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Volatility dynamics of the UK business cycle: A multivariate asymmetric GARCH approach

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Abstract. This paper analyses the volatility dynamics of the UK business cycle by proposing four new multivariate asymmetric GARCH models that not only capture asymmetric volatility but also time-varying correlations. The results indicate the existence of asymmetric volatility, but it is sensitive to the structure of the conditional variance. It is also found that correlations and volatility are usually higher around the recession phase of the UK economy. These have important implications for macroeconomic policy and forecasting for business cycle.

JEL Classification: E32; E37.
Keywords: Business Cycle Asymmetries; Constant Correlations; Multivariate Asymmetric GARCH; Time-Varying Correlations.

Résumé. Cet article étudie la dynamique de la volatilité du cycle au Royaume-Uni. À cette fin, l'étude s'appuie sur quatre nouveaux modèles GARCH multivariés qui prennent en compte non seulement le caractère asymétrique de la volatilité, mais aussi les corrélations variant dans le temps. Les résultats montrent que la volatilité est asymétrique, mais sensible à la structure de la variance conditionnelle. Les corrélations et la volatilité sont en général plus élevées au cours de la phase de récession. Ces différents points ont des conséquences importantes en matière de politique macroéconomique et de prévision faites sur le cycle.

Classification *JEL* : E32 ; E37.
Mots-clefs : Asymétrie du cycle ; corrélations constantes ; GARCH asymétrique multivarié ; corrélations variant dans le temps.

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1. Introduction

Two recurrent themes in macroeconomics are the existence of asymmetries or non-linearities in business cycles and the co-movements of macroeconomic variables. Knowing whether business cycles are asymmetric, understanding how the aggregate variables behave and characterizing the conditional variations between the GDP and its components is important for both academics and policymakers. Indeed, as suggested by Diebold and Rudebusch (1996), a successful model for business cycles should not only consider the non-linear behaviour of aggregate variables but also their co-movements.

A voluminous literature has developed on output and macroeconomic volatility and their implications for consumption, growth, and welfare. Among others, see Lucas (1987), Schwert (1989), Kose *et al.* (2003), Ramey and Ramey (1995), Loayza *et al.* (2005), Calderon and Fuentes (2006), Dopke *et al.* (2006), Paye (2006), and McKibbin *et al.* (2008). There are also studies which employ calibrating models to explain the variances and co-variances of macroeconomic variables, such as real output (see Kydland and Prescott, 1982; Canning *et al.*, 1988; Backus *et al.*, 1992). For example, Head (1995) and Canning *et al.* (1998) find evidence of a negative relationship between the volatility of real output and the country size.

Most of the aforesaid papers concentrate on the unconditional volatility of macroeconomic variables and do not capture temporal variations over time. Asymmetries in volatility (unlike business cycle asymmetries) and sectoral variations conditional on the available information are often ignored in these studies. However, it is important to examine the temporal variations of macroeconomic variables for at least two reasons. First, a deeper understanding of the issues of conditional heteroskedasticity and volatility asymmetry of aggregate output is important because of their implications on macroeconomic theory and forecasting. If aggregate output is conditionally heteroskedastic and exhibits volatility asymmetry, then any theory assuming the absence of either of these properties is most likely inadequate (see Valderrama, 2001). Second, economic theory frequently suggests that economic agents respond not only to the mean conditional on the available information, but also to higher moments of economic random variables such as conditional variance (see Engle, 1982). Some articles attempt to capture the property of asymmetric conditional volatility of real output include French and Sichel (1993), Brunner (1993), Henry and Olekalns (2000), Hamori (2000), and Ho and Tsui (2003). In particular, French and Sichel (1993) have detected significant cyclical asymmetry in the volatility of US real GNP. Similarly, Henry and Olekalns (2000) have noted that output volatility is the highest when the economy is contracting. One major drawback of these studies is the failure to examine how the variables are correlated over time, a salient feature emphasised by business-cycle researchers.

In this paper, we propose a variety of multivariate asymmetric generalised autoregressive conditional heteroskedasticity (GARCH) models that are capable of capturing not only non-linearities in the conditional variance of macroeconomic variables but also their time-varying correlations. We shall apply these models to three sectoral Index of Industrial Production (IIP) of the United Kingdom. The reason is that, to the best of our knowledge, there are so far no studies on the asymmetric volatility of the UK aggregate output and the correlations among various sectors. Moreover, even though the business cycle has become less volatile in the UK during the past decades compared with previous post-war business cycles, the key unanswered questions remains its sources and how the various sectors are correlated. This study reveals that correlations among the sectors are time-varying, and negative shocks can generate higher volatilities in the future compared to positive shocks of the same magnitude in some sectors, even though this can be sensitive to the model specified. This finding has important implications for macroeconomic policy and forecasting for business cycle.

The rest of the paper is organised as follows. Section 2 discusses the methodology employed in this paper, whilst Section 3 focuses on the data sets and the empirical results. Concluding remarks are given in Section 4.

2. Methodology

The problems of using multivariate GARCH models are that they not only increase the number of parameters to be estimated but also complicate the specifications of the conditional variance-covariance matrix. To circumvent these problems, Bollerslev (1990) introduced a multivariate GARCH model assuming constant correlations. Though this model satisfies the positive-definite condition for the variance-covariance matrix, its validity has been rejected in many contexts (Tsui and Yu, 1999). Furthermore, it fails to demonstrate how the correlations of the variables evolve over time. In addition, Bollerslev's (1990) constant-correlation GARCH (CC-GARCH) model does not capture asymmetries in the conditional variance.

