal capacity of the South compared to the North. This difference determines the strength and the direction of the effect of such an increase on the southern relative market size, s_E.

Proposition 2. *The effect of an increase in H_S on λ^* depends on the relative strength and direction of two effects: the "productivity" effect (PE) and the "demand" effect (DE). According to the former, H_S affects positively λ^* via its impact on labour productivity in the southern manufacturing sector; according to the latter, H_S affects λ^* via a reduction in the relative market size, s_E. If $s_F > s_L$ the "demand" effect is negative and the overall effect of an increase of H_S on λ^* could be positive, negative or nil depending on parameter values. If $s_F \leq s_L$ the "demand" effect is positive (or nil) and the overall effect of an increase of H_S on λ^* is positive.*

The impact of the provision of productive public goods on the equilibrium industrial location can be evaluated looking at the sign of the following derivative:

$$\frac{\partial\lambda^*}{\partial H_S} = -\frac{\phi}{(1-\phi z)^2}\left[1-\frac{(z^2-1)(1-\phi^2)}{(z-\phi)^2}s_E^*\right]\frac{\partial z}{\partial H_S} + z\frac{1-\phi^2}{(1-\phi z)(z-\phi)}\frac{\partial s_E^*}{\partial H_S} \qquad (37)$$

It is possible to identify neatly two effects that an increase in H_S could exert on the location of the manufacturing sector. We call the first one "productivity" effect (*PE*). It corresponds to the first term in equation (36):

$$PE = -\frac{\phi}{(1-\phi z)^2}\left[1-\frac{(z^2-1)(1-\phi^2)}{(z-\phi)^2}s_E^*\right]\frac{\partial z}{\partial H_S} \qquad (38)$$

19. In Commendatore, Kubin and Petraglia (2007), we also study the more complex case in which the investment sector is located in the South (case b) above).

According to this effect the provision of productive public goods in the South affects λ^* *via* its impact on the labour productivity in the southern manufacturing sector.

Since $\frac{\partial z}{\partial H_S} < 0$, the direction of the "productivity" effect is determined by the sign of the term in square brackets. To determine the sign of this term, we consider that a necessary condition for $0 < \lambda^* < 1$ is $\phi < z < \phi^{-1}$ (that is, the two regions should not differ too much in terms of provisions of public goods).[20] It is easy to verify that if the latter inequality holds, the term in square brackets is positive for any $0 \le s_E^* \le 1$. It follows that the "productivity" effect is positive.

We call the second effect "demand" effect (*DE*). It corresponds to the second term in expression (37):

$$DE = z\frac{1-\phi^2}{(1-\phi z)(z-\phi)}\frac{\partial s_E^*}{\partial H_S} \tag{39}$$

According to this effect, the provision of productive public goods in the South affects λ^* *via* a change in the relative market size. Since $\phi < z < \phi^{-1}$ and $\phi < 1$, the sign of the "demand" effect corresponds to the sign of $\frac{\partial s_E^*}{\partial H_S}$.

When the investment goods sector is located in the North, the impact of H_S on the regional distribution of consumption expenditures, s_E^*, is given by:

$$\frac{\partial s_E^*}{\partial H_S} = (s_L - s_F)\,\frac{\sigma(\rho+\delta)-\mu\rho}{\sigma(\rho+\delta)}\frac{\delta a_I^N L}{(L-\delta a_I^N K_G^*)^2} \quad \text{with} \quad \frac{\partial s_E^*}{\partial H_S} \ge (<)\, 0 \quad \textit{for} \quad s_L \ge (<)\, s_F \tag{40}$$

That is, the effect of an increase in the provision of productive public goods on s_E^* depends on the inequality between s_L and s_F.

If $s_F > s_L$, the tax burden necessary to finance public investment falls more on the South than it does on the North. An increase in the provision of public goods has a negative effect on s_E^*. On the other hand, if $s_F < s_L$ the "demand" effect is positive. Finally, the effect of an increase in the provision of productive public goods on the expenditure shares is nil at $s_L = s_F$.

In summary, the overall effect of an increase in the provision of public goods in the South is positive on λ^*, that is, $\frac{\partial \lambda^*}{\partial H_S} > 0$ when the following condition holds:

$$\frac{\partial s_E^*}{\partial H_S} > \frac{\partial z}{\partial H_S}\frac{\phi(z-\phi)}{z(1-\phi z)(1-\phi^2)}\left[1-\frac{(z^2-1)(1-\phi^2)}{(z-\phi)^2}s_E^*\right] \tag{41}$$

This condition is always satisfied when $\frac{\partial s_E^*}{\partial H_S} > 0$, that is, when $s_F < s_L$; otherwise its validity depends on parameter values.

