No 2015-32 – December

Working Paper

CEPII

Liquidity, Government Bonds and Sovereign Debt Crises

Francesco Molteni

Highlights

- We study the liquidity channel of the European financial crisis through the market of repurchase agreements (repos), where peripheral government bonds experienced a sharp increase in haircuts.
- This liquidity shock leads to a drop in economic activity and inflation, and a "flight-to-liquidity" towards the government bonds with lower haircuts amplifying the raise in peripheral bonds yields
- An unconventional policy which consists of purchasing illiquid bonds, similar to the expanded asset purchase program implemented by the ECB, is successful in alleviating the contractionary effect of the liquidity shock.

Abstract

This paper analyses the European financial crisis through the lens of sovereign bond liquidity. Using novel data we show that government securities are the prime collateral in the European repo market, which is becoming an essential source of funding for the banking system in the Euro area. We document that repo haircuts on peripheral government bonds sharply increased during the crisis, reducing their liquidity and amplifying the raise in the yields of these securities. We study the systemic impact of a liquidity shock on the business cycle and asset prices through a dynamic stochastic general equilibrium model with liquidity frictions. The model predicts a drop in economic activity, inflation and value of illiquid government bonds. We show that an unconventional policy which consists of purchasing illiquid bonds by issuing liquid bonds can alleviate the contractionary effect of liquidity shock. A Bayesian structural vector autoregressive model for the Irish economy confirms empirically the negative impact of a rise in haircuts on the value of government bonds.

Keywords

Repo, Haircuts, Government bonds, Liquidity shock, Quantitative Easing.

JEL

E44, E58, G12.

Working Paper

CEPII

CEPII (Centre d'Etudes Prospectives et d'Informations Internationales) is a French institute dedicated to producing independent, policy-oriented economic research helpful to understand the international economic environment and challenges in the areas of trade policy, competitiveness, macroeconomics, international finance and growth.

CEPII Working Paper
Contributing to research in international economics

Editorial Director:
Sébastien Jean

Production:
Laure Boivin

No ISSN: 1293-2574

CEPII
113, rue de Grenelle
75007 Paris
+33 1 53 68 55 00

www.cepii.fr
Press contact: presse@cepii.fr

RESEARCH AND EXPERTISE ON THE WORLD ECONOMY

Liquidity, Government Bonds and Sovereign Debt Crises[1]

Francesco Molteni*

1. Introduction

The recent financial crisis was characterised by the increase in the yield spreads of sovereign bonds issued by peripheral countries of the Eurozone (Greece, Ireland, Italy, Portugal and Spain), which has weakened their fiscal position and reduced their capacity to roll over debt. The creation of a monetary union has integrated the sovereign debt markets in the euro area and eliminated the exchange rate risk, since 90% of outstanding government debt is issued in the common currency (Eurostat, 2013). Two main factors may therefore explain why countries in the periphery of the Euro area have been paying higher interest rates on public debt than countries

[1]The views expressed in this paper are those of the authors and do not necessarily reflect those of the Institutions to which they belong. I am indebted with Nobuhiro Kiyotaki for its advices and supervision. I am grateful with Markus Brunnermeier, Qingqing Cao, Wei Cui, Arvind Krishnamurthy and Hyung Song Shin for their suggestions and insightful discussion while part of this reserach project was undertaken at the Department of Economics at Princeton University whose hospitality is gratefully acknowledged. I also thank Arpad Abram, Mackowiak Bartosk, Florin Bilbiie, Luca Benati, Jean-Bernard Chatelain, Fabrizio Coricelli, Stefano Corradin, Stephan Fahr, Marco Del Negro, Simone Manganelli, Giulio Nicoletti, Evi Pappa, Natacha Valla and all the participants to the Research Seminars at the European Central Bank, Banque de France, CEPII, the Macroeconomic Workshop at the Paris School of Economics, the Macro Workshop at the European University Institute, the 63rd Annual Meeting AFSE, in Lyon, the 6th IFABS Conference in Lisbon, the SSES Annual Congress Post-Crisis Macroeconomics in Berne, the 32nd International Symposium on Money, Banking and Finance in Nice and the 3rd Macro Banking and Finance workshop in Pavia for their comments. All remaining errors are my own. This paper is part of my PhD dissertation.
*CEPII, (francesco.molteni@cepii.fr)

in the core: credit risk and liquidity.

Credit risk derives from the government's probability of default. Weak fiscal and macroeconomic fundamentals of a country induce investors to ask higher compensations for holding government debt due to the possibility of suffering losses. In addition, fears of default and self-validating expectations may also drive up yields of government securities issued by countries that cannot press new currency as predicted by Calvo (1988), Cole and Kehoe (2000) and Corsetti and Dedola (2012).[2]

Liquidity is a broad concept that traditional theories by Keynes (1936) and Hicks (1967) refer to as the capacity of an asset to store wealth and protect its owner from a shortage of revenue. With the evolution of financial markets, modern corporate finance distinguishes between market liquidity and funding liquidity. Market liquidity is the facility to obtain cash by selling an asset; when it is difficult to find a buyer the price of the asset deviates from the fundamentals and its market liquidity is low.[3]

Funding liquidity is the ease with which investors can obtain funding; as they typically borrow against an asset, in this paper we consider funding liquidity as the "pledgeability" of an asset, that is its capacity to serve as collateral in the interbank market, especially in the market of repurchase agreements (repos). Following this interpretation, we investigate the role of sovereign bond liquidity in the European

[2] Tristani and Hördahl (2013) and Dewachter et al. (2013) find that economic fundamentals are the primary determinant of European bond yield spreads during the crisis, but their evolution is also associated with an unobservable non-fundamental factor that they interpret as the manifestation of self-fulfilling dynamics.

[3] Empirical works which disentangle credit and market liquidity risks in bond yields find compelling evidence that market liquidity has contributed to the rise of intra-euro spreads during the crisis (Manganelli and Wolswijk, 2009; Favero et al., 2010 and Monfort and Renne, 2013).

financial crisis.

We show that government bonds are the principal securities used to guarantee secured funding, which is becoming increasingly important for banks to meet their liquidity needs. The observed shift from unsecured to secured lending in the European money market during the crisis reflects a rise in agency frictions which obliges banks to post collateral securities whose value exceeds the loans by a determined amount that is the haircut or initial margin. This also provides government bonds with a special liquidity function. The role of government debt as collateral in the interbank market is also at the basis of the model of Bolton and Jeanne (2011) to study the contagion deriving from a sovereign default in an financially integrated economy. Gennaioli et al. (2014 a) provide empirical evidence that in a large panel of countries banks hold a sizeable amount of government securities because of their liquidity services. Further, Andritzky (2012) find that European banks increased their exposure to foreign government bonds before the onset of the financial crisis, involving a diversification of their collateral base.[4]

In the European money market, the collateral may be redeployed by the cash lender to secure other repos, this way stretching the intermediation chain.[5] For instance, Italian debt could be pledged by a French bank to borrow from a German bank, which in turn could re-use the same bond for a repo with a Dutch bank. Paraphrasing Kiyotaki and Moore (2003), before the crisis European banks held government securities not only for their maturity value but also for their exchange value and

[4]The European stress test in 2010 has revealed that in most countries of the Eurozone banks invested more than half of their government debt portfolio in foreign bonds.

[5]See Singh (2011) for the computation of the velocity of collateral.

these papers circulated as means of savings; hence they could be considered as money or a medium of exchange.

Nevertheless, we show that the increase in the sovereign risk during the crisis led to a rise in repo haircuts on government bonds issued by peripheral countries, reducing their liquidity and capacity to serve as collateral for secured borrowing. Consequently, leveraged investors were forced to sell illiquid bonds of the periphery and purchase liquid bonds of the core with lower haircuts in order to avoid a contraction of their funding, contributing to the widening of the yield spreads.[6] The pullback of foreign investors from the sovereign bonds of the periphery and the debt repatriation would derive not only from the rise in the sovereign risk and the tightening of prudential rules, but also from a reduction in their liquidity as collateral in the European interbank repo market.

Building on Del Negro, Eggertsson, Ferrero and Kiyotaki (2011), hereafter DEFK, we explore this liquidity channel of the Eurozone crisis through a DSGE model with liquidity frictions calibrated for Ireland.[7] Similarly to European banks, entrepreneurs choose to hold sovereign bonds as a way to store liquidity for financing future investments. Indeed, even if the returns on public bonds are lower than those on private assets, entrepreneurs can ease their funding constraint by borrowing against government securities. This echoes Hölmstrom and Tirole (1998) who show that

[6]Banks could alternatively pledge government bonds for ECB refinancing operation, but at higher interest rate than the market rate (see Manicini et al. (2014), Boissel et al. (2014).

[7]The model abstracts from the risk of sovereign default to focus on a pure liquidity channel. Several models introduce the probability of default. Coimbra (2014) analyses the feedback effects of the rise in the risk premium of government bonds in a model in which banks face a Value-at-Risk constraint. Bocola (2014) develops a quantitative model to estimate the balance sheet and credit risk channels in the pass-through of sovereign risk. Bi et al. (2014) endogenise the probability of default as a function of public debt.

firms that cannot pledge any of their future income are willing to pay a premium on assets that are able to store liquidity and help them in state of liquidity shortage.

However, a liquidity shock can suddenly reduce the amount of funding that entrepreneurs can obtain by pledging government bonds, equivalently to a rise in repo haircuts. As a result, they have less resources to investment and the value of illiquid bonds falls because they become less attractive.[8] We test the impact of a policy experiment consisting in purchasing illiquid long-term bonds by issuing liquid short-term bonds, which can be interpreted as the Expanded Asset Purchase program (APP) implemented by the European Central Bank. By providing entrepreneurs with an alternative liquid means of savings, this policy eases their funding constraint and reduces the contraction of investments, especially when the conventional monetary policy is constrained by the zero lower bound.

Finally, a structural vector autoregressive model confirms empirically the negative impact of a liquidity shock on the value of Irish government bonds. The model includes repo haircuts, CDS spreads and yields on 10-year bonds in order to identity separately a liquidity shock and a sovereign risk shock and the impulse response function suggests that both shocks lead to a rise in bond yields.

Related literature. This paper is related to different strands of research. Several works analyse the U.S. repo market and its role in the liquidity crisis of 2007-2008. Adrian and Shin (2009, 2010) show that the procyclical leverage of U.S. investment

[8] Acharya et al. (2015) present evidence that banks with large exposure on GIIPS sovereign bonds contracted corporate loans resulting in a fall in investment. De Marco (2015) finds that the contraction of bank lending was primarily due to a liquidty channel and the increased banks' cost of funding.

banks amplifies fluctuations in asset prices and that they adjust the balance sheets through variations in repos. Brunnermeier (2009) points out that prior to the crisis financial intermediaries were heavily exposed to maturity mismatch through their increased reliance on overnight and short-term repos and asset-backed commercial papers (ABCPs). Gorton and Metrick (2010, 2012) argue that the financial crisis was a run in the "securitized-banking system" characterised by a general rise in repo haircuts which shrank the banking liquidity in a similar way of massive deposit withdrawals in traditional bank runs and triggered a fire-sale of financial assets. In contrast to that study, Copeland et al. (2010) find that in tri-party repo market the haircuts were stable between 2008 and 2010 and Krishnamurthy et al. (2013) argue that the main cause of the liquidity crisis was a credit crunch in the ABCP market. Despite the interest towards the U.S. repo market, few academic articles explore the repo market in the Euro area, and we know little about the size, the evolution and implications for the sovereign-debt crises.[9] This paper attempts to fill this gap and examines the structural characteristics of the European repo market, the differences with the U.S. market and its development in time of crisis.

A second strand emphasises the negative feedback between the level of haircuts and the value of the underlying collateral. Gârleanu and Pedersen (2011) incorporate margin constraints into a consumption capital asset pricing model and show that when margin constraints bind, higher margins raise the required returns on

[9]Hördahl and King (2008) compare the development of the repo market in the Euro area, in U.S. and in UK in the first stage of the global financial crisis, but it does not cover the recent period characterised by tensions in sovereign-debt markets. Mancini et al. (2014) and Boissel et al. (2014) focus on the repo transactions performed on the principal European anonymous electronic trading platforms (Broker Tec, Eurex Repo and MTS), which account for less than one third of the total repo market according to the European Repo Market Survey.

assets, which lowers asset prices. Ashcraft et al. (2011) present an OLG model in which a reduction in the haircut on a security eases the funding constraint and lowers its required return. Empirically, they show that the Federal Reserve by cutting the haircuts through the Term Asset-Backed Securities Loan Facility (TALF) increased yields of the eligible securities. Brunnermeier and Pedersen (2009) endogenise changes in haircuts and changes in asset prices through a loss spiral and a margin spiral, which create a strong interaction between funding liquidity and market liquidity. The fall in the price of an asset reduces its value as collateral and force investors to sell some of its assets to obtain funding for maintaining other obligations, depressing its value even further (loss spiral). The higher volatility of the asset price leads to a rise in haircuts, which in turns strengthens the deleveraging (margin spiral).

We document this adverse feedback loop between levels of haircuts and value of underlying collateral during the most acute phase of the European financial crisis. Between 2010 and 2012 yields of peripheral government bonds and repo haircuts applied on these securities dramatically increased. Several models with financial frictions describe the effects of the fall in the value of the asset collateral, including Schleifer and Vishny (1992) in a static framework and Kiyotaki and Moore (1997) in a dynamic set-up.[10] The impact of a temporary shock which reduces the borrower's net worth and the value of the asset used simultaneously as input in the production process and as collateral translates in a persistent decline of output and asset

[10] In models in which the borrowing constraint derive from costly state of verification (Bernanke and Gertler, 1989; Carlstrom and Fuerst, 1997; Bernanke, Gertler and Gilchrist, 1999) the cost of external financing is increasing in the borrowing since higher leverage requires more monitoring. In Bernanke, Gertler and Gilchrist (1999) the negative shock hitting the net worth also reduces the price of capital arising an amplification mechanism similar to Kiyotaki and Moore (1997).

price. The surge in repo haircuts reinforces this mechanism as tightens the funding constraint and reduces the leverage of investors exacerbating the downturn. We study this mechanism by modelling a rise in the initial margins as a liquidity shock following DEFK, which incorporates the liquidity friction proposed by Kiyotaki and Moore (2012), hereafter KM,[11] into a DSGE model including nominal and real rigidities an dthe zero lower bound condition in line with Christiano et al. (2005) and Smets and Wouters (2007).[12]

We propose a model that departs from DEFK in two directions. First, equity is illiquid and public securities are almost completely - but not fully - liquid. Therefore funding constraints are tighter since entrepreneurs cannot acquire resources by disposing of private assets, emphasising the role of public bonds to store liquidity. In order to reproduce the rise in the repo haircuts on the peripheral sovereign debt, all the dynamics of the model takes place through a liquidity shock hitting public bonds. Second, the government can react by conducting an unconventional policy which consists of issuing liquid papers in the form of one-period bonds. These

[11]KM and DEFK focus on market liquidity and the liquidity shock is the result of a reduction in the "resaleability" of asset. Instead we focus on funding liquidity and the "pledgeability" of an asset in terms of level of repo haircut. Liquidity is modelled in different ways in macroeconomic models. In Gertler and Kiyotaki (2010) liquidity shocks affect interbank funds and variations in haircuts are modelled as tightening of the incentive constraint of banks. Jaccard (2013) considers a liquidity shock as the destruction of a fraction of the safe asset produced by the financial sectors which provide liquidity services fo firms and households. In Calvo (2009) liquidity is a parameter that enters the utility function making the land more valuable. Beningno and Nasticò (2013) define liquidity as the property of an asset to be exchanged for consumption goods. In Jermann and Quadrini (2012) the financial shock is equivalent to a reduction in the market liquidity of firm's capital which tightens its enforcement constraint, limiting the borrowing capacity of the firm.