In what follows, we propose a family of multivariate GARCH models by synthesising the methodologies of Tse and Tsui's (2002) varying-correlation GARCH (VC-GARCH)[2], Ding, Engle, and Granger's (1993) asymmetric power ARCH (APARCH), and Sentana's (1995) quadratic GARCH (QGARCH). Let $y_t = (\nu_{1t},\ \nu_{2t})'$ be the bivariate vector of interest with time-varying covariance matrix H_t, and let $\mu_t(\xi)$ be the arbitrary mean functions which depend on ξ, a column vector of parameters. Consider a trivariate GARCH model:

$$y_{it} = \mu_{it}(\xi)+\varepsilon_{it}, \quad i = 1,\ 2,\ 3 \tag{1}$$

$$\text{where } (\varepsilon_{1t},\varepsilon_{2t},\varepsilon_{3t})' \mid \Phi_{t-1} \sim MN\,(O,H_t) \tag{2}$$

2. We are grateful to one referee for pointing out the similarity between the Dynamic Conditional Correlations (DCC) model proposed by Engle (2002) and the Time Varying-Correlations (VC) model proposed by Tse and Tsui (2002), both published in the same issue of *Journal of Business and Economic Statistics*. In terms of parameter estimation, VC models jointly estimate the conditional variance and correlation equations whereas DCC models separate the estimation of the conditional mean and variance equations by using a two-stage process. We have also attempted to estimate the 2-step DCC and noted that the parameter estimates are somewhat similar to those reported in our paper. This is unsurprising, since we are already able to obtain convergence for the 1-step procedure of the VCC. In the case of the DCC, we have to go through the extra step of modifying the standard errors of the conditional correlation parameters in order to ensure the reliability of our statistical inference (see Engle and Sheppard, 2001).

Note that Φ_{t-1} is the σ-algebra generated by all the available information up to time (t–1). The random disturbance terms ε_{it} and the conditional variance equations h_{iit} are modelled as follows:

$$\varepsilon_{it} = \sqrt{h_{iit}}\, e_{it} \quad \text{where } \varepsilon_{it} \sim N(0,1), \quad i = 1,2,3 \tag{3}$$

$$h_{iit} = \eta_i + \alpha_i\, \varepsilon^2_{it-1} + \beta_i h_{iit-1}\,, \quad i = 1,2,3 \tag{4}$$

Denoting the ij-th element (i, j = 1,2,3) in H_t by h_{ijt}, the conditional correlation coefficients are given by $\rho_{ijt} = h_{ijt} \Big/ \sqrt{h_{iit}\, h_{ijt}}$. Tse and Tsui (2002) assumes that the time-varying conditional correlation matrix Γ_t is generated by the following recursion:

$$\Gamma_{t\,=}(1-\pi_1-\pi_2)\Gamma + \pi_1\Gamma_{t-1} + \pi_2\Psi_{t-1} \tag{5}$$

where $\Gamma_t = \begin{pmatrix} 1\rho_{12t}\,\rho_{13t} \\ \rho_{21t}\,1\,\rho_{23t} \\ \rho_{31t}\,\rho_{32t}1 \end{pmatrix}$, $\Gamma = \{\rho_{ij}\}$ is a (time-invariant) positive-definite correlation matrix, π_1 and π_2 are assumed to be nonnegative and sum up to less than 1, and Ψ_t is a function of standardised residuals. Denoting $\Psi_t = \{\Psi_{ijt}\}$, the elements of Ψ_{t-1} are specified as:

$$\Psi_{ij,t-1} = \frac{\sum_{h=1}^{M} e_{i,t-h}\, e_{j,t-h}}{\sqrt{(\sum_{h=1}^{M} e^2_{i,t-h})(\sum_{h=1}^{M} e^2_{j,t-h})}} \tag{6}$$

where M is set equal to the dimension of the GARCH model. The conditional log likelihood function is specified as:

$$l_t(\theta) = -\frac{1}{2}\log|H_t| - \frac{1}{2}(\varepsilon_{1t},\varepsilon_{2t},\varepsilon_{3t})\, H_t^{-1}(\varepsilon_{1t},\varepsilon_{2t},\varepsilon_{3t})' \tag{7}$$

where the conditional variance-covariance matrix H_t can be defined as:

$$H_t = \begin{pmatrix} h_{11t}\,h_{12t}\,h_{13t} \\ h_{21t}h_{22t}h_{23t} \\ h_{31t}h_{32t}h_{33t} \end{pmatrix} = D_t\,\Gamma_t D_t\,, \quad \text{and } D_t = \begin{pmatrix} \sqrt{h_{11t}}\,0\;0 \\ 0\;\sqrt{h_{22t}}\,0 \\ 0\;0\;\sqrt{h_{33t}} \end{pmatrix}$$

Consequently, the log likelihood can be rewritten as:

$$l_t(\theta) = -\frac{1}{2}\log|D_t\Gamma_t D_t| - \frac{1}{2}(\varepsilon_{1t},\varepsilon_{2t},\varepsilon_{3t})\, D_t^{-1}\Gamma_t^{-1}D_t^{-1}(\varepsilon_{1t},\varepsilon_{2t},\varepsilon_{3t})' \tag{8}$$

with Γ_t being defined using the recursion in (5). The foregoing equations (1) - (8) thus define the Varying-Correlations (VC)-GARCH model of Tse and Tsui (2002). Note that VC-GARCH nests

Bollerslev's (1990) CC-GARCH when $\pi_1 = \pi_2 = 0$. As such, we can apply the likelihood ratio test to compare the performance of both models.