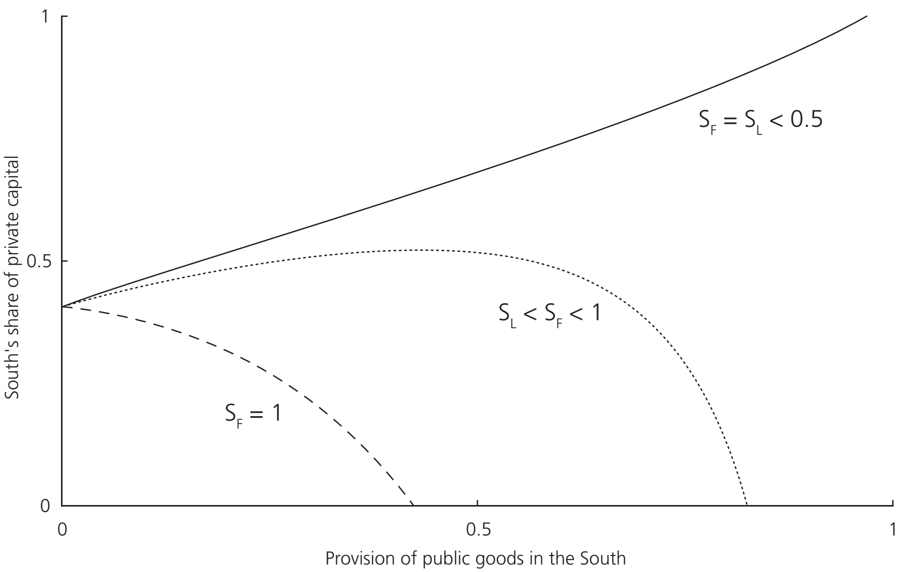
In order to disentangle the relative importance of "productivity" and "demand" effects, we employ numerical simulations. According to our stylized case the South is the backward region. In order to improve the situation in the South, productivity enhancing public expenditure is increased in this region. We study whether and how the effect of such a policy depends upon its financing scheme.

20. A necessary and sufficient condition for $0 < \lambda^* < 1$ is $\phi\,\frac{z-\phi}{1-\phi^2} < s_E^* < \frac{1}{z}\frac{z-\phi}{1-\phi^2}$.

FIGURE 1 summarizes the effects on λ^* of an increase in the provision of public goods in the South, within the interval $0 \leq H_S < \frac{L}{\delta a_I^N}$,[21] for different values of the regional tax burden necessary to finance it, s_F.[22] The solid line corresponds to the case $s_F = s_L < 0.5$ (in our simulation we assumed $s_L = 0.45$), that is, the burden of taxation is distributed among consumers in the two regions according to their contributive capacity. The "demand" effect is nil and only the "productivity" effect impact (positively) on the share of capital located in the South. The dotted line corresponds to the case $s_L < s_F < 1$ (we assumed $s_F = 0.55$). The "demand" effect is negative and after an initial range it overcomes the "productivity" effect. Finally, the dashed line corresponds to $s_F = 1$. The "demand" effect is stronger than the "productivity" effect for any H_S in the interval considered.

As shown in FIGURE 1, when $s_F = 1$, the "demand" effect could be so strong that λ^* decreases for any positive provision of productive public goods in the South. However this is not always the case as confirmed by the proposition below.

Figure 1 – Provision of public goods and agglomeration in the South: demand and productivity effects



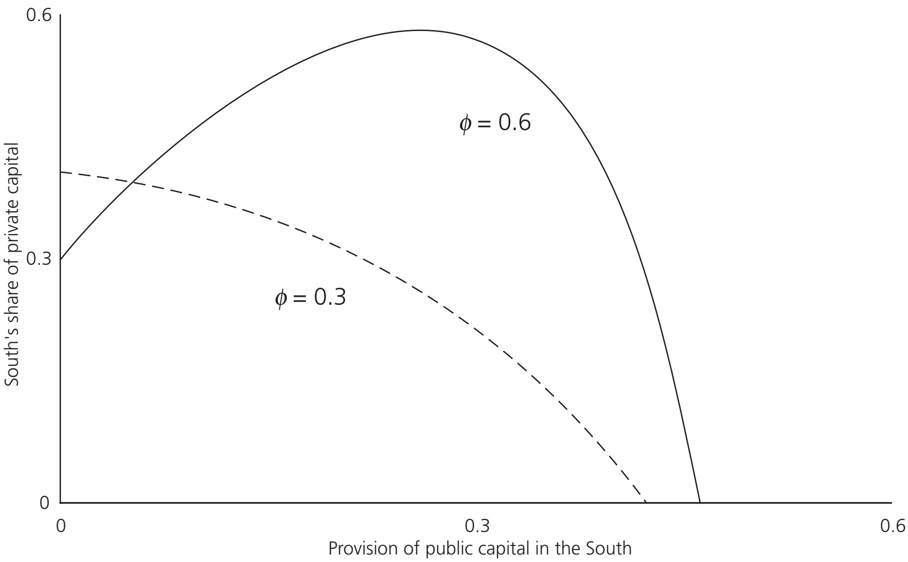
Proposition 3. *For $s_L < s_F$ and for z close to 1 (H_N close to H_S), there exists a value of trade freeness $\hat{\phi}$ such that if $\phi = \hat{\phi}$, the "productivity" effect outweighs the "demand" effect for an initial range of positive values of H_S. Within this range, λ^* increases with H_S. However, as we increase further H_S above the value $\hat{H}_S$, the "demand" effect is always sufficiently strong to outweigh the "productivity" effect. Thus, $\hat{H}_S$ represents the value of H_S which maximises λ^*. For a proof, see Appendix 1.*