[12]The same nominal rigidities are embedded also in other models of financial frictions such as Gertler and Karadi (2010), Getler and Kiyotaki (2011), Chen et al. (2012) and Christiano et al. (2013). Several models incorporate the KM type frictions: Ajello (2012), Bigio (2014), Cui and Guillen (2013), Cui and Radde (2013), Nezafat and Slavik (2011), Shi (2011). In all these models the liquidity friction applies on private financial assets and not on public bonds.

differences derive from the observation that while in the U.S. the liquidity of private financial assets, such as commercial papers and asset-backed securities suddenly dried up during the crisis, in Europe - where the market of private financial assets is far less liquid - the peculiar feature of the crisis was instead the reduction of sovereign bond liquidity.

The remainder of the paper is organized as follows: Section 2 defines the terms employed for the market of repurchase agreements, describes the data and provides a picture of the European repo market and its evolution during the crisis; Section 3 presents the model; Section 4 shows the results of the numerical simulation; Section 5 examines empirically the impact of a liquidity shock on Irish government bond yields and Section 6 concludes.

2. The European repo market

2.1. Definitions and data

We start by explaining the main characteristics of a repo contract, important in understanding the results of the analysis, and by describing the data used to investigate the European market of repurchase agreements. A repo transaction is an agreement between two parties on the sale and subsequent repurchase of securities at an agreed price. It is equivalent to a secured loan, with the difference that legal title of the securities passes from the seller to the buyer which may re-use them as collateral in other repo transactions. In order to protect the lender from the risk of a reduction in the value of collateral, repos involve overcollateralization and the difference between the value of the loan and the value of collateral is the haircut or initial margin. The haircut takes account of the unexpected loss that the lender in a repo may face due to the difficulty of selling that security in response to a default by the borrower. Accordingly, it is at the same time an indicator of funding liquidity from the perspective of the cash borrower and of market liquidity from the perspective of the cash lender. Figure 1 shows an example of bilateral repo.[13] At time t, the cash borrower (securities dealer, commercial bank, hedge fund) posts €100 securities as collateral and receives a €90 loan from the cash lender (commercial bank, investment fund, money market fund) with a haircut of 10%. At time t+k, the borrower returns the cash with an interest of 1.1% (the repo rate) and receives back the collateral. If repo is used to finance the purchase of a security, the haircut is equivalent to the inverse of the leverage. To hold €100 securities the investor can borrow up to €90 from the repo lender and must come up with €10 of its own capital, so the maximum leverage is 10. In this example, a rise in haircut by 10 percentage points reduces the borrower liquidity to €80 and the leverage to 5.

[Figure 1 here]

The determinants of haircuts vary according to the repo structure. In repos that are not cleared by a Central Clearing Counterparty (CCP), the haircut reflects mainly the creditworthiness of the borrower. Instead in repos involving a CCP which bears the counterparty credit risk, haircuts are settled on the basis of the CCP's internal

[13] According to the involvement of intermediaries between the lender and the borrower, repos can be distinguished in two types. In bilateral repos, the lender and the borrower transact directly with each other, selecting the collateral, initiating the transfer of cash and securities, and conducting collateral valuation. In tri-party repos, a third party intermediates the transaction providing operational services to the parties, in particular the selection and valuation of collateral securities, but does not participate in the risk of transaction.

rules and depend on the quality of the underlying collateral and its market risk.

Data. Because of the lack of comprehensive information on repos in the Eurozone, different sources are used in this study to shed light on this market, in particular Bankscope and the European Repo Market Survey (ERMS). Bankscope provides data on the banks' balance sheet at yearly frequency showing the stock of repos and reverse repos held by financial institutions.[14]

The ERMS presents information on the size and composition of the European repo market, including the type of repo traded, the rates, the collateral, the cash currency and the maturity. It is a semi-annual survey conducted by the International Capital Market Association (ICMA) starting from 2001. The survey asks a sample of banks in Europe for the value of their repo contracts that were still outstanding at close of a business day excluding the value of repos transacted with central banks as part of official monetary policy operations. The ERMS also reports the average of haircuts for various categories of collateral (governments bonds, public agencies, corporate bonds, covered bonds, mortgage-back securities, other asset-backed securities, convertible bonds, equity), but does not provide information about the haircuts on government bonds by nationality. For this reason, we collected data on the haircuts applied by the largest European clearing houses, LCH Clearnet Ltd and LCH Clearnet SA, to 10-year sovereign bonds issued by Italy, Ireland, Portugal and Spain by reading the RepoClear Margin Rate Circulars in the website of the

[14]The advantage of this database is that it allows to compare different sources of funding . Nevertheless, it presents three main limits: first, it lacks of important breakdowns, such as counterparty, maturity and currency, preventing a more granular analysis of the European repo market and it does not separate repos as interbank transactions from conventional monetary policy operations. Second, the period covered by the database starts from 2006. Lastly, for several banks data on repos are missing, making impossible an overview of the aggregated banking system.

clearing houses.

2.2. Structural characteristics

Figure 2 exhibits the extraordinary expansion of the European repo market in the last decade. The ERMS has collected data on repos held by 67 banks that continuously participated to all the surveys from which we have subtracted reverse repos in order to focus on their liabilities and avoid double counting. The repo market tripled in the run-up of the crisis and after a short contraction between 2008 and 2009 rapidly recovered, reaching around €3 trillion and approaching the about $4.4 trillion of the U.S. repo market estimated by the Federal Reserve of New York in 2009 based on the average daily amount outstanding of the primary dealers repo financing (Acharya and Öncü, 2012). Since data come from survey of banks the size of European repo market is underestimated.

[Figure 2 here]

Table 1 displays the funding structure of the largest European commercial banks, for which Bankscope reports data on repos.[15] For the biggest five institutions in the sample repos account for around 10% of their liabilities. Further, for those banks secured borrowing is larger than unsecured borrowing. This finding is in line with the European Money Market Survey conducted by the ECB, which reports that the largest ten banks account for 62% of the total repo turnover. These results confirm the importance of secured transactions in the European money market and

[15] Since Bankscope does not distinguish between repos in the private market and repos as monetary policy operations of the ECB, the table reports data on 2010 to exclude the large-scale LTROs in 2011 and 2012. The table reports only the banks for which data on repos are available.

point out that the largest European banks post collateral securities to access to the short-term wholesale funding.

[Table 1 here]

Figure 3 shows that sovereign bonds are the predominant assets in the collateral pool of the European repo market, accounting for around 80% of the total, and this share was stable during the crisis. Looking at the composition of government securities, German bonds account for the largest share, although their supply is lower than French and Italian bonds, because of their lower riskiness and suggesting that are the most liquid securities in the Euro area. As we will show, lower haircuts applied to German bonds in the repo market than to peripheral bonds also account for their largest share in the pool of collateral.

[Figure 3 here]

To summarize, government bonds are the fundamental collateral in the secured interbank market in the Eurozone. This is in contrast with the U.S. repo market, in which private assets are more largely pledged by financial institutions. Another difference between the two markets concerns their structure, since tri-party repos are the predominant arrangements in the U.S. while bilateral CCP-cleared repos constitute the biggest share in the Euro area. Consequently, increases of haircuts in the U.S. market can be explained by mechanisms of adverse selection (Gorton and Metrick, 2008) and information acquisition sensitivity (Dang et al., 2011 a and Dang et al. 2011 b), because the cash borrower may have more information about the quality of the asset used as collateral, e.g. mortgage-backed securities.

By contrast, in the European market the larger use of CCPs, which bear the counterparty credit risk, and of government securities as collateral reduces problems of asymmetric information between the borrower and the lender. On the other side, the market structure of the Eureopan repo market implies that haircuts are more sensitive to the credit risk of collateral security.

2.3. Development during the crisis

We next examine the evolution of the European repo market during the financial crisis. Figure 4 compares the dynamics of secured and unsecured borrowing in a sample of 101 European banks included in the European Money Market Survey. After the onset of the crisis we observe a shift of funding from the unsecured to the secured segment following the rise in counterparty risk.[16] Furthermore, bilateral CCP-cleared repos steadily increased within the secured segment.

The expansion of the European repo market in time of crisis is contrast with the evaporation of the U.S. market during the period 2007-2009. More than a consequence of the recent crisis, this evolution seems to derive from a structural transformation in the business model of the largest European financial institutions in the last decade, which rely more massively on short-term collateralised debt as a source of funding for their activities, similarly to the U.S. investment banks (see Adrian and Shin (2009) and Brunnermeier (2009)).

[Figure 4 here]

[16]Heider and Hoerova (2009) show the decoupling of interest rates between the unsecured and the market secured by government securities in the European interbank market during the crisis and present a model in which credit risk premia in the secured market affect the price of risk-free bonds through a no-arbitrage condition between the two markets.

The recent financial crisis also affected the composition of sovereign bonds within the pool of collateral (see figure 3). In particular, the share of government securities issued by peripheral countries reduced. Italian bonds fell from 10% of the total in June 2011 to 7% in December 2011 when the tensions on the sovereign debt reached the peak. Given the estimated amount of secured funding in December 2011 of around €3.1 trillion (see figure 3) and considering a conservative value of 5% as weighted average of the haircuts between June and December 2011, it follows that €100 billion of Italian government bonds flew out from the European repo market. This "collateral run" on Italian bonds may reflect not only the rise in the sovereign risk, but also their lower liquidity determined by increases in repo haircuts. For instance, LCH Clearnet SA on 8^{th} November 2011 published the variation in repo margins on Italian government securities by between 3 and 5.5 percentage points and accordingly on 9^{th} November the haircut on the 10-year Italian bonds increased from 6.65 to 11.65 percentage points. The yields of these securities increased from 6.39 to a high record level of 7.25 percentage points between 7^{th} November and 9^{th} November (figure 5).[17]

[Figure 5 here]

Larger increases in haircuts were experienced by Irish and Portuguese bonds in 2011. Figure 6 compares the evolution of the yields on 10-year bonds and the haircuts to these securities settled by LCH Clearnet Ltd. Following the rise in bond yields, the haircuts on both securities increased from a low level of 15% before the

[17] The Financial Times reported the following: "Italian bonds are in the perfect storm at the moment. Real money investors are running away and those investors using Italian bonds to finance will also be clearing the desk now.", Financial Times 9^{th} November 2011 "LCH Clearnet SA raises margin on Italian bonds",

crisis to 80%. The strong procyclicality of the haircut is due to the fact that LCH Clearnet Ltd systemically increases the haircut when the spread with the German government bonds exceeds 450 basis points.

However, the direction of causality is also reversed. Rises in haircuts to peripheral bonds in turn may have increased the pressure on the sovereign debt and yields of these assets. Higher initial margins diminish the ability of leveraged investors to borrow and tighten their funding constraint, increasing the shadow cost of capital. As a consequence, they reduce their position on the bonds with higher haircuts and shift their portfolio towards securities with lower margins to relax their funding constraint. At that point, bond prices are more driven by liquidity considerations rather than by movements in fundamentals. The next section presents a model to study this mechanism.

[Figure 6 here]

3. The model

The model is an infinite horizon economy populated by a continuum of households of measure one. The members of each household are either entrepreneurs or workers. The model incorporates nominal rigidities, since prices and wages are set in staggered contracts, and real rigidities with capital adjustment cost. Households allocate saving across three risk-free financial assets characterised by different degrees of liquidity: equity, long-term and short-term sovereign bonds. The government conducts fiscal policy collecting taxes and conventional monetary policy by setting the nominal interest rate. Long-term bonds are subject to a liquidity shock which is the only shock perturbing the economy. In response to this shock the government may implement an unconventional policy which consists of increasing the supply of one-period bonds that are more liquid than long-term bonds.

3.1. Households

Structure. Each household has a unit measure of members indexed $j \in [0, 1]$. At the beginning of each period all members are identical and hold an equal share of the household's assets. They receive an idiosyncratic shock, iid across members and across time, which determines their profession: entrepreneurs or workers

$$j = \begin{cases} Entrepreuneurs & \text{with probability } \gamma \\ Workers & \text{with probability } 1-\gamma \end{cases}$$

By the law of large number γ and $1-\gamma$ also represent respectively the fraction of entrepreneurs and workers in the economy. Each entrepreneur $j \in [0, \gamma)$ invests and each worker $j \in [\gamma, 1]$ supplies labour; both types return their earnings to the household. At the end of each period, all members share consumption goods and assets, but resources cannot be reallocated among members within the period.

Preferences. The household's objective is to maximize the utility function

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s, H_s(j)) = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{\xi}{1+\eta} \int_{\gamma}^{1} H_s(j)^{1+\eta} dj \right] \tag{1}$$

where $\mathbb{E}_t$ denotes the conditional expectation, β is the subjective discount factor, σ measures the degree of relative risk aversion, ξ is a scaling parameter that can be chosen to match a target value for the steady state level of hours and η is the inverse of the Frisch elasticity of the labour supply. Utility depends positively upon the sum of the consumption good bought by household members ($C_t = \int_0^1 C_t(j)dj$) and negatively upon the workers' labour supply H_t.

Portfolio. Households buy physical capital K_t at price q_t and they lend it to intermediate good producers earning a constant dividend stream r_t. They also own government securities B_t^L with price Q_t^L defined as perpetuities with coupons which decay exponentially as in Woodford (2001). A bond issued at date t pays λ^{k-1} at date t+k with $\lambda \in [0, \beta]$ is the coupon decay factor that parametrises the duration of government securities which corresponds to $(1 - \lambda\beta)^{-1}$.[18] We define this bond as long-term to differentiate it from the short-term bond B_t^S which is a one-period bond with zero coupon, i.e. $\lambda = 0$, with price Q_t^S. We assume that the supply of short-term sovereign debt is very limited and they account for a little share of the households' portfolio.[19] In addition, families hold N_t^O claims on other households'

[18] An alternative interpretation of the long-term debt is that λ is the fraction of the outstanding bonds paying a constant coupon of 1 and $(1 - \lambda)$ is the fraction of bonds which mature at each period and for which the government pays back the principal to the bond holder (Chatterjee and Eyingougor, 2012).