To incorporate asymmetric volatility in the VC-GARCH model, we modify the conditional variance equation in (4). The modifications we demonstrate below have the common advantage that they are less restrictive and nest several popular GARCH models in the literature. Details can be found in Sentana (1995), and Ding, Engle, and Granger (1993). One modification advocated by Sentana is the quadratic GARCH (QGARCH) model:

$$h_{iit} = \eta_i + \gamma_i \ \varepsilon_{it-1} + \alpha_{it-1}\,\varepsilon^2_{it-1} + \beta_i \ h_{iit-1}\,, \quad i = 1,2,3 \tag{9}$$

where γ is the asymmetric coefficient. As highlighted by Sentana (1995), a piecewise-quadratic spline approximation to the unknown conditional variance function would encompass QGARCH as a trivial smooth example, as well as the models of Taylor/Schwert (1986/1989), and Zakoian (1994). Therefore, QGARCH may provide a useful benchmark to assess the relative performance of these models.

Another modification to (4) is based on Ding, Engle, and Granger's (1993) asymmetric power ARCH (APARCH) model:

$$\begin{aligned} \varepsilon_{it} &= h_{iit}\, e_{it},\, e_{it} \sim N(0,1)\,, \quad i = 1,2,3 \\ h^{\delta}_{iit} &= \eta_i + \alpha_i \ (|\,\varepsilon_{it-1}| - \gamma\varepsilon_{it-1})^{\delta} + \beta_i \ h^{\delta}_{iit-1} \end{aligned} \tag{10}$$

When $\delta = 1$, this is the Threshold GARCH (TGARCH) model, which incorporates an asymmetric version of the Taylor/Schwert (1986/1989) model and Zakoian's (1994) Threshold ARCH (TARCH) model. Alternatively, as shown in Ding, Engle, and Granger (1993), when δ approaches 0, the logarithmic GARCH (LOGGARCH) model is obtained, and it incorporates an asymmetric version of the Geweke/Pantula (1986) model:

$$\log h_{iit} = \eta_i + \log(|\,\varepsilon_{it-1}| - \gamma\varepsilon_{it-1}) + \beta_i \ \log\ h_{iit-1}\,, \quad i = 1,2,3 \tag{11}$$

When δ is not restricted to any positive value, this is Ding *et al.*'s (1993) APARCH model.

3. Data and results

Our data sets comprise the three main sectoral IIP (Index of Industrial Production) series of UK: Intermediate Goods (INT), Investment Goods (INV), and Manufacturing (MFC). These (seasonally adjusted) series are obtained on-line from OECD's *Main Economic Indicators* and cover the period from January 1968 to December 2007 (480 observations).[3] [4] To calculate the monthly growth rates of these series on a continuously compounded basis, we take the first difference of the logarithmic IIP:

$$y_{it} = \log(\frac{Y_{it}}{Y_{it-1}}) \times 100, \quad i = 1,2,3 \tag{12}$$

where Y=the seasonally adjusted monthly IIP. Assume further that the conditional mean equation specified in (1) follows an autoregressive AR(k) structure:

$$y_{it} = \xi_o + \sum_{j=1}^{k} \xi_j \, y_{it-j} + \varepsilon_{it}, \quad i = 1,2,3 \tag{13}$$

Table 1 suggests that both linear and non-linear dependencies are detected in all the IIP series. In particular, the BDS test proposed by Brock *et al*. (1996) shows that all the series are not IID at the 1% level. As argued by Hsieh (1993), such departures from IID may be ascribed to the presence of conditional heteroskedasticity in all the series.

Before delving into the estimation results, we employ the augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) tests to check the stationarity of all the IIP growth rate series. Our findings, available upon request, show that all ADF and PP test statistics are significant at the 1% level, thereby indicating that all the series are stationary.

Based on a number of factors such as parsimony and residual diagnostics, we adopt the AR(1) model for the conditional mean equation. We have also experimented with longer lags but results are similar to those reported here. Due to space limitations, we only report in table 2 the results for the conditional variance and correlations equations from the VC-GARCH, VC-QGARCH, VC-LOGGARCH, VC-TGARCH, and VC-APARCH models. In addition, table 2 also reports the log-likelihood values and correlation coefficients obtained from the corresponding constant-correlations models to facilitate a comparison between the VC and CC approach (the complete results are available from the authors upon request).

3. We have extended the datasets to December 2007 in order to determine the robustness of our results. We did not include the more recent observations as they are subject to revision by OECD. In general, our original results still remain relatively robust.

4. Thanks to one of the referees for pointing out the issue of deterministic and stochastic seasonality. Most of the OECD countries (including UK) employ X11 ARIMA or X12 ARIMA model to calculate seasonally adjusted series (see Hong and Chavoix and Mannato, 2000). In particular, the UK IIP series in our paper are seasonally adjusted according to the X11 ARIMA method (UK version), not the method of deterministic seasonality. As such, stochastic seasonality has already been taken care of. The possible presence of deterministic seasonality in the series can be removed by including dummy variables in the conditional mean equation (see Pierce, 1978). This procedure has been implemented separately and we notice that the parameter estimates are generally similar to those we have reported in our paper, which is also consistent with the asymptotic result of Nelson and Foster (1994) that in general the conditional mean specification does not have significant impact on the specification of the conditional variance. Studies such as French and Sichel (1993), Hamori (1998), and Ho and Tsui (2003) also use seasonally adjusted data from sources like OECD and IMF to model business cycle fluctuations. As noted by Sims (1990), it is usually true that rational expectations modelling in macroeconomics with seasonally adjusted data (treating the adjusted data as if it were actual data) gives approximately correct results, and naïve extensions of standard modelling techniques to seasonally unadjusted data may give worse results than naïve use of adjusted data.