21. From (32) and (33), and considering that $K_G^* = H_S$, $\delta a_I^N H_S$ should be smaller than L in order to have positive overall consumption expenditures in the economy, $E^* > 0$.

22. To plot Figure 1, we set $L = 1$, $s_L = 0.45$, $\rho = 1.1$, $\delta = 0.2$, $\alpha_I = 5$, $\sigma = 2$, $\mu = 0.5$, $\phi = 0.3$, $C = 1$, $D = 1$, $H_N = 0$, i.e. the North receives no public goods.

FIGURE 2 illustrates Proposition 3. In FIGURE 2 the dashed line corresponds to the case presented in FIGURE 1 in which $s_F = 1$. In order to plot the solid line, we have increased the value of trade freeness. As shown in FIGURE 2, if trade freeness is increased sufficiently, for example, from $\phi = 0.3$ to $\phi = 0.6$, even when the tax burden needed to finance public investment falls entirely on the South, $s_F = 1$, increasing the provision of public goods up to $H_S = \widehat{H}_S$ favours agglomeration in that region.

Figure 2 – Trade freeness and agglomeration in the South ($s_F = 1$)



In summary, an increase in H_S can lead to a higher relative size of the southern manufacturing market – by relocation of firms from the North to the South – even in the presence of a non favorable "demand" effect as long as this effect is not too large.

The latter result tells us that policy measures aimed at enhancing labour productivity of southern manufacturing firms will be effective only if the North participates to the financing of such policies. If the South is "left alone" (that is, if public goods are financed solely by income of residents in the South), then the "demand" effect of an increase in H_S sooner or later prevails depending on the degree of trade freeness enjoyed by the economy under consideration. On the other hand, if the government sets s_F at sufficiently low value, letting northern tax payers contribute on the basis of their capacity, then the "demand" effect of public expenditures will be more than offset by the "productivity" effect.

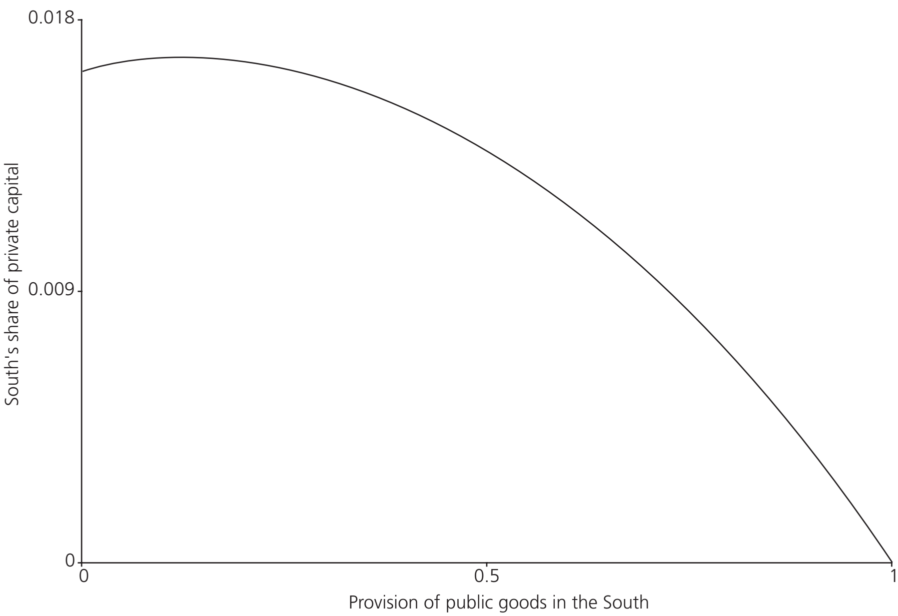
Finally, we consider that the objective of the government could be increasing the size of the manufacturing sector located in the South, corresponding to the level of private capital located in that region $\lambda^* K_p^*$, rather than its share, λ^*. The government, therefore, takes into account that the increase in public expenditure could induce a "crowding-out" effect on private capital. The following proposition 4 states that, when the tax burden is distributed between regions in such a way to neutralise the "demand" effect, $s_L = s_F$, it is possible to identify a level of public goods provision to the South which maximises $\lambda^* K_p^*$.