[19] This assumption avoids that household's members hold government bonds only with short maturity because more liquid. It is consistent with the evidence supported by Eurostat (2013) that in Europe one-year bonds account for just 5% of the total outstanding public debt and that the average maturity is between 6 and 7 years, which is matched in the calibration of the model. In equilibrium the returns on long-term and short-term bonds are linked by a non-arbitrage condition. Most of new Keynesian models present only a one-period bond rolled over every period. In Leeper and Zhou (2013) government may find optimal to issue bonds with long maturity because they facilitate the intertemporal smoothing of inflation and output gap. Cochrane (2001) analyses long-term debt and optimal policy in the fiscal theory of price level. Christiano et al. (2013) introduce both short-term and long-term corporate bonds assuming that the one-period bond is the source of funding for entrepreneurs while the long-term bond plays no direct role in resource allocation.

capital and sell claims on own capital to other households' N_t^I. The financing structure implies the households' balance sheet at the beginning of period t in the table below.

Household's balance sheet (financial assets)

Asset		**Liabilities**	
Capital stock:	$q_t K_t$	Equity issued:	$q_t N_t^I$
Other's equity:	$q_t N_t^O$		
Long-term bonds:	$Q_t^L \frac{B_t^L}{P_t}$		
Short-term bonds:	$Q_t^S \frac{B_t^S}{P_t}$	Net worth:	$q_t N_t + Q_t^L \frac{B_t^L}{P_t} + +Q_t^S \frac{B_t^S}{P_t}$

We assume that equity issued by the other households (N_t^O) and the unmortgaged capital stock ($K_t - N_t^I$) yield the same returns, have the same value and depreciate at the same rate, so they are perfect substitutes and can be summed together and defined as equity

$$N_t = N_t^O + K_t + N_t^I \tag{2}$$

At the end of each period households also receive profits Dt and DI t from intermediate goods producers and capital producers, respectively. The budget constraint of the typical household member j can be written as the following

$$C_t(j) + p_t^I I_t(j) + q_t[N_{t+1}(j) - I_t(j) - \lambda N_t] + Q_t^L \left[\frac{B_{t+1}^L(j)}{P_t} - \lambda \frac{B_t^L}{P_t} + Q_t^S \frac{B_{t+1}^L(j)}{P_t}\right]$$
$$= r_t N_t + \frac{B_t^L}{P_t} + \frac{B_t^S}{P_t} + \frac{W_t(j)}{P_t} H_t(j) + D_t + D_t^I - T_t \tag{3}$$

where P_t denotes the price level, p_t^I is the cost of one unit of new capital in terms of the consumption goods, differing from 1 because of capital adjustment cost, $H_t(j)$ and $W_t(j)$ are the working hours and nominal wage for workers j as discussed in the next section. According to the left side of the budget constraint, the household members allocate resources between purchase of non-storable consumption good, investment in new capital - if they are entrepreneurs - and net purchase of equity, long-term bonds and short-term bonds. They finance their activities in the right side of the budget constraint with returns on equity, long-term bonds, short-term bonds, wages of differentiated labour - if they are workers - and the dividends net to taxes.

A key assumption of the model is the presence of the following funding constraints which limit the financing of new investments by entrepreneurs and determine the different degree of liquidity of the assets

$$N_{t+1}(j) \geq (1-\theta)I_t(j) + \lambda N_t \tag{4}$$

$$B_{t+1}^L(j) \geq (1-\phi)B_t^L \tag{5}$$

$$B_{t+1}^S(j) \geq 0 \tag{6}$$

Inequality 4 means that the entrepreneur can issue claims on the future output of investment but only for a fraction $\theta \in [0,1]$. This borrowing constraint implies that investment is partially funded internally and entrepreneurs have to retain 1θ as own equity. In addition, equity is assumed to be completely illiquid and entrepreneurs cannot sell it to obtain more resources to invest. Hence, the entrepreneurs' equity holding at the start of the period t+1 must be at least the sum of the downpayment $(1-\theta)I_t$ and depreciated equity λN_t, where λ is the inverse depreciation rate.[20]

The entrepreneur can acquire additional resources by disposing of a fraction $\phi_t \in [0,1]$ of long-term bonds, so a resaleability constraint imposes to keep the residual $(1-\phi_t)$ of bonds in its portfolio (inequality 5). $(1-\phi_t)$ is equivalent to the haircut in a repo transaction since determines the amount of liquidity that the entrepreneur can obtain by pledging sovereign securities in secured borrowing. In other words, the entrepreneur cannot borrow against the value of the entire bond holding because of the presence of the haircut. The assumption on the diverse resaleability of equity and bonds reflects the different liquidity of private assets and sovereign bonds in the European repo market: the first are scarcely used as collateral, while the latter are largely pledged by European banks for repo transactions to lever up.

Inequality 6 implies that short-term bonds are not subject to resaleability constraint and are fully liquid, but entrepreneurs cannot borrow from the government.[21] ϕ_t is the key parameter of the model characterising the liquidity of financial assets. We can think that it takes value 0 for equity, value 1 for short-term bonds and an intermediate value for long-term bonds. The dynamic of the model follows a

[20]Nezafat and Slavik (2011) model a financial shock as a tightening in the credit conditions and a drop in θ and assume that equity/capital is completely liquid. In our set-up the assumption that equity is illiquid means that entrepreneurs cannot issue equity on the unmortgaged capital stock and cannot sell any of others' equity remained.

[21]Similarly, inequalities 4 and 5 ensure that receipts from trading equity and long-term bonds are strictly positive, which prevents the entrepreneur from going short on these securities.

reduction in ϕ_t, which is paramount of a rise in the repo haircut on sovereign bonds.

At the end of the period, the assets of households are given by

$$N_{t+1} = \int N_{t+1}(j)\ dj \tag{7}$$

$$B^S_{t+1} = \int B^S_{t+1}(j)\ dj \tag{8}$$

$$B^L_{t+1} = \int B^L_{t+1}(j)\ dj \tag{9}$$

$$K_{t+1} = K_t + \int I_t(j)\ dj \tag{10}$$

Next, we can take into account the specific functions of entrepreneurs and workers.

3.1.1. Entrepreneurs

Entrepreneur $j \in [0, \gamma)$ does not supply labour, so we set $H_t(j) = 0$ in equation 3 to get its budget constraint. In order to acquire new equity he can either produce it at price p^I_t or buy it in the market at price q_t. For the rest of the model we assume that $q_t > p^I_t$ to focus on the economy where the liquidity constraints bind limiting the ability of the entrepreneur to finance investments. In this case, the entrepreneur will use all the available liquidity for new investment projects to maximise the households' utility. Accordingly, they minimise the equity holding by issuing the maximum amount of claims on the investment return to reduce the size of the downpayment as implied by constraint 4. The entrepreneur also sells the maximum amount of bonds as allowed by constraint 5, because their expected returns are lower than the ones on new investment. As a result, in equilibrium the liquidity constraints all bind and the entrepreneur does not consume goods within the period:

$$N_{t+1}(j) = (1-\theta)I_t(j) + \lambda N_t \tag{11}$$

$$B^L_{t+1}(j) = (1-\phi)B^L_t \tag{12}$$

$$B^S_{t+1}(j) = 0 \tag{13}$$

$$C_t(j) = 0 \tag{14}$$

Since we have the solutions for entrepreneurs, $N_{t+1}(j), B^L_{t+1}(j), , B^S_{t+1}(j), C_t$ for $j \in [0, \gamma)$, we can plug equations 11, 12, 13 and 14 into equation 3 to derive the function of investment for entrepreneurs

$$I_t(j) = \frac{r_t N_t + [1 + \lambda\phi_t Q^L_t]\frac{B^L_t}{P_t} + Q^S_t \frac{B^S_t}{P_t} + D_t + D^I_t - T_t}{p^I_t - \theta q_t} \tag{15}$$

The nominator represents the maximum liquidity available for the entrepreneurs deriving from the return on papers (equity and long-term bonds), sales of the resaleable fraction of long-term bonds after depreciation, sales of short-term bonds and the dividends net to taxes. The denominator is the difference between the price of one unit of investment goods and the value of equity issued by the entrepreneur which indicates the amount of own resources necessary to finance one unit of investment. Equation 15 shows that a drop in ϕ_t not only increases the haircut to long-term bonds but also reduce the leverage of entrepreneurs and impact directly on their investment. Aggregating by entrepreneurs we obtain total investment

$$I_t = \int_0^\gamma I_t(j)\ dj = \frac{r_t N_t + [1 + \lambda \phi_t Q_t^L] \frac{B_t^L}{P_t} + Q_t^S \frac{B_t^S}{P_t} + D_t + D_t^I - T_t}{p_t^I - \theta q_t} \tag{16}$$

3.1.2. Workers

Workers $j \in [\gamma, 1]$ do not invest, so $I_t(j) = 0$. They supply labour as demanded by firms at a fixed wage, as the union who represents each type of worker sets wages on a staggered basis. To determine the asset and consumption choices of workers, we first derive the household's decision for N_{t+1}, B_{t+1}^L, B_{t+1}^S and C_t, taking W_t and H_t as given. Knowing the solution for entrepreneurs, we can determine $N_{t+1}(j)$, $B_{t+1}^L(j)$, $B_{t+1}^S(j)$ and $C_t(j)$ for workers, given the aggregate consumption and asset holding.

3.1.3. Household's problem

To solve the model for the household, we aggregate the workers' and entrepreneurs' budget constraint

$$C_t + p_t^I I_t + q_t[N_{t+1} - I_t - \lambda N_t] + Q_t^L \left[\tfrac{B_{t+1}^L}{P_t} - \lambda \tfrac{B_t^L}{P_t} + Q_t^S \tfrac{B_{t+1}^L}{P_t}\right]$$
$$= r_t N_t + \frac{B_t^L}{P_t} + \frac{B_t^S}{P_t} + \int_\gamma^1 \frac{W_t(j)}{P_t} H_t(j) + D_t + D_t^I - T_t \qquad (17)$$

Households maximise the utility function (2) by choosing C_t, N_{t+1}, B_{t+1}^L and B_{t+1}^L subject to the aggregate budget constraint and the investment constraint. The first order conditions for equity, long-term bonds and short-term bonds are respectively

$$U'_{c,t} = \beta \mathbb{E}_t \left\{ U'_{c,t+1} \left[\frac{r_{t+1} + \lambda q_{t+1}}{q_t} + \frac{\gamma(q_{t+1} - p_t^I)}{p_t^I - \theta q_{t+1}} \frac{r_{t+1}}{q_t} \right] \right\} \qquad (18)$$

$$U'_{c,t} = \beta \mathbb{E}_t \left\{ \frac{1}{\pi_{t+1}} U'_{c,t+1} \left[\frac{1 + \lambda Q_{t+1}^L}{Q_t^L} + \frac{\gamma(q_{t+1} - p_t^I)}{p_t^I - \theta q_{t+1}} \frac{1 + \lambda \phi_{t+1} Q_{t+1}^L}{Q_t^L} \right] \right\} \qquad (19)$$

$$U'_{c,t} = \beta \mathbb{E}_t \left\{ \frac{1}{\pi_{t+1}} U'_{c,t+1} \left[\frac{1}{Q_t^S} + \frac{\gamma(q_{t+1} - p_t^I)}{p_t^I - \theta q_{t+1}} \frac{1}{Q_t^S} \right] \right\} \qquad (20)$$

where π_t is the inflation rate defined as $\pi_t = \frac{P_{t+1}}{P_t}$. The choice of sacrificing one unit of consumption today to purchase a paper gives a payoff which is composed by two parts. The first is the returns on the asset: $\frac{r_{t+1}+\lambda q_{t+1}}{q_t}$ for equity, $\frac{1+\lambda Q_{t+1}^L}{Q_t^L}$ for long-term bonds and 1 for short-term bonds. The second part is a "liquidity premium", deriving from the fact that papers in the entrepreneurs' portfolio relax their investment constraint. This premium is a function of the leverage $\frac{\gamma}{p_t^I-\theta q_t}$, the distance between the price of equity and the price of capital goods, and the liquid returns of each asset. Thus, the bond holding eases the financing constraints more than the equity holding which makes bonds more valuable for entrepreneurs.

3.2. Firms

Competitive final good producers combine differentiated intermediate goods Y_{it}, for $i \in [0,1]$ into a single homogeneous final good Y_t, using a constant return to scale technology in the form of Dixit and Stiglitz (1977)

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{1}{1+\lambda_f}} di\right]^{1+\lambda_f} \tag{21}$$

where $\lambda_f > 0$. They take input prices P_{it} and output prices P_t as given and their demand for the generic i^{th} intermediate good is

$$Y_{it} = \left[\frac{P_{it}}{P_t}\right]^{-\frac{1+\lambda_f}{\lambda_f}} Y_t \tag{22}$$

The zero profit condition for competitive final-goods producers implies that the aggregate price level is

$$P_t = \left[\int_0^1 P_{it}^{-\frac{1}{\lambda_f}} di\right]^{-\lambda_f} \tag{23}$$

The monopolist firm i hires labour services and rent capital from households to produce the intermediate good using the following technology

$$Y_{it} = K_{it}^{\alpha} H_{it}^{1-\alpha} \tag{24}$$

where $\alpha \in (0,1)$. K_{it} denotes the capital services and H_{it} the quantity of labour hired by the i^{ht} intermediate-good producer, which sets prices P_{it} subject to Calvo scheme frictions, taking the rental rate of capital r_t and the real wage $\frac{W_t}{P_t}$ as given. With probability $1-\zeta_p$, the firm can reset its price and with probability ζ_p cannot. By the law of large numbers, the probability of changing the price corresponds to the fraction of firms that rest the price, so each period a randomly selected fraction of firms $1-\zeta_p$ can reoptimize the price $\tilde{P}_{it}$ to maximize the present discount value of profits.

3.2.1. Labour market

The labour markets mirrors the structure of the good market along the lines of Erceg et al. (2000). Competitive labour agencies aggregate differentiated j labour inputs into a homogeneous single labour service H_t according to the technology

$$H_t = \left[\left(\frac{1}{1-\gamma} \right)^{\frac{\lambda_w}{1+\lambda_w}} \int_{\gamma}^{1} H_t(j)^{\frac{1}{1+\lambda_w}} \right]^{1+\lambda_w} \tag{25}$$

where $\lambda_w > 0$. Labour agencies sell labour services H_t to intermediate good producers for the nominal wage rate W_t. The first order condition for labour services determines the demand curve for the j_{th} labour type:

$$H_t(j) = \frac{1}{1-\gamma}\left[\frac{W_t(j)}{W_t}^{-\frac{1+\lambda_w}{\lambda_w}}\right] H_t \tag{26}$$

Labour unions represent all types of workers and set the wage rate $W_t(j)$ for the specific labour input j taking as given the demand for their specific labour input and subject to the Calvo scheme frictions on a staggered basis, taking as given the demand for their specific labour input. Each period, labour agencies are able to reset the wage $W_t(j)$ with probability $1-\zeta_w$ and with probability ζ_w they cannot and the wage remains fixed. By the law of large number, the probability of changing the wage corresponds to the fraction of workers whose wages change. Households supply whatever labour is demanded at that wage. If labour agencies can modify the wage, they will chose the wage $\tilde{W}_t$ to maximize their utility function.