Table 1 - Summary statistics of the UK IIP growth rates, 1968-2007

Variables	INT	INV	MFC
Panel A: Moments, maximum, minimum			
Mean	0.0010	0.0010	0.0006
Median	0.0011	0.0000	0.0011
Maximum	0.1727	0.0643	0.0895
Minimum	−0.1529	−0.0885	−0.1013
Standard Deviation	0.0204	0.0180	0.0148
Skewness	0.1600	−0.3889	−0.7599
Kurtosis	22.0286	6.0789	14.2319
Observations	479	479	479
Panel B: Jarque-Bera test			
Test Statistic	7228.6856**	201.2743**	2563.9524**
Panel C: Ljung-Box Q-statistic			
4 lags	18.4873**	17.3453**	35.7008**
8 lags	23.8259**	26.4795**	44.7457**
12 lags	28.2916**	37.0562**	49.6019**
Panel D: McLeod-Li test			
4 lags	112.5710**	45.1429**	92.0526**
8 lags	113.1199**	73.9381**	97.0250**
12 lags	113.2979**	88.6533**	115.1539**
Panel E: ARCH LM test			
4 lags	127.5105**	38.0435**	118.2648**
8 lags	128.4300**	54.0368**	120.8293**
12 lags	128.8417**	57.2243**	128.3589**
Panel F: QARCH LM test			
1 lag	159.1991**	33.9545**	95.5265**
4 lags	238.4979**	57.1335**	136.1378**
Panel G: BDS test			
e=3, l=1.5	7.8320**	4.8423**	7.2643**
e=4, l=1.5	7.8193**	5.3216**	7.4278**
e=5, l=1.5	7.8674**	5.3907**	7.4092**
e=3, l=1.0	7.5407**	3.4622**	8.0061**
e=4, l=1.0	7.8478**	3.9281**	8.9298**
e=5, l=1.0	8.4557**	3.9743**	9.5681**
Panel H: Runs test			
R1	1.2641	1.0545	2.0044*
R2	−4.4579**	−0.6418	−3.9235**
R3	−3.2681**	−1.3825	−5.5900**

Notes:

1. INT = Intermediate Goods, INV = Investment Goods, MFC = Manufacturing.

2. The Jarque-Bera statistic follows the chi-square distribution with 1 degree of freedom.

3. The QARCH LM test statistic is due to Sentana (1995) and it is distributed as chi-squared with $q(q+3)/2$ degrees of freedom, where q is the number of lags.

4. For the BDS Test, e represents the embedding dimension whereas l represents the distance between pairs of consecutive observations, measured as a multiple of the standard deviation of the series. Under the null hypothesis of independence, the test statistic is asymptotically distributed as standard normal.

5. For the Runs Test, R_i for i = 1, 2, 3 denote the runs tests of the series R_t, $|R_t|$, and R_t^2 respectively. Under the null hypothesis that successive observations in the series are independent, the test statistic is asymptotically standard normal.

6. * and ** stand for significance at the 5% and 1% level respectively.

Table 2 - Estimation results of GARCH, QGARCH, LOGGARCH, TGARCH, and APARCH models

Variables	η	β	α	γ	δ	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
Panel A: *GARCH* $h_t = \eta + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$												
INT	1.2850	0.2901	0.3183	-	-	$\rho_{int,\,inv}$ = 0.4470	0.9431	0.0211	–994.1406	$\rho_{int,\,inv}$ = 0.3893	–1002.7147	17.1482**
	(0.3139)	(0.0979)	(0.0840)			(0.0698)	(0.0193)	(0.0074)		(0.0495)		
INV	0.2388	0.8246	0.1032	-	-	$\rho_{int,\,mfc}$ = 0.6919				$\rho_{int,\,mfc}$ = 0.6312		
	(0.0957)	(0.0403)	(0.0280)			(0.0724)				(0.0494)		
MFC	0.0777	0.8680	0.0940	-	-	$\rho_{inv,\,mfc}$ = 0.8109				$\rho_{inv,\,mfc}$ = 0.7647		
	(0.0540)	(0.0451)	(0.0340)			(0.0361)				(0.0273)		
Panel B: *QGARCH* $h_t = \eta + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} - \gamma\varepsilon_{t-1}$												
INT	1.2107	0.3720	0.2645	0.3726	-	$\rho_{int,\,inv}$ = 0.4727	0.9453	0.0225	–988.2249	$\rho_{int,\,inv}$ = 0.3973	–998.7054	20.961**
	(0.2702)	(0.0900)	(0.0724)	(0.1928)		(0.0799)	(0.0137)	(0.0072)		(0.0534)		
INV	0.2571	0.8238	0.1015	0.1070	-	$\rho_{int,\,mfc}$ = 0.7129				$\rho_{int,\,mfc}$ = 0.6344		
	(0.1006)	(0.0430)	(0.0301)	(0.0883)		(0.0764)				(0.0502)		
MFC	0.0665	0.8887	0.0793	0.0620	-	$\rho_{inv,\,mfc}$ = 0.8202				$\rho_{inv,\,mfc}$ = 0.7669		
	(0.0392)	(0.0362)	(0.0298)	(0.0638)		(0.0373)				(0.0269)		
Panel C: *LOGGARCH* $\log h_t = \eta + \log(-\lvert\varepsilon_{t-1}\rvert - \gamma\varepsilon_{t-1}) + \beta \log h_{t-1}$												
INT	0.4419	0.3879	0.1522	0.6879	-	$\rho_{int,\,inv}$ = 0.4468	0.9590	0.0101	–1004.9755	$\rho_{int,\,inv}$ = 0.4053	–1008.0890	6.227*
	(0.1024)	(0.1308)	(0.0332)	(0.1492)		(0.0629)	(0.0189)	(0.0060)		(0.0487)		
INV	0.0705	0.8912	0.0475	0.3118	-	$\rho_{int,\,mfc}$ = 0.6844				$\rho_{int,\,mfc}$ = 0.6442		
	(0.0181)	(0.0265)	(0.0102)	(0.2969)		(0.0520)				(0.0402)		