Proposition 4. *When $s_L = s_F$ and z is close to 1, it is possible to identify a value of trade freeness in correspondence of which there exists a positive value of H_S, denoted by $\bar{H}_S$, which maximizes $\lambda^* K_p^*$. For a proof, see Appendix 2.*

According to proposition 4, even if the "crowding-out" effect on private expenditure induced by an increase in public expenditure is taken into account, there is still large scope for public intervention in order to reduce inter-regional inequalities. The optimal level of public goods provision needed to reduce regional disparities depends on several factors which determine the relative strength of the "demand", "productivity" and "crowding-out" effects. FIGURE 3 summarizes the effects on $\lambda^* K_p^*$ of an increase in the provision of public goods in the South, within the interval $0 \leq H_S < \dfrac{L}{\delta a_I^N}$ for $s_F = 1$.[23]

However, a couple of notes of caution are in order. First, even when the "demand" effect is nil, the "crowding-out" effect induced by the increase in public investment sooner or later will outweigh the "productivity" effect. Moreover, an excessive increase in the provision of public goods aiming to bring down regional inequalities could cause the undesired result of reducing the size of the manufacturing sector in both regions. That is, a trade-off between regional equity and overall efficiency is revealed.

Figure 3 – Provision of public goods and level of private capital in the South ($s_L = s_F$)



23. The other parameter values are the same used to plot Figure 1.

6. Welfare analysis

In this section we present a brief analysis concerning the welfare effects of an increase in the provision of public goods in the South. For the sake of simplicity, we assume that $s_L = s_F$, i.e., we rule out by assumption the "demand" effect on the interior equilibrium value of λ. It follows $s_E = s_L$. Moreover, we assume that the investment sector is located in the North (analogous to case a) in the previous section).[24] Our results are summarised in the following proposition.

Proposition 5. *Let $s_L = s_F$, an increase in the provision of public goods in the South, H_S, is welfare improving for local residents if and only if the positive effect on industrial agglomeration in the South outweighs the negative effect on consumption expenditure and on the overall size of the manufacturing sector. The welfare of northern residents, instead, is unambiguously reduced by an increase in H_S.*

In order to demonstrate proposition 5, we follow the indirect utility approach of welfare analysis. Indirect utilities of representative households living respectively in the South and in the North correspond to:

$$V_S = \left((1-\mu)\frac{E_S}{\overline{P}_S L_S}\right)^{1-\mu}\left(\mu\,\frac{E_S}{\overline{P}_S L_S}\right)^{\mu} = \vartheta\,\frac{E_S}{P_S^{\mu} L_S} \tag{42}$$

$$V_N = \left((1-\mu)\frac{E_N}{\overline{P}_N L_N}\right)^{1-\mu}\left(\mu\,\frac{E_N}{\overline{P}_N L_N}\right)^{\mu} = \vartheta\,\frac{E_N}{P_N^{\mu} L_N} \tag{43}$$

where $\overline{P}_r = p_A^{1-\mu} P_r^{\mu} = P_r^{\mu}$ is the perfect price index in region r, which takes into account agricultural prices, $p_A = 1$, E_r/L_r is the expenditure per household in region r and $\vartheta \equiv (1-\mu)^{1-\mu}\mu^{\mu}$.

We start looking at the utility of a household living in the North. A change in H_S affects V_N *via* the northern price index of manufacturing goods P_N or *via* consumption expenditure in the North E_N. That is,

$$\frac{\partial V_N}{\partial H_S} = \frac{\partial E_N}{\partial H_S}\frac{V_N}{E_N} - \mu\,\frac{\partial P_N}{\partial H_S}\frac{V_N}{P_N} \tag{44}$$

Consider $P_N = p\Delta_N^{\frac{1}{1-\sigma}} K_P^{\frac{1}{1-\sigma}}$ first. An increase in the provision of public goods in the South determines the following change in the northern price index of manufacturing goods:

$$\frac{\partial P_N}{\partial H_S} = -\frac{P_N}{\sigma-1}\left(\frac{\partial \Delta_N}{\partial H_S}\Delta_N^{\sigma} + \frac{\partial K_P}{\partial H_S}K_P^{\sigma}\right) \tag{45}$$

At the interior equilibrium corresponding to $s_E = s_L$, we have that $\Delta_N = (1-s_L)(1-\phi^2)\dfrac{z}{1-\phi z}$ and $\dfrac{\partial \Delta_N}{\partial H_S} = \dfrac{(1-s_L)(1-\phi^2)}{(1-\phi z)^2}\dfrac{\partial z}{\partial H_S} < 0$. Moreover, since $\dfrac{\partial K_P}{\partial H_S} < 0$, it follows that $\dfrac{\partial P_N}{\partial H_S} > 0$.