3.2.2. Capital-good producers

The creation of new capital is delegated to competitive capital-good producers who transforms consumption goods into investment goods. They choose the amount of investment goods to maximize the profits taking the price of investment goods p_t^I as given

$$D_t^I = \left\{p_t^I - \left[1 + \Gamma\left(\frac{I_t}{I}\right)\right]\right\} I_t \tag{27}$$

The price of investment goods differ from the price of consumption goods because of the adjustment costs, which depends on the deviations of actual investment from its steady-state value. $\Gamma(.)I_t$ reflects the adjustment cost and $\Gamma(.)$ is a measure of technology illiquidity, capturing the difficulty to undo investment. We assume that $\Gamma(1) = \Gamma'(1) = 0$ and $f''(1) > 0$. The first order condition for this problem is

$$p_t^I = 1 + \Gamma\left(\frac{I_t}{I}\right) + \Gamma\left(\frac{I_t}{I}\right)\frac{I_t}{I} \tag{28}$$

The law of motion of capital is

$$K_{t+1} = \lambda K_t + I_t \tag{29}$$

We consider the following identity equation between capital and net equity

$$K_{t+1} = N_{t+1} \tag{30}$$

The resource constraint can be expressed as

$$Y_t = C_t + \left[1 + \Gamma\left(\frac{I_t}{I}\right)\right] I_t \tag{31}$$

Finally, considering the aggregate expression for D_t and D_t^I the investment function can be written as

$$I_t = \chi \frac{r_t N_t + [1 + \lambda \phi_t Q_t^L] \frac{B_t^L}{P_t} + Q_t^S \frac{B_t^S}{P_t} + Y_t - w_t H_t - r_t K_t + p_t^I I_t - I_t [1 + \Gamma \left(\frac{I_t}{I}\right)] - T_t}{p_t^I - \theta q_t} \tag{32}$$

3.3. The government

The government conducts monetary and fiscal policy separately following exogenous policy rules and faces the following intertemporal budget constraint

$$Q_t^L \frac{B_t^L}{P_t} + Q_t^S \frac{B_t^S}{P_t} + T_t = (1 + \lambda Q_t^L) \frac{B_t^L}{P_t} + \frac{B_t^S}{P_t} \tag{33}$$

The debt repayment is financed by the issue of new debt and a net taxes T_t. A solvency condition links taxes with the outstanding beginning-of-period government debt in term of deviation from the steady state

$$T_t - T = \psi_T \left(\frac{B_t^L}{P_t} - \frac{B^L}{P} \right) \tag{34}$$

where $\psi_T > 0$ measures the elasticity of fiscal policy to variations in the size of the debt. The government sets the nominal interest rate following the feedback rule constrained by the zero lower bound condition

$$R_t = \max \left(\pi_t^{\psi_\pi}, 1 \right) \tag{35}$$

where $\psi_\pi > 1$. Unconventional policy consists of purchasing illiquid long-term bonds by issuing liquid short-term bonds. The supply of one-period bonds (in term of equity) is a function of the proportional deviations of liquidity from the steady state

$$\frac{B^S_{t+1}}{N} = \psi_B \left(\frac{\phi_t}{\phi} - 1 \right) \tag{36}$$

The price of the nominal short-term bond is the inverse of the nominal rate, so the government by setting the nominal interest rate, also sets the price of short-term bonds.

$$Q^S_t = \frac{1}{R_t} \tag{37}$$

4. Numerical simulation

4.1. Calibration

The model is calibrated at quarterly frequency to match the economy of Ireland because of the rich dynamics of the haircuts on Irish bonds used to calibrate the process of the liquidity parameter ϕ_t. Table 2 reports the calibrated values. The steady state of liquidity ϕ is 0.75, equivalent to one minus the level of the haircut on 10-year Irish bonds before the crisis (25%). We estimate the stochastic process for ϕ_t as an AR(1) process from the dynamics of the Irish haircut during the period of crisis. The autoregressive coefficient ρ^ϕ is found to be 0.99 and the standard deviation of the residuals σ^{ϕ} is 1.3. These values measure the persistence of the freeze in the repo market and the size on the liquidity shock, respectively. The other parameter characterising the financial frictions θ describes the fraction of investment financed externally. Since entrepreneurs represent broadly the banking system in channelling resources to the production sector of the economy, we consider θ as the ratio of banks' external finance (defined as the sum of deposits, long-term debt and equity) over total assets. We construct the average of this ratio for the financial institutions included in our sample in Table 1 and we set $\theta = 0.5$. The liquidity share in this economy is defined as:

$$ls_t = \frac{\phi Q_t^L B_{t+1}^L}{\phi Q_t^L B_{t+1}^L + q_t P_t K_{t+1}} \tag{38}$$

The nominator is the liquid part of public debt computed as the total of Irish government gross liabilities times the liquidity parameter. The denominator is equal to the value of the total productive capital (OECD Economic Outlook, 2013). The average of this ratio during the period 2000 and 2011 is 0.43, which is taken as the steady state value of the liquidity share. The parameter λ pins down the duration of long-term bonds given by $(1-\lambda)^{-1}$. We set $\lambda = 0.973$ to match the average maturity of the Irish debt which is 6.9 years (Eurostat, 2013).

Other parameters are standard in the literature: the discount factor $\beta = 0.99$, the inverse Frish elasticity of labour supply $\eta = 1$, the capital share $\alpha = 0.4$, the arrival rate of investment opportunity in each quarter $\gamma = 0.05$ The degree of monopolistic competition in labour and product markets are calibrated symmetrically assuming a steady state markup of 10% ($\delta_p = \delta_w = 0.1$). The average duration of price and wage contracts is 4 quarters $\zeta_p = \zeta_w = 0.1$). Concerning the policy rules, feedback coefficient on inflation in the monetary policy rule ψ_π is 1.5 to guarantee a uniquely determined equilibrium. Transfers slowly adjust to the government debt ($\psi_B = 0.1$). For the sake of comparison, the coefficient of the intensity of government intervention ψ_B is -0.127 the same as in DEFK.

[Table 2 here]

4.2. Results

The impact of a liquidity shock. We first analyse the economy in which the government does not respond to the negative liquidity shock. Figure 7 shows the response of real and financial variables to a drop in ϕ_t for the first twelve quarters. Output, consumption and investment simultaneously fall by 8.01%, 7.45% and 10.92%, respectively. The magnitude of the reduction in investment is close to that observed in the Irish economy in 2011 (-7.21%). The contraction of liquidity impacts directly investments by tightening the entrepreneurs' funding constraint and shrinks the funds they can obtain from the sale of the bonds. Moreover, it is amplified by the fall in the value of equity that increases the required downpayment reducing the leverage of entrepreneurs. The drop in the price of equity (-14.23%) and long-term bonds (-8.42%) has a negative wealth effect on consumption. In particular, the presence of nominal rigidities is a key element for the fall in consumption, because with flexible price the contraction in the economic activity would generate deflationary expectations leading to negative real interest rate and boost-ing consumption, as observed in the model of KM. The liquidity shock causes a large decline in inflation (-6.12%) which is very persistent.

Concerning asset prices, the contraction in the resaleability of long-term bonds leads to a "flight to liquidity" towards the more liquid short-term bonds, as indicated by the jump in the liquidity spread (13.78%), defined as the difference between the prices of the two government securities. In addition, the strong persistence of the liquidity shock induces entrepreneurs to anticipate lower future resaleability reinforcing this mechanism, consistent with the observed widening of the yield spreads between liquid bonds of the core and illiquid bonds of the periphery during the European financial crisis. Shi (2012) notes that when the resaleability constraint applies to equity, a negative liquidity shock in the KM setting leads to an increase in equity price. A reduction in the resaleability reduces the supply of equity, while the demand is not affected since there is no change in the quality of investment projects. In this model, the fall in the demand for long-term bonds more than offset the contraction of their supply following the reduction in their resaleability, bringing about the drop in their price.

[Figure 7 here]

The effects of the policy intervention. We consider now the scenario in which the government reacts to the drop in ϕ_t by issuing more public liquidity. Figure 8 compares the impulse responses to a negative liquidity shock in case the government does not intervene (blue line) and in case in which does react (red line). The model predicts that this unconventional policy alleviates the contractionary effect of the shock substantially. Output decrease by 5.73%, consumption by 6.17%

and investment by 5.35%. The fall in consumption drastically lessens, mainly because the reduction in the price of equity is less pronounced, which also reduces the deleveraging of entrepreneurs. The increased supply of liquid short-term bond relaxes the funding constraint of entrepreneurs. Moreover, the implemented policy succeeds in reducing the deflationary effect of the liquidity shock (- 2.89%). Regarding financial variables, the impact of the unconventional measure on the liquidity premium is very limited as shown by the response of the liquidity spread.

[Figure 8 here]

The role of financial frictions. We compare the consequences of the policy intervention with tighter financial frictions. We set $\phi = 0.65$, keeping the same size of the liquidity shock. The purpose of this exercise is to assess the impact of unconventional policy when the liquidity of government bond is lower. Figure 9 shows the impulse response function with the higher value (red line) and the lower value (green line) of the liquidity parameter. The difference is considerable both for real and financial variables. when the government acquires less liquid bonds the reduction in output, consumption and investment following the liquidity shock is much lower (respectively -4.32%, -4.16% and -2.68%) and the drop in inflation is more contained (-1.73%). The main difference concerns the response of the value of government securities. The variation of the prices of long-term bonds and short-term bonds is lower. The unconventional policy is more effective when the liquidity of the asset purchased is lower, since reduces the liquidity premium with a positive impact on the economic activity and inflation. This is consistent with the empirical findings on the consequences of the large scale asset purchases conducted by the Federal Reserve in the aftermath of the crises (Krishnamurthy and Vissing-Jorgensen, 2011; Gagnon et al. 2011). The first stage of the policy intervention, the QE I, by targeting less liquid assets such as agencies reduced the yield spreads of these securities with Treasuries having the same maturity and credit risk, since both are debt issued by the U.S. government. This liquidity channel was much weaker in the second stage of the intervention, the QE II, since involved the purchase of liquid Treasuries. Furthermore, expected inflation increased substantially in the wake of the QE I, but modestly after the QE II.

[Figure 9 here]

The zero lower bound. We show that when monetary policy is constrained by the zero lower bound the impact of the liquidity shock and the effect of unconventional policy are both amplified. Figure 10 displays the impulse response to the liquidity shock if the zero lower bound is binding with (red dashed line) and without (blue dashed line) the policy intervention. On impact, the drop in output and consumption is similar to that we obtain ignoring the zero lower bound as showed by figure 7 (-8.55% and -7.19%, respectively), but far more persistent. The nominal interest rate cannot reach the negative region and conventional monetary policy cannot boost consumption expenditure through a reduction in the real interest rate mitigating the impact of the liquidity shock as in the case in which the zero lower bound does not bind. This in part explains the strong deflationary pressure following the liquidity shock. The fall in investment is deeper(-15.80%) and also persistent. Concerning asset prices, the price of equity falls more steeply following the stronger reduction in the demand for new capital, while the magnitude of the liquidity spread is similar to the scenario without zero lower bound. Because of the zero lower bound constraint, the effectiveness of unconventional policy is greater than it would be if it were possible for the government to lower the policy rate below zero, as indicated by figures 8 and 9. In particular, this measure avoids that the economy remains in a region of low consumption and deflation.

[Figure 10 here]

5. Empirical analysis

In this section we study the dynamic relationship between liquidity, sovereign risk and yields of government bonds during the period of crisis and we examine whether the data confirms that increases in repo haircuts have a negative impact on the value of underlying government bonds, as shown by the model in the previous section. While in the theoretical analysis we abstracted from the probability of a government default, we include in the empirical investigation an indicator of sovereign risk in order to disentangle the liquidity and credit risk channel and to better identify a liquidity shock. Funding liquidity is measured by the haircuts applied by LCH Clearnet Ltd and data of bond yields are taken from Global Finance Data.[22] Sovereign risk is measured by the spread of a senior five-year dollar-denominated CDS contract obtained from Datastream. All series are at daily frequency.

First, we test if credit risk leads funding liquidity and if funding liquidity leads the yields with a Granger causality test. Second, we estimate the impulse response

[22]Empirical literature generally considers the cost of funding in the unsecured market, such as the Euribor-Eonia spread, as proxy for funding liquidity. However, in the context of the recent crisis the unsecured market shut down. Consistent with our initial interpretation of funding liquidity, we propose an alternative indicator which is specific to a security and not a market rate.

functions (IRF) of a liquidity shock and credit risk shock with a structural vector autoregressive (SVAR) model. The analysis focusses on the Irish sovereign debt market to compare the predictions of the model. Furthermore, the evolution of 10-year bond yields and the repo haircut to these securities provide us with a laboratory to study the interaction between those variables. From November 2010 to July 2011 yields rose from around 8% to 14% and then decreased until 7% in February 2012. During the same period, the haircut applied by by LCH Clearnet Ltd on this security grew until 80% before scaling down to 25%. The sample is censored from 01-11-2010 to 01-12-2011, in order to exclude the 3-year LTROs in December 2011 and February 2012 which could have altered the relationship between the variables since banks of the periphery had an incentive to purchase domestic sovereign bonds in carry trade operations (Acharya and Steffen, 2015). During the period of time included in the sample the Security Market Program (SMP) was already active. Even though purchases of Irish bonds in the secondary market through the SMP affected yields and credit risk of Irish bonds, we exclude the shift in market sentiment deriving from the announcement of the adoption of this policy in May 2010.[23]

As preliminary analysis, we start by studying the copula function between haircuts and yields. The marginal cumulative distribution function and the joint cumulative distribution functions are estimated non parametrically by kernel methods. Figure 11 displays the scatter. There is a strong and positive link between the two variables, in particular for the upper-tail probabilities as we can see at the top-

[23]Ghyseler et al. (2014) and Easer and Shwaab (2014) find that the SMP was successful in reducing bond yields of Irish bonds and other peripheral bonds as well as Italian, Portuguese, Greek and Spanish bonds.

right corner of the graph. This is confirmed by the coefficient of linear correlation parameter of copula which is 0.59.

[Figure 11 here]

We next analyse the dynamic interaction between credit risk, funding liquidity and bond yields. In order to investigate whether there exists a lead-lag relationship between the variables, we use a Granger causality test to assess if past value of the variable i helps predict the variable j, above and beyond the information contained in past values of j alone. To do so, we estimate the following reduced-form VAR including in the following order haircut, CDS spreads and yields of 10-year sovereign bond yields. Because of the small size of the sample, the model is estimated with Bayesian techniques, using diffuse priors taking the Normal and inverse Wishart distributions for the estimation of the coefficients and volatilities. The appendix presents the details of the estimation strategy.

Table 4 shows the F statistics for the test at 5%. If the statistics is higher than the critical value we reject the null hypothesis that variable x does not Granger cause variable y. Changes in CDS spreads and haircut are significant to predict variations in the bond yields, suggesting that credit risk and funding liquidity lead the value of the government securities. In turn, bond yields Granger cause credit risk and funding liquidity. The latter results may be explain by the fact that LCH Clearnet Ltd changes the haircut when the spread with the German bond yields exceed 450 basis points and take into account other indicators for credit risk, that may explain the significance of the CDS spreads variations on changes in haircuts. All in all, the test shows the rich dynamics between funding liquidity, credit risk and bond yields. This exercise is close to Pelizzon et al. (2014) who examine the dynamic interaction between credit risk and market liquidity in the Italian sovereign debt market; they find that credit risk Granger causes market liquidity, but the opposite direction is not significant. We show that credit risk has an indirect impact on the yields not only via market liquidity, but also via funding liquidity.