Table 2 - continued

Variables	η	β	α	γ	δ	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
MFC	0.0580	0.9464	0.0385	0.8098	-	$\rho_{inv, mfc}$ = 0.7971				$\rho_{inv, mfc}$ = 0.7714		
	(0.0151)	(0.0102)	(0.0084)	(0.1561)		(0.0314)				(0.0226)		
Panel D: *TGARCH* $h_t = \eta + \alpha(\lvert\varepsilon_{t-1} - \gamma\varepsilon_{t-1}\rvert) + \beta h_{t-1}$												
INT	0.6243	0.4587	0.2499	0.4430	-	$\rho_{int, inv}$ = 0.4666	0.9476	0.0185	–980.9058	$\rho_{int, inv}$ = 0.4022	–988.5937	15.3758**
	(0.1423)	(0.0972)	(0.0562)	(0.1606)		(0.0715)	(0.0161)	(0.0068)		(0.0514)		
INV	0.1301	0.8449	0.1058	0.3250	-	$\rho_{int, mfc}$ = 0.7050				$\rho_{int, mfc}$ = 0.6392		
	(0.0445)	(0.0340)	(0.0240)	(0.2306)		(0.0669)				(0.0478)		
MFC	0.0367	0.9038	0.0980	0.3847	-	$\rho_{inv, mfc}$ = 0.8119				$\rho_{inv, mfc}$ = 0.7706		
	(0.0205)	(0.0260)	(0.0254)	(0.2205)		(0.0339)				(0.0245)		
Panel E: *APARCH* $h_t^\delta = \eta + \alpha(\lvert\varepsilon_{t-1}\rvert - \gamma\varepsilon_{t-1})^\delta + \beta h_{t-1}^\delta$												
INT	0.7372	0.4343	0.2562	0.3842	1.2604	$\rho_{int, inv}$ = 0.4638	0.9462	0.0191	–980.5561	$\rho_{int, inv}$ = 0.4013	–988.1435	15.1748**
	(0.2996)	(0.1188)	(0.0664)	(0.1810)	(0.4720)	(0.0710)	(0.0160)	(0.0078)		(0.0518)		
INV	0.1289	0.8451	0.1062	0.3299	0.9958	$\rho_{int, mfc}$ = 0.7015				$\rho_{int, mfc}$ = 0.6366		
	(0.0693)	(0.0361)	(0.0252)	(0.2398)	(0.6579)	(0.0658)				(0.0473)		
MFC	0.0364	0.9015	0.1017	0.4019	0.9503	$\rho_{inv, mfc}$ = 0.8120				$\rho_{inv, mfc}$ = 0.7711		
	(0.0221)	(0.0297)	(0.0285)	(0.4008)	(0.6321)	(0.0336)				(0.0240)		

Note: LL (VC) and LL (CC) refer to the log-likelihood values obtained from the VC and CC models respectively. Corr (CC) refers to the conditional correlation coefficients from the CC models, LR refers to the likelihood ratio test statistic and it is asymptotically distributed as chi-squared with 2 degrees of freedom. All standard errors are computed based on the Bollerslev-Wooldridge (1992) heteroskedastic-consistent errors.

* and ** means statistical significance at the 5% and 1% respectively.

The results indicate the existence of varying correlations among the different sectors. In most cases, π_1 and π_2 are individually significant at the 5% level, and the likelihood ratio test suggests that the VC models outperform the restrictive CC models. The exception is the VC-LOGGARCH model, whose π_2 is significant only at the 10% level. This finding is consistent with our casual observation that the lower volatility in both price and output and quicker recovery during the recent UK recession are essentially driven by private consumption and investment, in addition to the flexible regulation of labour, product and financial markets. The recent data indicate that the recovery taking place in the UK economy was initially fuelled by a sharp increase in durable consumption from the last quarter of 1992, and then a sharp pick up in residential investment from the second quarter of 1993. Using a vector-autoregression (VAR) approach, Catao and Ramaswamy (1996) also find that the recent recession in the UK was precipitated primarily by shocks to consumption and investment, and consumption shocks have a long lasting impact on the economy. Hence, consumption was expected to drive the recovery in the initial stages once these negative shocks dissipate away.