That is, since part of the production of manufacturing goods has moved to the South, their consumption is more expensive for the residents in the North because of transport costs. Moreover, if public expenditure involves a reduction in the size of the overall manufacturing sector – that is, the number of manufacturing firms becomes smaller – the northern price index increases and welfare decreases.

24. In Commendatore, Kubin and Petraglia (2007), we explore also the case in which the investment sector is located in the South (analogous to case b) in the previous section).

Consider now that $E_N^* = (1 - s_L)E^*$, where from (32) and (33) $E^* = \dfrac{\sigma(\rho+\delta)(L-\delta a_I^N H_S)}{(\sigma-\mu)(\rho+\delta)+\mu\delta}$. We have that $\dfrac{\partial E^*}{\partial H_S} = -\delta a_I^N \dfrac{\sigma(\rho+\delta)}{(\sigma-\mu)(\rho+\delta)+\mu\delta} < 0$ and therefore $\dfrac{\partial E_N^*}{\partial H_S} < 0$. That is, as a consequence of the increase in the tax burden needed to finance public expenditure, consumption expenditure and welfare in the North decrease.

Thus, the welfare of a representative household in the North unambiguously declines following an increase in the provision of public goods in the South.

Moving on to the utility of a household living in the South, a change in H_S affects V_S *via* the southern price index of manufacturing goods P_S or *via* consumption expenditure in the South E_S. That is:

$$\frac{\partial V_S}{\partial H_S} = \frac{\partial E_S}{\partial H_S}\frac{V_S}{E_S} - \mu\frac{\partial P_S}{\partial H_S}\frac{V_S}{P_S} \tag{46}$$

Consider $P_S = p\Delta_S^{\frac{1}{1-\sigma}} K_P^{\frac{1}{1-\sigma}}$ first. An increase in the provision of public goods in the South determines the following change in the southern manufacturing price index:

$$\frac{\partial P_S}{\partial H_S} = -\frac{P_S}{\sigma-1}\left(\frac{\partial \Delta_S}{\partial H_S}\Delta_S^{\sigma} + \frac{\partial K_P}{\partial H_S}K_P^{\sigma}\right) \tag{47}$$

At the interior equilibrium corresponding to:

$$s_E = s_L,\ \Delta_S = s_L(1-\phi^2)\frac{z}{z-\phi} \quad \text{and} \quad \frac{\partial \Delta_S}{\partial H_S} = -s_L\frac{1-\phi^2}{(z-\phi)^2}\frac{\partial z}{\partial H_S} > 0.$$

It follows that if $\dfrac{\partial K_P}{\partial H_S}$ is sufficiently small in absolute value, then $\dfrac{\partial P_S}{\partial H_S} < 0$. More specifically, $\dfrac{\partial P_S}{\partial H_S} < 0$ when $\dfrac{\partial K_W}{\partial H_S} > -\dfrac{\partial \Delta_S}{\partial H_S}\left(\dfrac{\Delta_S}{K_W}\right)^{\sigma}$.

That is, if more goods are produced in the South, consumption of manufactured goods is cheaper for the residents in the South because they pay less for transport costs. However, this can only occur if the reduction of K_P is not so large that the manufacturing production declines also in the South. If the reduction of K_P is sufficiently large also the price index in the South increases.

Considering now that $E_S^* = s_L E^*$, as above we have that since $\dfrac{\partial E^*}{\partial H_S} < 0$, then $\dfrac{\partial E_S^*}{\partial H_S} < 0$. That is, also in the South consumption and welfare are negatively affected by the increase in the tax burden needed to finance public expenditure.

Therefore, the impact on southern welfare of an increase in the provision of public goods in the South depends on the counterbalancing of two negative effects, i.e., the reduction in consumption expenditure and the reduction of the size of the overall manufacturing sector; and one positive effect, i.e., the agglomeration of industrial activity.

7. Conclusions

The NEG model presented in this paper allows for private capital accumulation and capital mobility between a "rich" and a "poor" region. In addition, a central government decides upon the level and the regional and sectoral distribution of productivity enhancing public investments. We derived results on how the provision and the financing of productive public investments affect the overall private capital stock as well as its regional distribution. Two alternative financing schemes have been considered, i.e. whether or not the regional tax burden corresponds to the expenditures for local public goods.

We have shown that the regional and sectoral distribution of public investment matters in determining the final impact of an increase in public capital on the level of overall private capital. The "productivity" effect tends to increase private capital, while a "crowding-out" effect between public and private investments works in the opposite direction.