[Table 4 here]

In order to assess the impact of an increase in haircuts on government bond yields avoiding endogeneity problems and reverse causality issues, we estimate the IRF of a structural liquidity shock and credit risk shock. Structural shocks are recovered using the triangular Cholesky decomposition placing haircuts before CDS spread and yields. This ordering implies two main identification assumptions. First, haircuts respond to variations in yields with one lag. Second, the information used by LCH Clearnet to set the haircuts consists of past values of the CDS spreads and yields. This identification approach finds a justification in the way the clearing house decides to modify the haircuts and communicate it to the market participants. LCH Clearnet Ltd settles the level of haircut applied on government bonds on the basis of their yield-spreads of German bonds and indicators of credit risk. When the risk management changes the haircut, the variation is published at the end of the day when the financial market is closed. Hence, market participants may react to this decision with one day of delay, eliminating possible announcement effects. Moreover, although the decision of changing haircuts is based on financial indicators which are public available, it remains a discretionary decision of the risk management, so we can exclude perfect foresight of private agents on the variation in haircuts.

Figure 12 and 13 show the response of yields to a liquidity shock and a credit risk shock, respectively. Yields rise significantly following the two shocks , suggesting that the both the liquidity and the credit risk channels are important for the dynamics of sovereign bond yields. In particular, these results confirm the mechanism of margin spiral for Irish government securities during the most acute phase of tensions in the sovereign-debt market in 2011 and a negative feedback loop between the price of government bonds and the haircuts, as predicted by the model.

[Figures 12 and 13 here]

6. Conclusions

This paper has explored the liquidity channel of the European sovereign debt crisis. It has shown that government securities play a key role as collateral in the European repo market easing the interbank lending in the Eurozone. Nevertheless, during the crisis repo haircuts on peripheral government bonds substantially augmented following the raise in sovereign risk. The combined escalation of sovereign bond yields and haircuts suggest an alternative channel for the European "twin crises" - the combination of sovereign weakness and banking fragility - in addition to the balance sheet channel (Gennaioli et al., 2014 b) and the bail-out channel (Acharya et al., 2014). Before the crisis, banks accumulated government bonds to store liquidity and use them as collateral to lever up, increasing their exposure on foreign bonds due to the European financial integration. When the sovereign risk of peripheral countries appeared, repo haircuts to public bonds surged drying up interbank liquidity. Banks in the core reduced simultaneously their position on peripheral bonds whose haircuts augmented causing a fire sale which depressed the value of these securities even more and resulting in further increases in haircuts.

We studied the consequences of a reduction in the liquidity of public bonds with a model incorporating liquidity frictions to simulate the impact of a raise in haircuts. The model exhibits a fall in output, investment, consumption and a raise in the liquidity spreads. The contractionary effects of this shock can be alleviated by a policy response consisting in issuing new short-term bonds to provide the investor with an alternative liquid asset that relaxes their funding constraint. This policy is more effective when the government purchases bonds with a lower degree of liquidity and when the monetary policy is constrained by the zero lower bound, providing a rationale for unconventional policy implemented by the European Central Bank (the expanded asset purchase program) and stressing the importance of the composition of government bond purchased for the success of this measure.

We have assessed empirically the main results of the model by estimating the impulse response function of a liquidity shock in a SVAR model including haircuts, CDS spreads and yields for Irish government bonds to disentangle the liquidity and credit risk effects.

The theoretical model can be extended in several dimensions. One is to introduce risk of sovereign default to endogenise the liquidity parameter as a function of the debt riskiness and a Value-at-Risk constraint for entrepreneurs. When the probability of default rises, the haircut also increases creating a feedback between the liquidity and riskiness of the government bonds, as shown in the empirical investigation. A second extension would be that of consider an open economy with two countries conducting independent fiscal policy and sharing monetary policy in a typical framework of a monetary union in order to study the impact of a liquidity shock on the bond issued by one country and the possible policy responses of the central bank. We leave this extensions for further research.

References

Acharya, V.V., Drechsler I. and Schnabl, P. 2014."A Pyrrhic Victory? - Bank Bailouts and Sovereign Credit Risk," *Journal of Finance*, 69: 2689-2739.

— and Öncü, T.S. 2012. "A proposal for the resolution of systematically important assets and liabilities: the case of the repo market," CEPR Discussion Paper 8927.

— and Pedersen, L. H. 2005, "Asset pricing with liquidity risk", *Journal of Financial Economics*, 77(2), 375-410.

— and Steffen, S. 2013. "The "Greatest" Carry Trade Ever? Understanding Eurozone Bank Risks," NBER Working Papers 19039, National Bureau of Economic Research, Inc.

Adrian, T., Begalle, B., Copeland, A., and Martin, A. 2012. "Repo and Securities Lending," NBER Working Papers 18549, National Bureau of Economic Research, Inc.

— and Shin, H. S. 2009. "Money, Liquidity, and Monetary Policy," *American Economic Review*, American Economic Association, vol. 99(2), pages 600-605, May.

— and — . 2010. "Liquidity and Leverage," *Journal of Financial Intermediation*, 19, 418-437.

Ajello, A. 2012. "Financial intermediation, investment dynamics and business cycle fluctuations," Finance and Economics Discussion Series 2012-67, Board of Governors of the Federal Reserve System.

Allen, W.A. and Moessner, R. 2012. "The liquidity consequences of the euro area sovereign debt crisis," BIS Working Paper No 390, October.

Andritzky, J. 2012. "Government Bonds and Their Investors: What Are the Facts and Do They Matter?," IMF Working Paper No. WP/12/158, Washington D.C.

Angelini, P., Grande, G. and Panetta, F. 2014. "The negative feedback loop between banks and sovereigns," Bank of Italy Occasional Papers, No. 213.

Ashcraft, A., Gârleanu, N. and Pedersen, L. H. 2011. "Two Monetary Tools: Interest Rates and Haircuts," NBER Chapters, in: NBER Macroeconomics Annual 2010, Volume 25, pages 143-180 National Bureau of Economic Research, Inc.

Avouyi-Dovi, S. and Idier, J. 2012. "The impact of unconventional monetary policy on the market for collateral: The case of the French bond market," *Journal of Banking & Finance*, Elsevier, vol. 36(2), pages 428-438.

Bank for International Settlements, 2010. "Strengthening repo clearing and settlements arrangements," Working Paper (September 2010).

Benigno, P. and Nasticò, S. 2013. "Safe assets, Liquidity and Monetary Policy," CEPR Discussion Paper 9767.

Bernanke, B.S. and Gertler, S. 1989. "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79(1), 14-31.

— and Gilchrist, S. 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework", in John Taylor and Micheal Woodford, eds., Handbook of Macroeconomics, Amsterdam: North Holland.

Bi, H., Leeper, E. M. and Leith, C. 2014. "Financial Intermediation and Government Debt Default," Mimeo.

Bigio, S. 2014. "Endogenous Liquidity and the Business Cycle," *American Economic Review*, 105(6): 1883-1927.

Bocola, L. 2014. "The Pass-through of Sovereign Default Risk", Mimeo.

Boissel, C., Derrien, F., Oors, E., and Thesmar, D. 2014. "Sovereign Crises and Bank Financing: Evidence from the European Repo Market," Mimeo.

Bolton, P. and Jeanne, O. 2011. "Sovereign Default Risk and Bank Fragility in Financially Integrated Economies," IMF Economic Review, Palgrave Macmillan, vol. 59(2), pages 162-194, June.

Broner, F., Erce, A., Martin, A. and Ventura, J. 2014 "Sovereign Debt Markets in Turbulent Times: Creditor Discrimination and Crowding-Out Effects," *Journal of Monetary Economics*, 61: 114-142.

Brunnermeier, M.K. 2009. "Deciphering the Liquidity and Credit Crunch 2007- 2008," *Journal of Economic Perspectives*, American Economic Association, vol. 23(1), pages 77-100, Winter.

— and Pedersen, L. H. 2009. "Market Liquidity and Funding Liquidity," *Review of Financial Studies*, Society for Financial Studies, vol. 22(6), pages 2201-2238, June.

— and Sannikov, Y. 2009. "A Macroeconomic Model with a Financial Sector", Mimeo, Princeton.

Calvo, G. 1983. "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics* 12(3), 383-398.

— . 1988. "Servicing the Public Debt: The Role of Expectations," *American Economic Review* 78(4), 647-661.

— . 2009. "Financial crises and liquidity shocks: a bank-run perspective," *European Economic Review*, Elsevier, vol. 56(3), pages 317-326. *American Economic Review* 78(4), 647-661.

Chatterjee, S. and Eyigungor, B. 2012. "Maturity, Indebtedness, and Default Risk," *American Economic Review*, American Economic Association, vol. 102(6), pages 2674-99, October.

Chen, H., Curdia, V. and Ferrero, A. 2012. "The Macroeconomic Effects of Large-scale Asset Purchase Programmes," *Economic Journal*, Royal Economic Society, vol. 122(564), pages F289-F315, November.

Christiano, L., Eichenbaum, M., Evans, C.L. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113, 1-45.

— , Motto, R. and Rostagno, M. 2013. "Risk Shocks", *American Economic Review*, 104(1): 27-65.

Cochrane, J.H. 2001. "Long-term Debt and Optimal Policy in the Fiscal Theory of the Price Level," *Econometrica*, 69, 69-116.

Cole, H. L. and Kehoe, T. 2000. "Self-Fulfilling Debt Crises," *Review of Economic Studies*, 67, 91-116.

Copeland, A., Martin, A. and Walker, M. 2010. "The Tri-Party Repo Market before the 2010 Reforms," Federal Reserve Bank of New York Staff Report, No. 477.

Corsetti G. and Dedola, L. 2012. "The Mystery of the Printing Press: Self-fulfilling Debt Crises and Monetary Sovereignty," mimeo, Cambridge University.

Cui, W. and Guillen, D. 2012. "Optimal Monetary Responses to Asset Price Levels and Fluctuations: The Ramsey Problem and A Primal Approach," 2012 Meeting Papers 1106, Society for Economic Dynamics.

— and Radde, S. 2013. "Search-Based Endogenous Illiquidity, Business Cycles and Monetary Policy," 2013 Meeting Papers 1009, Society for Economic Dynamics.

Curdia, V. and Woodford, M. 2011. "The central-bank balance sheet as an instrument of monetary policy," *Journal of Monetary Economics*, Elsevier, vol. 58(1), pages 54-79, January.

Dang, T.V., Gorton, G. and HolmstrÂĺom B. 2011 a. "Ignorance, Debt and Financial Crises," working paper

—, — and —. 2011 b. "Repo, Haircut and Liquidity," working paper

Dewachter, H., Iania, L., Lyrio, L. and Perea M.d.S. 2013 "A Macro-Financial Analysis of the Euro Area Sovereign Bond Market," *Journal of Banking & Finance*, Elsevier, vol. 50(C), pages 308-325.

Dixit, A.K. and Stiglitz, J. 1977. "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67, 297-308.

Doms, M.E. and Dunne, T. 1998. "Capital Adjustment Patterns in Manufacturing Plants," Review of Economic Dynamics, 1998, 1(2), 409-429.

Duffie, D. 1996. "Special Repo Rates," FRBNY Policy Review, 27-42. May.

Eberly, J.E. 1997. "International Evidence on Investment and Fundamentals", *European Economic Review*, 41(6), 1055-1078.

Eggertsson, G. 2008. "Great Expectations and the End of the Depression," *American Economic Review*, 98(4), 1476-1516.

Erceg, C.J., Henderson, D.W. and Levin, A.T. 2000. "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46(2), October, pages 281-313.

Ericsson, J. and O. Renault: 2006, "Liquidity and Credit Risk," *Journal of Finance* 61(5), 2219-2250.

Eurostat. 2013. "Structure of Government Debt in Europe in 2013".

Eser, F., and Schwaab, B. 2014. "Assessing asset purchases within the ECB's Securities Markets Programme," European Central Bank Working Paper.

Favero, C., M. Pagano, and E.-L. von Thadden: 2010, "How Does Liquidity Affect Government Bond Yields?," *Journal of Financial and Quantitative Analysis* 45(01), 107-134.

Ferrero, A., Eggertsson, G., Del Negro, M., and Kiyotaki, N. 2011. "The great escape? A quantitative evaluation of the Fed's liquidity facilities", Staff Reports 520, Federal Reserve Bank of New York.

Gagnon, J., Raskin, M., Remache, J. and Sack, B. 2011. "Large-scale asset purchases by the Federal Reserve: did they work?", *Economic Policy Review*, Federal Reserve Bank of New York, issue May, pages 41-59.

Gârleanu, N. and Pedersen, L. H. 2011. "Margin-based Asset Pricing and Deviations from the Law of One Price," Review of Financial Studies, Society for Financial Studies, vol. 24(6), pages 1980-2022.

Ghysels, E., Idier, J., Manganelli, S., Vergote, O. 2014. "A high frequency assessment of the ECB securities markets programme," Working Paper Series 1642, European Central Bank.

Geanakoplos, J. 2003. "Liquidity, Default and Crashes, Endogenous Contracts in General Equilibrium," in in Advances in Economics and Econometrics: Theory and Applications, Eighth World Conference, ed. by L.P.H.M. Dewatripont, and S.J. Turnovsky, vol.2, pp.170-205, Cambridge University Press, Cambridge, MA.

Gennaioli, N., Martin, A. and Rossi, S. 2014 a. "Banks, government bonds, and default: what do the data say?," Economics Working Papers 1378, Department of Economics and Business, Universitat Pompeu Fabra.

— , — and —. 2014 b. "Sovereign Default, Domestic Banks, and Financial Institutions," *Journal of Finance*, American Finance Association, vol. 69(2), 816-866.

Gertler, M. and Karadi, P. 2011. "A model of unconventional monetary policy," *Journal of Monetary Economics*, Elsevier, vol. 58(1), pages 17-34, January.

—, and Kiyotaki, N. 2010. "Financial Intermediation and Credit Policy in Business Cycle Analysis," Handbook of Monetary Economics, in Benjamin M. Friedman & Michael Woodford (ed.), Handbook of Monetary Economics, edition 1, volume 3, chapter 11, pages 547-599 Elsevier.

— and Metrick, A. 2010. "Haircuts," Federal Reserve Bank of St. Louis Review 92, 507-519.

— and —. 2012. "Securitized banking and the run on repo," *Journal of Financial Economics* 104(3), 425-451.

Gurio, F. and Kashyap 2007. "Investment Spikes: New Facts and a General Equilibrium Exploration," *Journal of Monetary Economics*, 54(Supplement), 1-22.

Heider, F. and Hoerova, M. 2009. "Interbank Lending, Credit-Risk Premia, and Collateral", *International Journal of Central Banking*, vol. 5(4), pages 5-43, December.

— and —. 1998. "LAPM: A liquidity-based Asset Pricing Model," NBER working paper 6673, National Bureau of Economic Research, Inc.

— and —. 2010. Inside and Outside Liquidity, Cambridge: MIT Press.