The log-likelihood values also indicate that the VC-TGARCH model outperforms the rest of the models. Even though the VC-APARCH model has a slightly higher log-likelihood value than VC-TGARCH, the difference is statistically insignificant, which can be shown using the likelihood ratio test. As noted by Nelson and Foster (1994), the TGARCH model is a consistent estimator of the conditional variance of near diffusion processes, and, in the presence of leptokurtic error distributions, the TGARCH model is a more efficient filter of the conditional variance than Bollerslev's (1986) GARCH model.

As regards the existence of asymmetric volatility, the coefficient γ is significant at the 5% level for the Intermediate Goods (INT) series in the VC-QGARCH, VC-TGARCH, VC-LOGGARCH and VC-APARCH models. However, for the Manufacturing (MFC) sector, significant asymmetric volatility is detected only in the VC-LOGGARCH model.

On the diagnostics front, we employ a battery of diagnostic tests, such as the Ljung-Box Q-statistic and the runs test, to check for model adequacy (the results are available upon request). It is found that most Ljung-Box Q-statistics based on the cross product of the standardised residuals suggest the absence of serial correlation. However, the diagnostic results for the constant-correlation models are less favourable, as there is some evidence of serial correlation in the cross product of the standardised residuals.

Finally, the upper panels of FIGURES 1 and 2 show that volatility is usually higher around the recessionary phase of the UK business cycle, such as the periods of 1974-1976 and 1979-1981. This accords with the view that economic downturns are periods of increased uncertainty and volatility. Furthermore, output volatility has declined in the 1980s and 1990s compared with the early 1970s, and this is consistent with the recent finding of Stock and Watson (2002). Undoubtedly the structural flexibility and the stabilization and growth policy implemented in the recent years have contributed to this low volatility of business cycle. On the other hand, as shown in the lower panels of FIGURES 1 and 2, correlations are usually stronger when output volatility is higher. In passing, we note that this observation further reinforces the advantage of a multivariate GARCH model with time-varying correlations, because it would not be obvious in a constant- correlation framework.

Figure 1 - Conditional standard deviation and conditional correlations: VC-APARCH