As for the impact of public investment on industrial agglomeration, we have shown that an increase in productivity enhancing public capital in the "poor" region does not always increase the share of manufacturing in that region. The final result depends on the relative strength of two effects arising from public policy decisions on expenditures: the "productivity" and the "demand" effects, which have been studied separately in the literature so far. The "productivity" effect works through relative productivity of labour in the manufacturing sector and it tends to increase the share of manufacturing. The "demand" effect influences location *via* the relative size of local market and – depending on the tax scheme – can work in the opposite direction. Only if the "rich" region contributes to the financing of the public capital in the "poor" region, public investments unambiguously increase the southern share of manufacturing.

Relaxing the assumption that public goods affect productivity in the manufacturing and investment goods sectors, a natural extension of our model that we leave for future research is to consider productivity enhancing public investments which are specific to each sector. In doing so, the model will be enriched by contemplating the possibility to design appropriate public policies – in terms of sectoral specific public investments – aimed at mitigating the trade-off between regional equity and overall efficiency that we have depicted in our framework.

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Appendix 1
Proof of Proposition 3

In order to prove this proposition we follow two steps. As first step we search for a value of H_S which satisfies the following first order condition:

$$\frac{\partial \lambda^*}{\partial H_S} = PE + DE = 0 \qquad \text{(A1.1)}$$

where $PE = -\frac{\phi}{(1-\phi z)^2}\left[1-\frac{(z^2-1)(1-\phi^2)}{(z-\phi)^2}s_E^*\right]\frac{\partial z}{\partial H_S}$ is the "productivity" effect

and $DE = z\frac{1-\phi^2}{(1-\phi z)(z-\phi)}\frac{\partial s_E^*}{\partial H_S}$ is the "demand" effect.

Given the complexity of the above expression which does not allow to solve for H_S, we proceed *via* an alternative route searching for a value of ϕ, denoted by $\widehat{\phi}$, which satisfies (A1.1) for a given value of public goods provision, that is, $H_S = \widehat{H}_S$. If $\widehat{\phi}$ exists, then $\widehat{H}_S$ also satisfies condition (A1.1).

Differentiating *DE* and *PE* with respect to ϕ, we obtain:

$$\frac{\partial DE}{\partial \phi} = \frac{1-\phi^2}{(z-\phi)^2(1-\phi z)^2}z\frac{\partial s_E}{\partial H_S}$$

$$\frac{\partial PE}{\partial \phi} = -\frac{(z^2-1)\left\{\phi(1+\phi^2)(1+z^2)+z\left[(1-\phi)^2-4\phi\right]\right\}s_E^*+(z-\phi)^3(1+\phi z)}{(z-\phi)^3(1-\phi z)^3}\frac{\partial z}{\partial H_S}$$

$\frac{\partial DE}{\partial \phi} < 0$ for $\frac{\partial s_E}{\partial H_S} < 0$, that is, for $s_L < s_F$.

The sign of $\frac{\partial PE}{\partial \phi}$ depends on the sign of its numerator. It is easy to prove that it is always positive.

We know that $\phi < z < \phi^{-1}$, therefore $\frac{\partial PE}{\partial \phi} > 0$, for:

$$(z^2-1)\left\{\phi(1+\phi^2)(1+z^2)+z\left[(1-\phi)^2-4\phi\right]\right\}s_E^*+(z-\phi)^3(1+\phi z) > 0 \qquad \text{(A1.2)}$$

We distinguish three cases depending on the value of z:

Case 1: $z > 1$.
If $\phi(1+\phi^2)(1+z^2)+z\left[(1-\phi)^2-4\phi\right] > 0$ the inequality (A1.2) holds for any $s_E^* \geq 0$
If $\phi(1+\phi^2)(1+z^2)+z\left[(1-\phi)^2-4\phi\right] < 0$ the inequality (A1.2) holds for any $s_E^* \leq 1$.

Case 2: $\phi < z < 1$.
If $\phi(1+\phi^2)(1+z^2)+z\left[(1-\phi)^2-4\phi\right] > 0$ the inequality (A1.2) holds for any $s_E^* \leq 1$
If $\phi(1+\phi^2)(1+z^2)+z\left[(1-\phi)^2-4\phi\right] < 0$ the inequality (A1.2) holds for any $s_E^* \geq 0$.

Case 3: $z = 1$. The inequality (A1.2) always holds.

Therefore as long as $0 \leq s_E^* \leq 1$, $\frac{\partial PE}{\partial \phi} > 0$.

This proves that by increasing ϕ both the "demand" effect and the "productivity" effect increase monotonically in absolute value.

Note also that when $\phi = 0$, $PE = 0$ and $DE = \frac{\partial s_E^*}{\partial H_S} < 0$, which implies $-DE > 0$. Therefore as we increase ϕ, if PE and $-DE$ intersect once at $\phi = \hat{\phi}$, we have that $PE < -DE$ for $0 \le \phi < \hat{\phi}$ and $PE > -DE$ for $\hat{\phi} < \phi \le 1$. However, given that both PE and DE have an asymptote at $\min(z, z^{-1})$, it follows that the intersection point $\hat{\phi}$ may not exist or may not be unique.