Hördahl, P. and King, M. R. 2008. "Developments in repo markets during the financial turmoil," BIS Quarterly Review, Bank for International Settlements, December.

International Monetary Fund. 2010. Global Financial Stability Report (October)

Jaccard, I. 2013. "Liquidity constraints, risk premia, and the macroeconomic effects of liquidity shocks," Working Paper Series 1525, European Central Bank.

Jermann, U. and Quadrini, V. 2012. "Macroeconomic Effects of Financial Shocks", *American Economic Review*, American Economic Association, vol. 102(1), pages 238-71, February.

Kiyotaki, N. and Moore, J. 1997 "Credit Cycles," *Journal of Political Economy*, 1997, 105(2), 211-248.

— and —. 2003. "Inside Money and Liquidity," Mimeo. London School of Economics.

— and —. 2012. "Liquidity, Business Cycles and Monetary Policy," Mimeo, Princeton University.

Koop, G. and Korobilis, D. 2009. "Bayesian Multivariate Time Series Methods for Empirical Macroeconomics," Working Paper Series 49-07, The Rimini Center for Economic Analysis

Krishnamurthy, A. 2010. "How Debt Markets have Malfunctioned in the Crisis," *Journal of Economic Perspectives*, 24(1), 3-28.

—, Nagel, S. and Orlov D. 2013. "Sizing up Repo," *Journal of Finance*, American Finance Association, vol. 69(9), 2381-2417.

—, Vissing-Jorgensen, A. 2011. "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy", Brookings Papers on Economic Activity, Fall 2011.

Nezafat, P. and Slavik, C. 2011. "Asset Prices and Business Cycles with Financial Shocks," mimeo Goethe University.

Leeper, E.M. and Zhou, X. 2013. "Inflation's Role in Optimal Monetary-Fiscal Policy," NBER Working Papers,19686, National Bureau of Economic Research, Inc.

Liu, Z., Wang, P. and Zha T. 2013. "Land Price Dynamics and Macroeconomic Fluctuations," *Econometrica*, Econometric Society, vol. 81(3), pages 1147-1184, 05.

Lucas, R. 1990. "Liquidity and interest rates", *Journal of Economic Theory*, 1990, 50(2), 237-264.

Mancini, L., Ranaldo, A. and Wrampelmeyer, J. 2014. "The Euro Interbank Repo Market," mimeo.

Manganelli, S. and Wolswijk, G. 2009. "Euro bond spreads," *Economic Policy* 24, 191-240.

Monfort, A. and Renne, J.P. 2013. "Decomposing euro-area sovereign spreads: credit and liquidity risks," *Review of Finance*, 18(6), 2103-2151.

Pelizzon, L., Subrahnmanyam M.G., Tomio, D., Uno, J. 2014. "Sovereign Credit Risk, Liquidity and ECB Intervention: Deus Ex Machina?," MIMEO.

Reinhart C.M. and Rogoff, K. S.. 2011. "From Financial Crash to Debt Crisis," *American Economic Review*, American Economic Association, vol. 101(5), pages 1676-1706, August.

Singh, M. "Velocity of pledged collateral: analysis and implication," IMF Working Papers (2011):1-24

Shi, S. 2012 "Liquidity, Assets and Business Cycles", Mimeo, University of Toronto.

Shleifer, A. and R. W. Vishny. 1992. "Liquidation Values and Debt Capacity: A Market Equilibrium Approach," *The Journal of Finance*, 47(4), pp. 1343âĂŞ1366.

Smets, F. and Wouters, R. 2007 "Shocks and Frictions in US Business Cycles: a Bayesian DSGE Approach," *American Economic Review*, 97(3), 586-606.

Tristani, O. and Hördahl, P. 2013 "Macro factors and sovereign bond spreads," Mimeo.

Wallace, N. 1981 "A Modigliani-Miller Theorem for Open-Market Operations," *American Economic Review*, 71: 267-274.

Woodford, M. 2003. "Interest and Prices: Foundations of a Theory of Monetary Policy", Princeton, N.J.: Princeton Univerity Press.

—. 2011. "Optimal Monetary Stabilization Policy," in Handbook of Monetary Economics, ed. by B.M. Friedman and M. Woodford, Amsterdam: Elsevier, vol. 3B, 723-828.

Figures and tables

Figure 1 – Bilateral repo contract

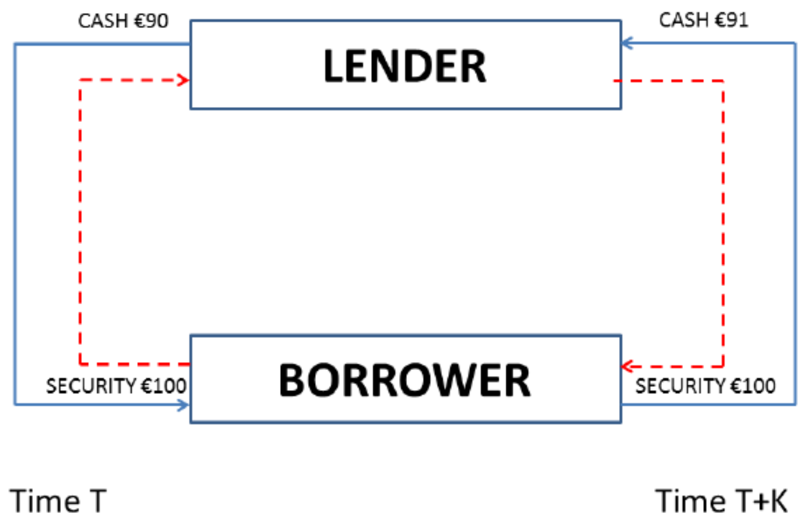
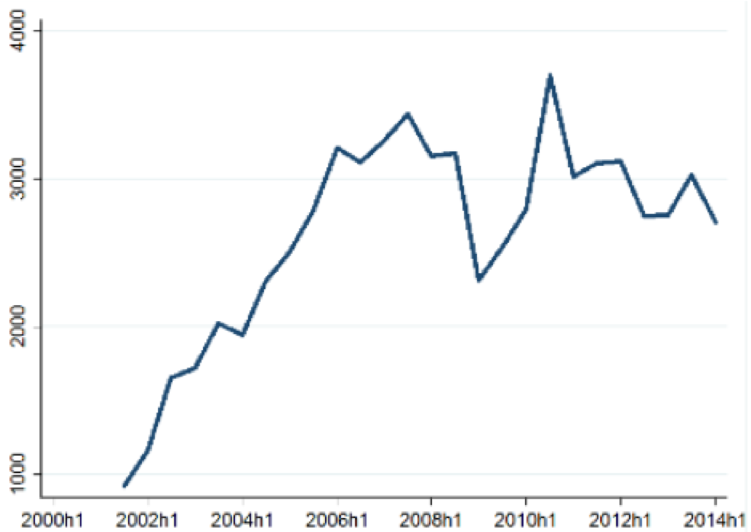


Figure 2 – European repo market (billions of euros)



Note: repos reported by banks which have participated continously to all the surveys (borrowing activty) Source: Eurpean repo survey (ICMA)

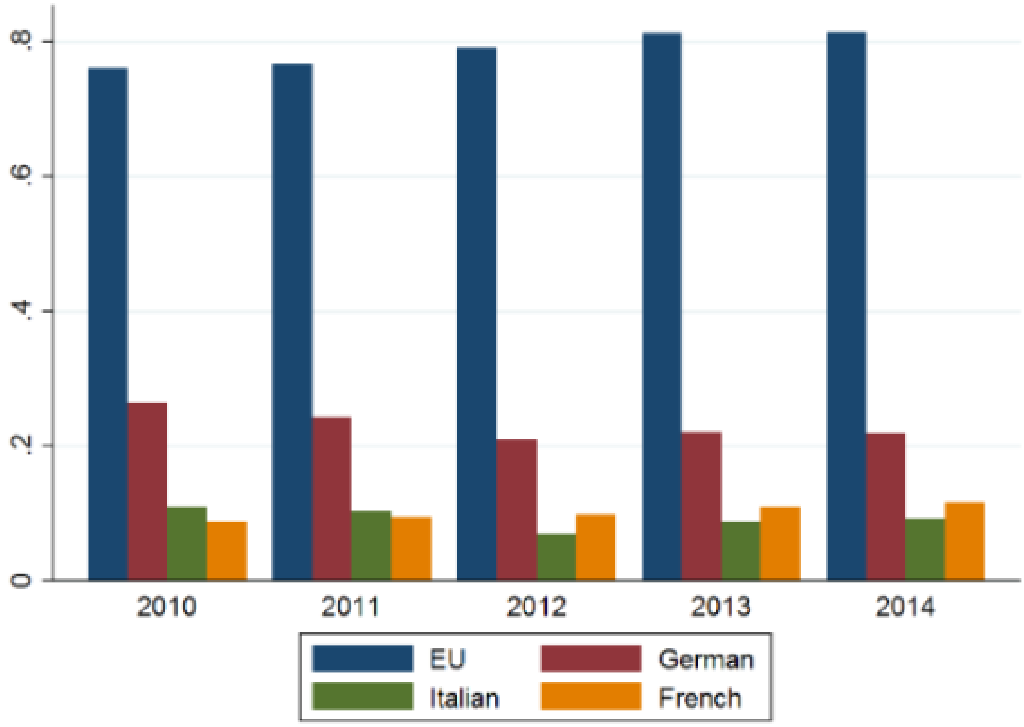
Table 1 – Funding structure of european commercial banks in percentage of total liabilities (2010)

Bank	Deposits	Interbank	LT debt	Repos
BNP Paribas	26.62	7.07	6.19	10.48
Barclays Bank Plc	23.41	5.89	9.89	13.26
Banco Santander	45.04	4.69	16.92	9.60
Société Generale	24.47	7.62	8.74	9.58
UBS AG	24.13	2.13	10.04	12.52
UniCredit SpA	42.99	10.83	16.96	3.39
Credit Agricole Corporate	14.43	10.63	0.70	5.61
Intesa Sanpaolo	30.84	10.32	27.99	1.99
Banco Bilbao	43.79	6.01	14.16	8.89
Commerzbank AG	34.32	12.01	13.72	7.13
Danske Bank	23.22	5.19	27.66	7.87
Skandinaviska Enskilda Banken	35.5	7.42	14.01	2.14
Bankia SA	36.78	3.43	29.6	11.56
Svenska Handelsbanken	29.21	8.06	30.27	0.49
Fortis Bank	41.18	7.68	5.50	4.30
Abbey National Treasury Services Plc	52.99	13.42	13.85	2.79
KBC	50.76	7.35	10.58	8.53
Banca Monte dei Paschi	32.35	9.41	24.56	9.86

Note: Deposits = costumer deposits; Interbank = interbank deposits; LT debt = long-term debt

Source: Bankscope

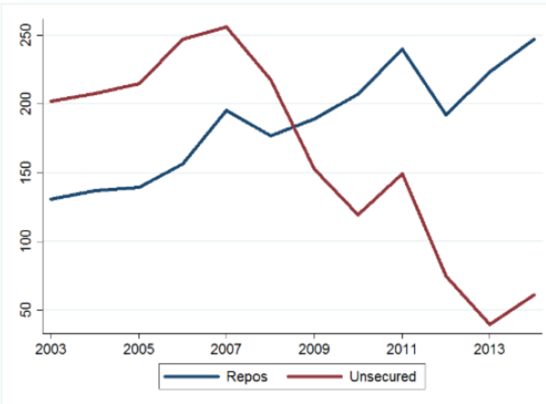
Figure 3 – Share of collateral in the European repo market



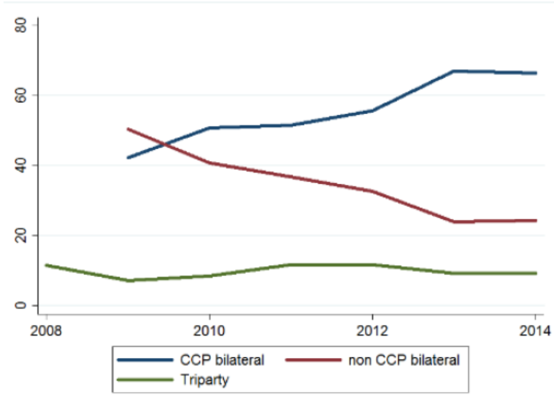
Source: Eurpean repo survey (ICMA)

Figure 4 – Interbank transactions in the European money market

Repos and unsecured borrowing (total turnover)

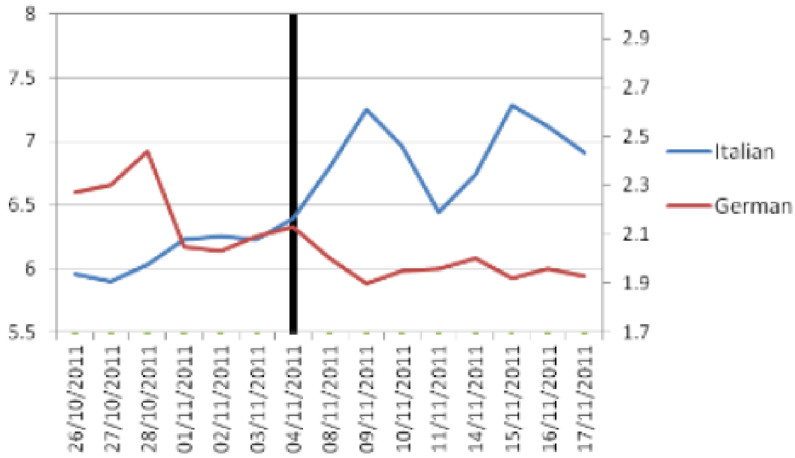


Shares of bilateral and tri-party repos (in percent of the total)