Figure 1a – Conditional standard deviation: VC-APARCH

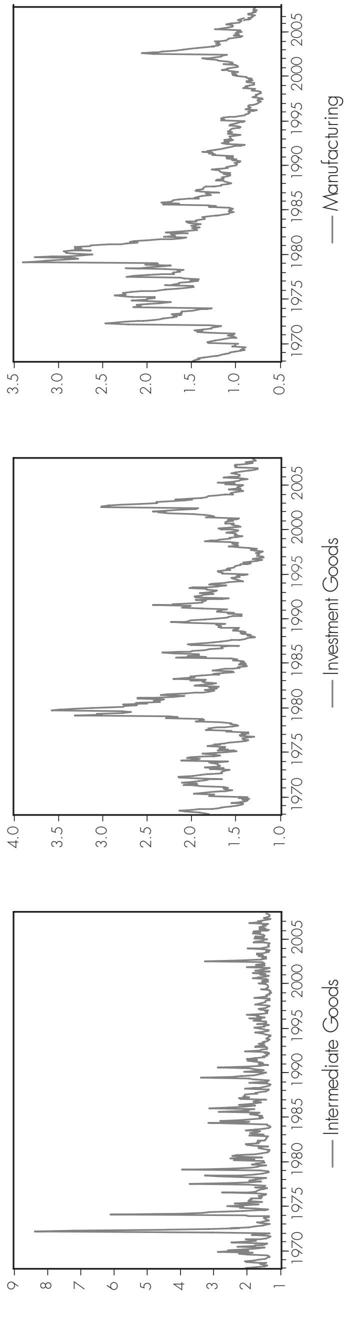


Figure 1b – conditional correlation: VC-APARCH

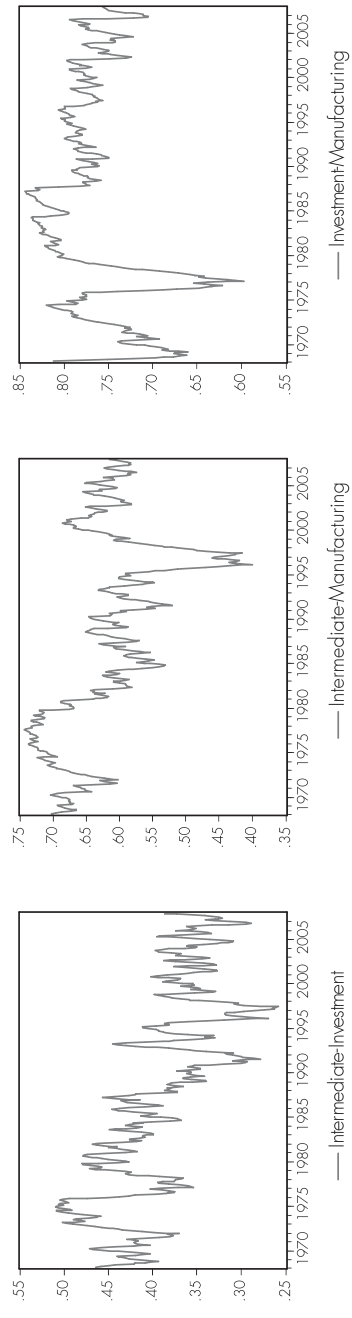
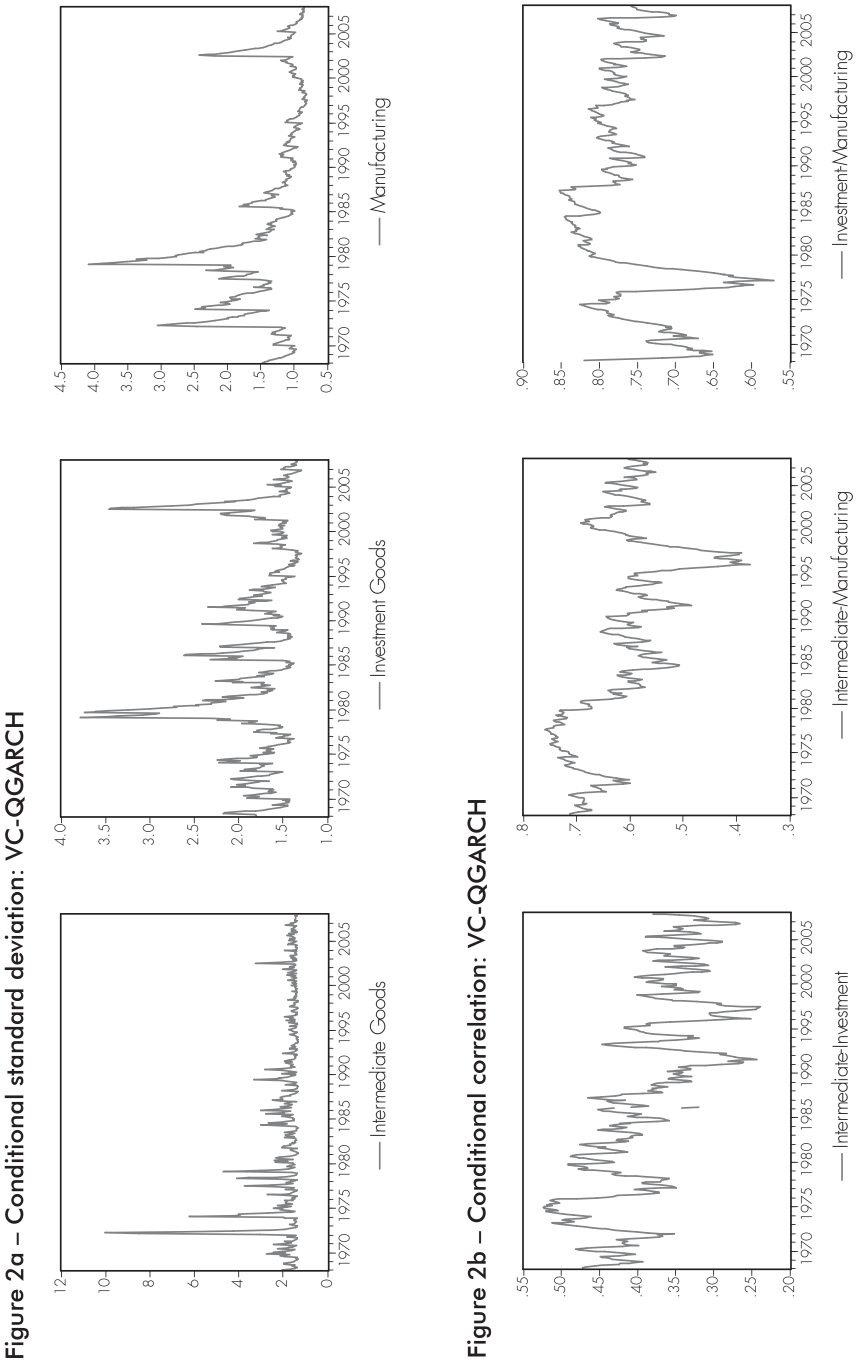


Figure 2 - Conditional standard deviation and conditional correlations: VC-QGARCH



4. Conclusion

We have employed a variety of multivariate asymmetric GARCH models with time-varying correlations to characterise the output volatility dynamics of three UK sectors. The results indicate the existence of asymmetric volatility, and it is sensitive to the structure of the conditional variance. The correlations and volatility of the variables are found usually higher around the recession phase of the UK economy. This finding is consistent with our casual observation that the lower volatility in both price and output and quicker recovery during the recent UK recession are essentially driven by private consumption and investment, in addition to the flexible regulation of labour, product and financial markets. It is indicated in the recent data that the recovery was initially fuelled by a sharp increase in durable consumption from the last quarter of 1992, followed up by a sharp pick up in residential investment from the second quarter of 1993. The findings of Catao and Ramaswamy (1996) also confirm that the recent recession in the UK was precipitated primarily by shocks to consumption and investment, and hence consumption was expected to drive the recovery in the initial stages once these negative shocks dissipate away.

The estimated conditional-correlation path provides an interesting time history that would otherwise be lost in a constant-correlation framework.

The finding of asymmetric volatility has important policy implications. When negative output shocks induce greater future volatilities, it may vindicate the government's role in stabilising the macroeconomic environment during recessions. This is because the adverse impact of negative shocks could be mitigated through effective counter-cyclical measures. Furthermore, negative economic disturbances arising from one sector may spill over to another sector through strong sectoral linkages. As such, economic policy co-ordination becomes imperative to ameliorate the effect of shocks originating from one sector.

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References

Backus, D.K., Kehoe, P.J., Kydland, F.E., 1992. International real business cycles, *Journal of Political Economy* 101, 745-75.

Beaudry, P., Koop, G., 1993. Do recessions permanently change output?, *Journal of Monetary Economics* 31, 149-63.

Bollerslev, T., 1986. Generalised autoregressive conditional heteroskedasticity, *Journal of Econometrics* 31, 307-27.