To verify if there exists a value of trade freeness in correspondence of which DE and PE intersects, we need to solve a quartic equation (four degree equation):

$$\phi^4 + \left[\psi\left(\frac{1 - s_E^* + z^2 s_E^*}{z^2}\right) - \left(z + \frac{1}{z}\right)\right]\phi^3 - 2\psi\phi^2 + \left[\psi\left(\frac{z^2 + s_E^* - z^2 s_E^*}{z^2}\right) - \left(z + \frac{1}{z}\right)\right]\phi - 1 = 0 \quad \text{(A1.3)}$$

where $\psi = \frac{\partial z}{\partial H_S} \Big/ \frac{\partial s_E^*}{\partial H_S}$.

However, given the complexity of the solutions of a quartic equation, we are not able to find an explicit expression for $\hat{\phi}$ nor we are able to say if there exists a corresponding value $\hat{H}_S$ which is economically significant and unique or if this value maximises λ^*. Therefore, we proceed as follows. We set $z = 1$ in equation (A1.3), which is equivalent to assume $H_S = H_N$, then we prove that it exists a value of trade freeness $0 < \hat{\phi} < 1$ that guarantees the existence of an intersection point for $z = 1$; by continuity such a value of ϕ exists also for a z different but sufficiently close to 1.

When $z = 1$, equation (A1.3) can be reduced to

$$\phi^2 + \psi\phi - 1 = 0 \quad \text{(A1.4)}$$

There is a unique value $0 < \hat{\phi} < 1$, which satisfies equation (A1.4), that is,

$$\hat{\phi} = -\frac{\psi}{2} + \sqrt{1 + \frac{\psi^2}{4}} \quad \text{(A1.5)}$$

From (A1.5) it is possible to deduce that to each $\hat{\phi}$ corresponds a unique $\hat{H}_S$. Therefore, we can construct a functional relationship between $\hat{\phi}$ and $\hat{H}_S$, i.e., $\hat{\phi}(\hat{H}_S)$, which is invertible. Moreover, since $\psi'(\hat{H}_S) < 0$, then $\hat{\phi}'(\hat{H}_S) = \frac{\psi'(\hat{H}_S)\,\hat{\phi}(\hat{H}_S)}{\sqrt{\psi(\hat{H}_S)^2 + 4}} > 0$. This suggests that by increasing sufficiently ϕ it is possible to find value of $\hat{H}_S > 0$, which satisfies condition (A1.1).

As second step we verify that $\hat{H}_S$ is a maximum. We reason as follows.

Choose three values of ϕ close to each other and such that $\hat{\phi}_1 < \hat{\phi}_2 < \hat{\phi}_3$. Set $\phi = \hat{\phi}_1$, from (A1.5) the corresponding value of public expenditure that satisfies the condition $\frac{\partial \lambda^*}{\partial H_S} = PE + DE = 0$, is $\hat{H}_{S,1} = \hat{\phi}^{-1}(\hat{\phi}_1)$. Now, if we increase trade freeness from $\phi = \hat{\phi}_1$ to $\phi = \hat{\phi}_2$, at $H_S = \hat{H}_{S,1}$ we have that $PE > -DE$. In correspondence of the new value of trade freeness, the condition $\frac{\partial \lambda^*}{\partial H_S} = 0$ is satisfied at $\hat{H}_{S,2} = \hat{\phi}^{-1}(\hat{\phi}_2) > \hat{H}_{S,1}$.

Now set $\phi = \hat{\phi}_3$, the corresponding value of public expenditure that satisfies the condition $\frac{\partial \lambda^*}{\partial H_S} = PE + DE = 0$ is $\hat{H}_{S,3} = \hat{\phi}^{-1}(\hat{\phi}_3)$. If we decrease trade freeness from $\phi = \hat{\phi}_3$ to $\phi = \hat{\phi}_2$, at $H_S = \hat{H}_{S,3}$ it occurs that $PE < -DE$. In correspondence of the new value of trade freeness the condition $\frac{\partial \lambda^*}{\partial H_S} = PE + DE = 0$ is satisfied for $\hat{H}_{S,2} < \hat{H}_{S,3}$. Therefore, since for values of H_S on the left of $\hat{H}_{S,2}$ we have that $\frac{\partial \lambda^*}{\partial H_S} = PE + DE > 0$ and for values on the right we have that $\frac{\partial \lambda^*}{\partial H_S} = PE + DE < 0$, it follows that $\hat{H}_{S,2}$ maximises λ^*.

Q.E.D.