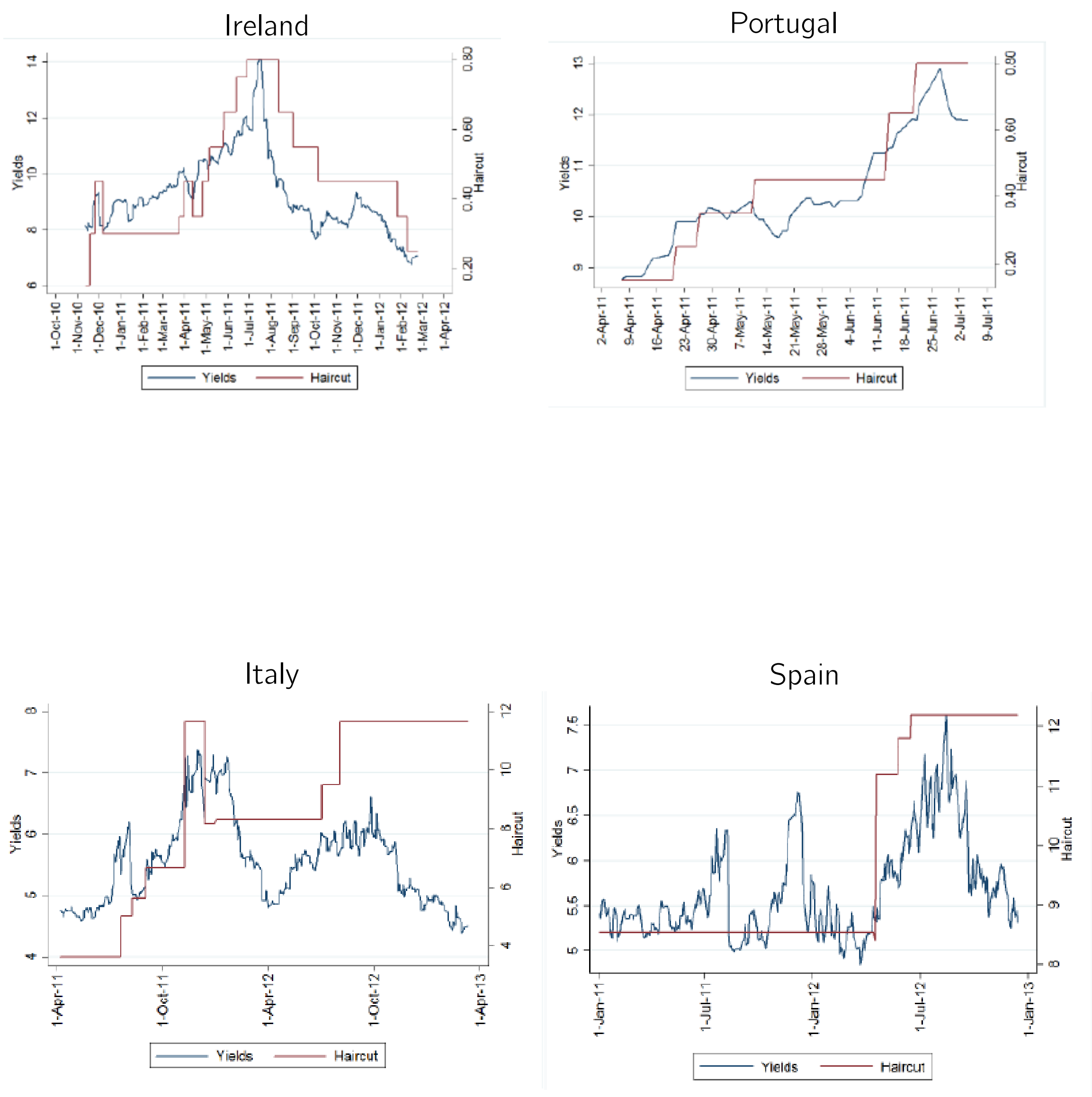
Source: European Money Market Survey (ECB)

Figure 5 – Yields on 10-year Italian (LHS) and German (RHS) government bonds



Note: the black line indicates the change in the haircut on Italian bonds by LCH Clearnet SA

Figure 6 – Yields (LHS) and haircuts (RHS) on 10-year government bonds



Source: Global finance data and LCH Clearnet website

Table 2 – Calibration

Definition	Parameter	Value
Preferences		
Household discount factor	β	0.99
Realtive risk aversion	σ	1
Inverse Frish elasticity	η	1
Production and investment		
Capital share of output	α	0.4
Adjustment cost parameter	Γ	1
Probability of investment opportunity	γ	0.05
Inverse depreciation rate / Bond maturity parameter	λ	0.973
Nominal frictions		
Price and wage calvo probability	$\zeta_\pi = \zeta_w$	0.75
Price and wage steady state markup	$\delta_\pi = \delta_w$	0.5
Financial frictions	θ	0.5
Borrowing constraint	θ	0.5
Resaleability constraint	ϕ	0.75
Autoregressive coefficient of liquidity	ρ^ϕ	0.99
Size of liquidity shock	σ^ϕ	1.3
Steady-state of liquidity share	ls	0.43
Policy rule		
Monetary policy rule coefficient	ψ_π	1.5
Transfer rule coefficient	ψ_T	0.1
Government intervention coefficient	ψ_B	-0.127

Figure 7 – Impulse response function to a negative liquidity shock

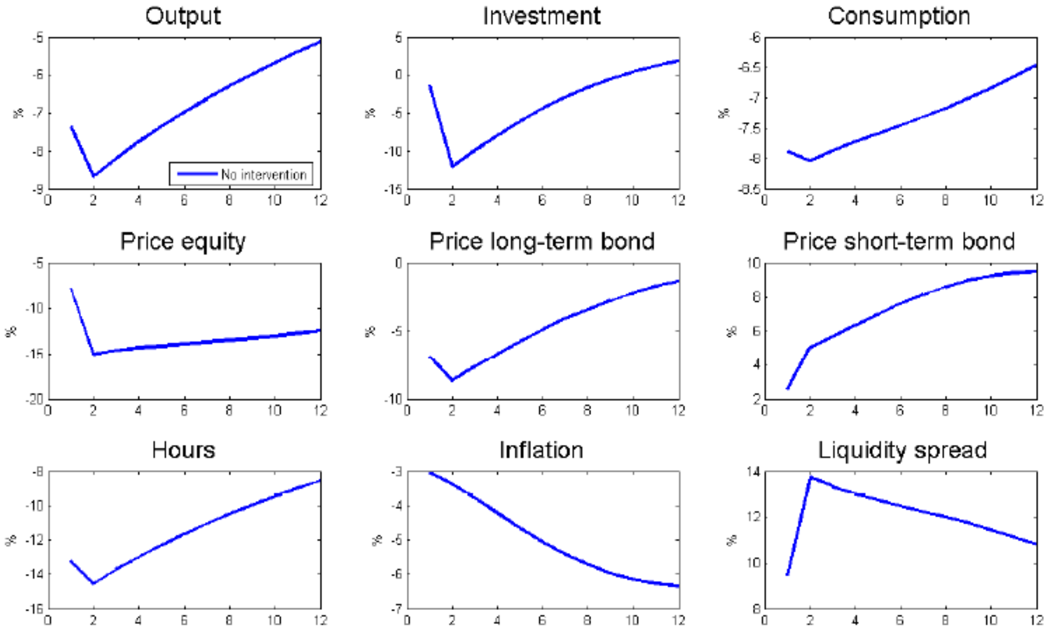


Figure 8 – The effects of policy intervention

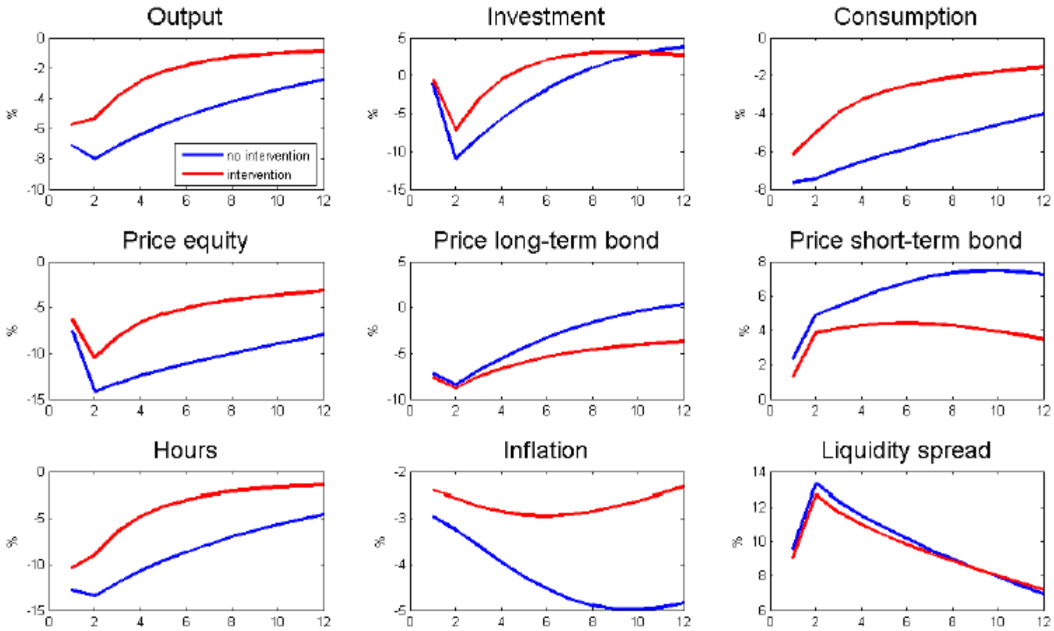


Figure 9 – The role of financial frictions

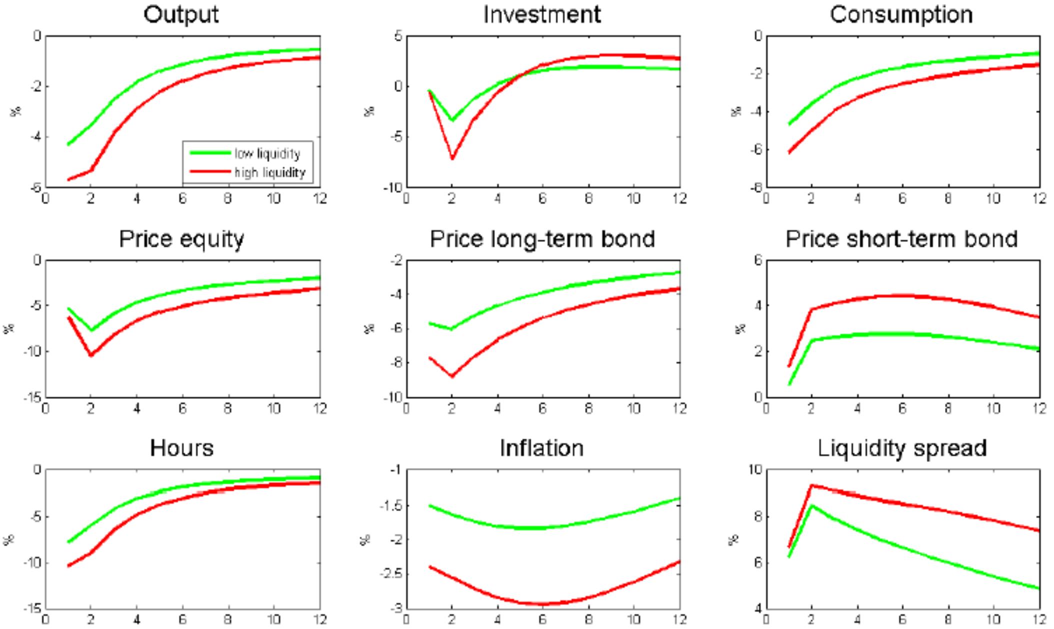


Figure 10 – The zero lower bound

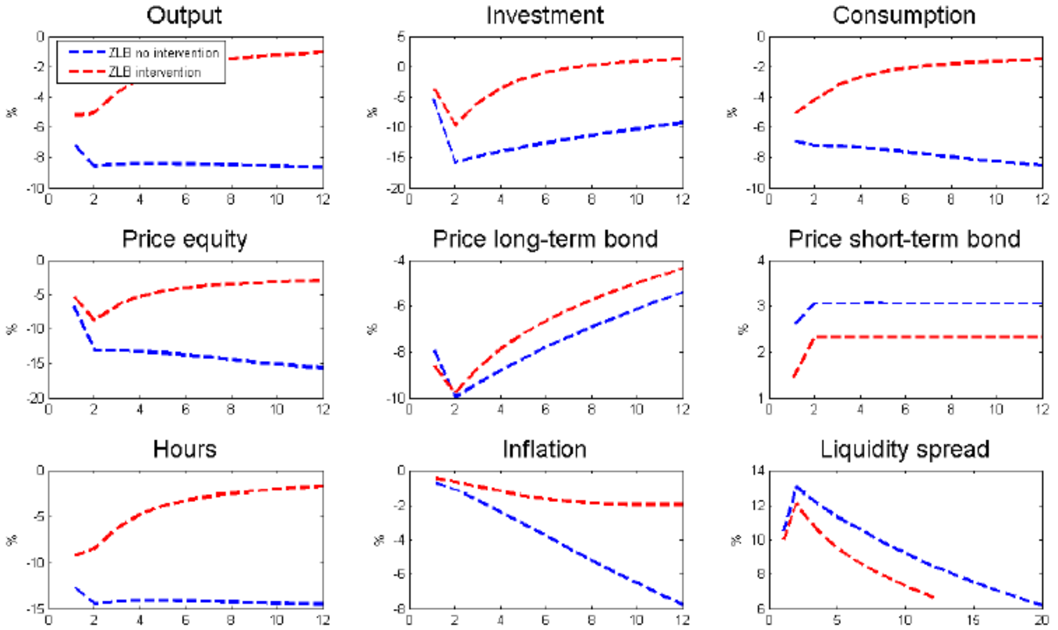


Figure 11 – Copula of kernel distributions of yields and haircuts

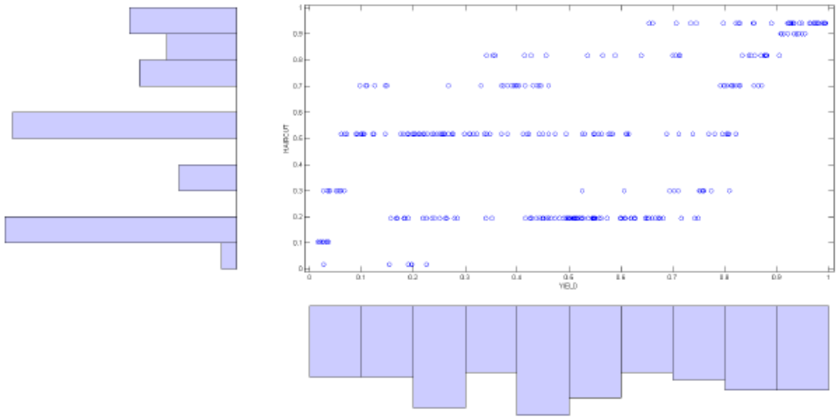


Table 3 – Granger causality test

Test	F-statistics	Critical value
$\Delta yields \rightarrow \Delta haircut$	9.582	0.908
$\Delta haircut \rightarrow \Delta yields$	3.305	0.456
$\Delta yields \rightarrow \Delta CDSspread$	6.222	0.546
$\Delta CDSspread \rightarrow \Delta yields$	9.205	0.695

Figure 12 – Impulse response function of a liquidity shock

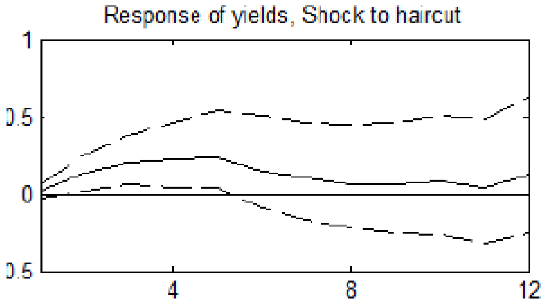
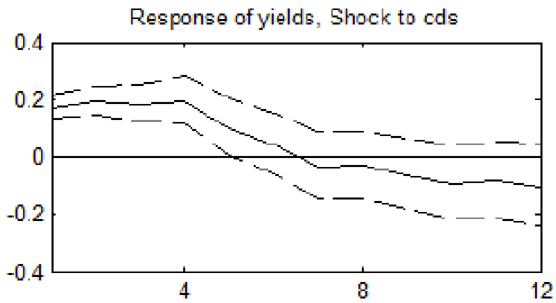