Bollerslev, T., 1990. Modelling the coherence in short-run nominal exchange rates: A multivariate generalised ARCH model, *Review of Economics and Statistics* 72, 498-505.

Bollerslev, T., Wooldridge, J., 1992. Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances, *Econometrics Review* 12, 143-72.

Brock, W., Dechert, D., Sheinkman, J., Le Baron, B., 1996. A test for independence based on the correlation dimension, *Econometrics Review* 15, 197-235.

Brunner, A., 1997. On the dynamic properties of asymmetric models of real GNP, *Review of Economics and Statistics* 79, 321-26.

Calderon, C., Fuentes, R., 2006. Characterizing the business cycles of emerging economies, Center for Economic Policy Research, London.

Canning, D., Amaral, L., Lee, Y., Meyer, M., Stanley, H.E., 1998. Scaling the volatility of GDP growth rates, *Economics Letters* 60, 335-41.

Catao, L., Ramaswamy, R., 1996. Recession and recovery in the United Kingdom in the 1990s: A vector autoregression approach, in Diebold, F., Rudesbusch, G. (Eds.), Measuring Business Cycles: A Modern Perspective, *Review of Economics and Statistics* 78, 67-77.

Ding, Z., Engle, R., Granger, C., 1993. A long memory property of stock market returns and a new model, *Journal of Empirical Finance* 1, 83-106.

Dopke, J., Hartmann, D., Pierdzioch, C., 2006. Forecasting stock market volatility with macroeconomic variables in real time, Deutsche Bundesbank Discussion Paper 01/2006, Series 2: Banking and Financial Studies.

Engle, R., 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of the UK inflation, *Econometrica* 50, 987-1008.

Engel, R., 2002. Dynamic conditional correlation - A simple class of multivariate GARCH models, *Journal of Business and Economic Statistics* 20, 339-50.

Engle, R., Sheppard, K., 2001. Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH, NBER Working Paper 8554, National Bureau of Economic Research, Inc.

Geweke, J., 1986. Modelling the persistence of conditional variances: Comment, *Econometrics Review* 5, 57-61.

Hamori, S., 2000. Volatility of real GDP: Some evidence from the United States, the United Kingdom, and Japan, *Japan and the World Economy* 12, 143-52.

Head, A., 1995. Country size, aggregate fluctuations, and international risk sharing, *Canadian Journal of Economics* 28(4), 1096-1119.

Hsieh, D.A., 1993. Implications of non-linear dynamics for financial risk management, *Journal of Financial and Quantitative Analysis* 28, 41-64.

Kose, M., Prasad, E., Terrones, M., 2003. Financial integration and macroeconomic volatility, *IMF Staff Papers* 50.

Kydland, F.E., Prescott, E.C., 1986. Time to build and aggregate fluctuations, *Econometrica* 50, 1345-70.

Loayza, N., Oviedo, A.M., Serven, L., 2005. Regulation and macroeconomic performance, Policy Research Working Paper 3469, World Bank, Washington, DC.

Lucas, R.E., 1987. *Models of Business Cycles*, New York: Basil Blackwell.

McKibbin, W., Morris, A., Wilcoxen, P., 2008. Expecting the unexpected: Macroeconomic volatility and climate policy, The Harvard Project on International Climate Agreements, Harvard Kennedy School Discussion Paper 08-16, November.

Nelson, D., Foster, D., 1994. Asymptotic filtering theory for univariate ARCH models, *Econometrica* 62, 1-41.

Pantula, S., 1986. Modelling the persistence of conditional variances: Comment, *Econometrics Review* 5, 79-97.

Paye, B., 2006. Do macroeconomic variables predict aggregate stock market volatility?, Rice University Working Paper.

Pierce, D.A., 1978. Seasonal adjustment when both deterministic and stochastic seasonality are present, Zellner, A. (Eds.), *Seasonal Analysis of Economic Time Series*, Proceedings of a Bureau of the Census-NBER-ASA Conference, U.S. Department of Commerce, U.S. Bureau of the Census, Washington, D.C.

Ramey, G., Ramey, V.A., 1995. Cross-country evidence on the link between volatility and growth, *American Economic Review* 85(5), 1138-51.

Schwert, W., 1989. Why does stock market volatility change over time?, *Journal of Finance* 44, 1115-53.

Sentana, E., 1995. Quadratic ARCH models, *Review of Economic Studies* 62, 639-61.

Sims, C., 1990. Rational expectations modelling with seasonally adjusted data, Federal Reserve Bank of Minneapolis Discussion Paper 35, December.

Stock, J., Watson, M., 2002. Has the business cycle changed and why?, in Gertler, M., Rogoff, K.S. (Eds.), *NBER Macroeconomics Annual*, The MIT Press, 59-218.

Taylor, S.J., 1986. *Modelling Financial Time Series*, UK: John Wiley.

Tse, Y., Tsui, A., 2002. A multivariate GARCH model with time-varying correlations, *Journal of Business and Economic Statistics* 20, 351-62.

Tsui, A.K., Yu, Q., 1999. Constant conditional correlation in a bivariate GARCH model: Evidence from the stock markets of China, *Mathematics and Computers in Simulation* 48, June, 503-9.

Valderrama, D., 2001. Statistical nonlinearities in the business cycle: A challenge for the canonical RBC model, Duke University Economics Department Unpublished Manuscript, August 24.

Zakoian, J., 1994. Threshold heteroskedastic models, *Journal of Economic Dynamics and Control* 18, 931-55.