Appendix 2
Proof of Proposition 4

We proceed following the same steps as in Proposition 3. First, we search for a value of H_S which maximises $\lambda^* K_p^*$ when $s_F = s_L = s_E^*$ ($DE = 0$). That is, $\bar{H}_S$ should satisfy the following first order condition:

$$\frac{\partial}{\partial H_S}\left(\lambda^* K_P^*\right) = \left(PE\right) K_P^* + \frac{\partial K_P^*}{\partial H_S}\lambda^* = 0 \qquad \text{(A2.1)}$$

Given the complexity of the above expression which does not allow to solve for H_S, as before we search for a value of ϕ, denoted by $\bar{\phi}$, which satisfies (A2.1) for $H_S = \bar{H}_S$. If $\bar{\phi}$ exists, then also $\bar{H}_S$ satisfies condition (A2.1). Differentiating expression (A2.1) with respect to ϕ, we obtain

$$\left(\frac{\partial PE}{\partial \phi}\right) K_P^* + \frac{\partial K_P^*}{\partial H_S}\frac{\partial \lambda^*}{\partial \phi} \qquad \text{(A2.2)}$$

where $\frac{\partial PE}{\partial \phi} > 0$, $\frac{\partial K_P^*}{\partial H_S} < 0$ and

$$\frac{\partial \lambda^*}{\partial \phi} = z\left[\frac{s_E^*}{(1-\phi z)^2} - \frac{1-s_E^*}{(z-\phi)^2}\right] > (\leq 0) \quad \text{for} \quad s_E^* \geq (<0)\, \frac{(z-\phi)^2}{(1-\phi z)^2 + (z-\phi)^2}.$$

We need to distinguish two cases:

a) $s_E^* > \frac{(z-\phi)^2}{(1-\phi z)^2 + (z-\phi)^2}$. It follows that $\frac{\partial K_P^*}{\partial H_S}\frac{\partial \lambda^*}{\partial \phi} < 0$ and therefore that $-\frac{\partial K_P^*}{\partial H_S}\frac{\partial \lambda^*}{\partial \phi} > 0$. Since at $\phi = 0$, $PE = 0$ and $-\frac{\partial K_P^*}{\partial H_S}\lambda^* = -\frac{\partial K_P^*}{\partial H_S} s_E^* > 0$ and since both PE and λ^* have an asymptote at $\min(z, z^{-1})$, it follows that $\bar{\phi}$ may not exist or may not be unique;

b) $s_E^* \leq \dfrac{(z-\phi)^2}{(1-\phi z)^2+(z-\phi)^2}$, then $\dfrac{\partial K_P^*}{\partial H_S}\dfrac{\partial \lambda^*}{\partial \phi} \geq 0$; it follows that since at $\phi = 0$, $PE = 0$ and $-\dfrac{\partial K_P^*}{\partial H_S}\lambda^* = -\dfrac{\partial K_P^*}{\partial H_S} s_E^* > 0$ and since both PE and λ^* have an asymptote at min(z, z^{-1}), $(PE)K_p^*$ and $-\dfrac{\partial K_P^*}{\partial H_S}\lambda^*$ meet necessarily once at $\bar{\phi}$. However, given the complexity of expression (A2.2) we cannot solve explicitly for $\bar{\phi}$.

In what follows, we confine our demonstration to the special case corresponding to a value of z close to 1 (and therefore to a value of H_S close to H_N).

When $z = 1$, the value of $\bar{\phi}$ which satisfies expression (A2.1) solves also the following second degree equation:

$$-(1-s_L)\theta\phi^2 + \left(\theta - \frac{\partial z}{\partial H_S}\right)\phi - s_L\theta = 0$$

where $\theta = \dfrac{a_I^N \delta}{L - H_S a_I^N \delta}$.

There is only one value of $0 < \phi < 1$, which satisfies the above expression, that is:

$$\bar{\phi} = \alpha - \sqrt{\alpha^2 + \beta} \tag{A2.3}$$

where $\alpha = \dfrac{1}{2(1-s_E^*)}\left(1 - \dfrac{\partial z}{\partial H_S}\dfrac{1}{\theta}\right)$ and $\beta = \dfrac{s_E^*}{1-s_E^*}$.

From (A2.3) it is possible to deduce that there exists a functional relationship between $\bar{\phi}$ and $\bar{H}_S$ which is invertible, i.e. $\bar{\phi}(\bar{H}_S)$. This follows from the fact that $\dfrac{\partial z}{\partial H_S}$ is a monotonically increasing function of H_S. Moreover, since $\dfrac{\partial^2 z}{\partial H_S^{\,2}} > 0$, it follows $\alpha'(\bar{H}_S) < 0$ and therefore $\bar{\phi}'(\bar{H}_S) > 0$.

Second, given the properties of $\bar{\phi}(\bar{H}_S)$, the same reasoning adopted in proposition 3 applies. It follows that $\bar{H}_S$ maximises λ^*.

Q.E.D.

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