Figure 13 – Impulse response function of a credit risk shock



Appendix

A. Solving the model

A.1. Optimality conditions in good markets and labour market

At each period $1 - \zeta_p$ intermediate goods firms set the price $\tilde{P}_{it}$ to maximize the present discounted value of profits

$$D_{it+k} = P_{it+k}Y_{it+k} - w_{t+k}H_{it+k} - r_{t+k}K_{it+k} \tag{39}$$

subject to the demand for its own good (22) and conditional on not changing its price. Intermediate good producers, first choose the optimal amount of inputs (capital and labour) and they minimize the costs, $w_tH_{it} - r_tK_{it}$, subject to the production of intermediate goods (24). The first order condition is

$$\frac{K_{it}}{H_{it}} = \frac{K_t}{H_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t} \tag{40}$$

Since the marginal capital-labour ratio is independent of firm-specific variables, then the marginal cost,mc_{it}, i.e. the Lagrange multiplier on the constraint, is also independent of firm-specific variables

$$mc_{it} = mc_t = \left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \tag{41}$$

In a second step, the $(1 - \zeta_p)$ firms that can change the price, will choose $\tilde{P}_{it}$ to maximize

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\zeta_p)^{s-t} C_s^{-\sigma} \left[\frac{\tilde{P}_{it}}{P_s} - (1+\lambda_f) mc_s \right] Y_s(i) = 0 \tag{42}$$

We focus on a symmetric equilibrium in which all firms choose the same price $\tilde{P}_{it} = \tilde{P}_t$. Let $\tilde{p}_t = \tilde{P}_t / P_t$ the optimal price level. The first order condition for optimal price settings becomes

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\zeta_p)^{s-t} C_s^{-\sigma} \left[\frac{\tilde{p}_{it}}{\pi_s} - (1+\lambda_f) mc_s \right] \left(\frac{\tilde{p}_{it}}{\pi_{t,s}} \right) Y_s = 0 \tag{43}$$

By the law of large numbers, the probability of changing the price coincides with the fraction of firms who change the price in equilibrium. From the zero profit condition, 23, inflation depends on the optimal reset price according to

$$1 = (1-\zeta_p)\tilde{p}_t^{-\frac{1}{\lambda_f}} + \zeta_p \left(\frac{1}{\pi_t} \right) \tag{44}$$

Finally, since the ratio of capital-output is independent of firm-specific factors, the aggregate production function is

$$K_t^{\gamma} H_t^{1-\gamma} = \int_0^1 Y_{it}\; di = \sum_{s=0}^{\infty} \zeta_p (1-\zeta_p)^{t-s} \left(\frac{\tilde{p}_{t-s}}{\pi_{t-s,t}} \right)^{-\frac{1+\lambda_f}{\lambda_f}} Y_t \tag{45}$$

where $K_t = \int_0^1 K_{it}\; di$ and $H_t = \int_0^1 H_{it}\; di$. Each period, $1-\zeta_w$ labour agencies are able to reset the wage $W_t(j)$ to minimise the present discount value of disutility of work

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\zeta_p)^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{\xi}{1+\eta} \int_{\gamma}^{1} H_s(j)^{1+\nu} \, dj \right] \tag{46}$$

subject to (17 and 26). the first order condition for this problem is

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\zeta_p)^{s-t} C_s^{-\sigma} \left[\frac{W_{t+s}(j)}{P_s} - (1+\lambda_w) \frac{\omega H_s(j)^{\nu}}{C_s^{-\sigma}} \right] H_s(j) = 0 \tag{47}$$

We focus on a symmetric equilibrium in which all agencies choose the same wage. Let $w_t = \frac{W_t}{P_t}$. From equation 26 the law of motion of real wage is

$$w_t^{\frac{1}{\lambda_w}} = (1-\zeta_w)\tilde{w}_t^{-\frac{1}{\lambda_w}} + \zeta_w \left(\frac{w_{t-1}}{\pi_t} \right)^{-\frac{1}{\lambda_w}} \tag{48}$$

A.2. Equilibrium condition

To solve the model we define $L_{t+1} = \frac{B_{t+1}^L}{P_t}$ as real long-term bonds. The liquidity parameter ϕ_t follows an exogenous AR(1) process and there are 4 endogenous state variables: the aggregate capital stock, the nominal short-term bond, the real long-term bon and the real wage rate from the previous period $(K_t, B_t^S, L_t, w_{t-1})$. The recursive competitive equilibrium is defined as 9 endogenous quantities $(I_t, C_t, Y_t, K_{t+1}, N_{t+1}, B_{t+1}^S, L_{t+1}, H_t, T_t)$ and 11 prices $(q_t, Q_t^L, Q_t^S, p_t^I, r_t, R_t, \tilde{w}_t, w_t, \tilde{p}_t, \pi_t, mc_t)$ as a function of state variables $(K_t, B_t^S, L_t, w_{t-1}, \phi_t)$, which satisfies the 19 equilibrium conditions (16, 18, 18, 18, 18, 18, 18, 18, 18, 19, 20, 28, 29, 30, 31, 33, 34, 35, 36, 39, 40, 42,43,44,46, 47). Once all market clearing conditions and the government budget constraint are satisfied, the household budget constraint is satisfied by Walras' Law.

A.3. Steady state

In the steady state economy there is change in the resaleability of bonds, nominal price level, prices and endogenous variables. The steady state version of the Euler conditions are respectively

$$\beta^{-1} = \frac{r + \lambda q}{q} + \frac{\gamma(q-1)r}{q(1-\theta q)} \tag{A.1}$$

$$\beta^{-1} = \frac{1 + \lambda Q^L}{Q^L} + \frac{\gamma(q-1)}{q(1-\theta q)} \frac{1 + \lambda \phi Q^L}{Q^L} \tag{A.2}$$

$$\beta^{-1} = \frac{1}{Q^S} + \frac{\gamma(q-1)}{q(1-\theta q)} \frac{1}{Q^S} \tag{A.3}$$

where in the steady state $p^I = 1$ because $\Gamma(1) = \Gamma'(1) = 0$. the non-arbitrage condition between short-term and long-term bonds in steady state implies

$$\frac{1}{Q^S} = \frac{1 + \lambda Q^L}{Q^L} \tag{A.4}$$

The capital-labour ratio is

$$\frac{K}{H} = \frac{\alpha}{1-\alpha}\frac{w}{r} \tag{A.5}$$

Since in the steady state all firms charge the same price, $\tilde{p} = 1$ and real marginal cost is equal to the inverse of markup

$$mc = \left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha} = \frac{1}{1+\delta_f} \tag{A.6}$$

Plugging these two equations into the production function at the steady state we obtain the capital-output ratio which is a function of the rental rate of capital.

$$\frac{Y}{K} = \frac{(1+\zeta_f)r}{\alpha} \tag{A.7}$$

Equation (A.6) can be rewritten as a function of the rental rate

$$w = (1-\alpha)\left(\frac{1}{1+\delta_f}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} \tag{A.8}$$

In the steady state, the real wage is equal to a markup over the marginal rate of substitution between labour and consumption

$$w = (1+\delta_w)\frac{[H\ (1-\gamma)]^{\eta}}{C^{-\sigma}} \tag{A.9}$$

Assuming that $B^S = 0$ and considering $K = N$, the investment function in steady state is

$$I = \gamma \frac{rK + (1 - \lambda\phi Q^L)B^L + \frac{\delta_f}{1+\lambda_f} - T}{1 - \theta q} \tag{A.10}$$

Steady-state investment is the depreciated steady-state capital

$$\frac{I}{K} = (1 - \lambda) \tag{A.11}$$

The resource constraint is

$$Y = C + I \tag{A.12}$$

Finally, from the government budget constraint the steady state tax is

$$T = B(Q^B - \lambda Q^B - 1) \tag{A.13}$$

A.4. Log-linear approximation

Let $\hat{x}_t = log(\frac{x_t}{x})$, where x is the steady state value of x_t. The log-linearized equilibrium conditions are the following:

Investment

$$(1-\gamma)\lambda\hat{p}^I_t + (1-\theta q)\lambda\hat{I}_t - \theta\lambda q\hat{q}_t - \gamma\lambda\phi q\hat{\phi}_t - \gamma\lambda\phi q\hat{Q}^L_t - \gamma(1+\lambda\phi Q^L)\frac{L}{K}\hat{L}_t + \gamma(1+\lambda\phi Q^L)\frac{L}{K}$$
$$-\gamma Q^S\hat{B}^S_t - \gamma r\hat{N}_t + \gamma\frac{T}{K}\hat{T}_t - \gamma\frac{Y}{K}\hat{Y}_t + \gamma\frac{(1-\alpha)r}{\alpha}(\hat{w}_t + \hat{H}_t) + \gamma r\hat{K}_t = \tag{B.1}$$

Euler equation for equity

$$-\sigma\hat{C}_t = -\sigma\mathbb{E}_t[\hat{C}_{t+1}] - \hat{q}_t + \beta\frac{r}{q}\left(1+\gamma\frac{q-1}{1-\theta q}\right)\mathbb{E}_t[\hat{r}_{t+1}] + \beta\gamma r\frac{1-\theta}{(1-\theta q)^2}\mathbb{E}_t[\hat{q}_{t+1}]$$
$$-\beta\gamma r\frac{1-\theta}{(1-\theta q)^2}\mathbb{E}_t[\hat{p}^I_{t+1}] \tag{B.2}$$

Euler equation for long-term bonds

$$-\sigma\hat{C}_t = -\sigma\mathbb{E}_t[\hat{C}_{t+1}] - \hat{Q}^L_t - \mathbb{E}_t[\hat{\pi}_{t+1}] + \beta\lambda\gamma\frac{q-1}{1-\theta q}\phi\mathbb{E}_t[\hat{\phi}_{t+1}] + \beta\left[\lambda + \gamma\frac{q-1}{1-\theta}\phi\right]\mathbb{E}_t[\hat{Q}^L_{t+1}]$$
$$\beta\left[\gamma\left(\frac{1}{Q^L} + \lambda\phi\frac{1-\theta}{(1-\theta q)^2}\right)\right]\mathbb{E}_t[\hat{q}_{t+1}] - \beta\left[\gamma\left(\frac{1}{Q^L} + \lambda\phi\frac{1-\theta}{(1-\theta q)^2}\right)\right]\mathbb{E}_t[\hat{p}^I_{t+1}] \tag{B.3}$$

Euler equation for short-term bonds

$$-\sigma\hat{C}_t = -\sigma\mathbb{E}_t[\hat{C}_{t+1}] - \hat{Q}^S_t - \mathbb{E}_t[\hat{\pi}_{t+1}] + \beta\gamma\frac{(1-\theta)q}{(1-\theta q)^2}\mathbb{E}_t[\hat{q}_{t+1}] - \beta\gamma\frac{(1-\theta)q}{(1-\theta q)^2}\mathbb{E}_t[\hat{q}_{t+1}] \tag{B.4}$$

The resource constraint

$$\hat{Y}_t = \frac{I}{Y}\hat{I}_t + \frac{C}{Y}\hat{C}_t \tag{B.5}$$

The marginal cost

$$\hat{mc}_t = (1-\alpha)\hat{w}_t + \alpha\hat{r}_t \tag{B.6}$$

The Phillips curve

$$\hat{\pi}_t = \frac{(1-\zeta_f\beta)(1-\zeta_f)}{\zeta_f}\hat{mc}_t + \beta\mathbb{E}_t[\hat{C}_{t+1}] \tag{B.7}$$

The capital-labour ratio

$$\hat{K}_t = \hat{w}_t - \hat{r}_t + \hat{H}_t \tag{B.8}$$

The law of motion for aggregate wages

$$\hat{w}_t = (1-\zeta_w)\hat{\tilde{w}}_t + \zeta_w(\hat{w}_{t-1}) \tag{B.9}$$

Wage-setting decisions

$$\left(1+\eta\frac{1+\delta_w}{\delta_w}\right)\hat{\tilde{w}}_t-(1-\zeta_w\beta)\,\eta\frac{1+\delta_w}{\delta_w}\hat{w}_t=(1-\zeta_\beta)\left(1+\eta\frac{1+\delta_w}{\delta_w}\right)\mathbb{E}_t[\hat{\tilde{w}}_t+\hat{\pi}_{t+1}] \quad \text{(B.10)}$$

Aggregate production function

$$\hat{Y}_t=\alpha\hat{K}_t+(1-\alpha)\hat{H}_t \quad \text{(B.11)}$$

The first order condition for capital producers

$$\hat{p}^I_t=\Gamma''(1)\hat{I}_t \quad \text{(B.12)}$$

Identity condition of capital

$$K_{t+1}=(1-\lambda)\hat{I}_t+\lambda\hat{K}_t \quad \text{(B.13)}$$

Government budget constraint

$$\frac{T}{K}\hat{T}_t=\frac{L}{K}(1+\lambda Q^L)\hat{L}_t-\frac{L}{K}(1+\lambda Q^L)\hat{\pi}_t+\hat{B}^S_t+(1+\lambda)(Q^L\frac{L}{K})\hat{Q}^L_t+Q^L\frac{L}{K}\hat{L}_{t+1}+Q^S\hat{B}^S_{t+1} \quad \text{(B.14)}$$

Tax rules

$$\frac{T}{K}\hat{T}_t = \psi_T \left[\frac{L}{K}(\hat{L}_t - \hat{\pi}_t)\right] \tag{B.15}$$

The interest rule rules

$$\hat{R}_t = \psi_\pi \hat{\pi}_t \tag{B.15}$$

Government rule for issuing short-term bonds

$$\hat{B}_t^S = \psi_k \hat{\phi}_t \tag{B.15}$$

Price of short-term bonds

$$\hat{R}_t^S = -log(Q^s) \tag{B.15}$$

B. Bayesian VAR

Consider a VAR(p) model

$$Y_t = \alpha_0 + \sum_{j=1}^{p} A_j Y_{t-j} + \epsilon_t \tag{49}$$

where Y_t is a (3 x 1) vector, α_0 is (3 x 1) vector of intercepts, A_j is a (3 x 3) matrix of coefficients, ϵ_t is a (3 x 1) vector of residuals and ϵ_t is i.i.d. $\mathcal{N}(0, \Sigma)$. An alternative way to write the VAR is the following; let y be a MT x 1 vector $y = (y_1', ..., y_T')$ and similarly $\epsilon = (\epsilon_1', ..., \epsilon_T')$. let $x_t = (1, y_1', ..., y_{t-p}')$ and $X' = [x_1, x_2, ..., x_T]$. K = 1 + Mp is the number of coefficients in each equation of VAR and X is a T x K matrix. The VAR can be written as

$$y = (I_M \otimes X)\alpha + \epsilon \quad ; \quad \epsilon \sim \mathcal{N}(0, \Sigma \otimes I_M) \tag{50}$$

conjugate priors with Normal and inverse Wishart distributions are used for estimation of α and ϵ

$$\alpha|\Sigma \sim \mathcal{N}(\alpha^*, \Sigma \otimes V^*) \tag{51}$$

$$\Sigma^{-1} \sim \mathcal{W}(S^{-1*}, \nu^*) \tag{52}$$

where $\alpha^*, V^*, S^{-1*}, \nu^*$ are the hyperparameters. the posterior distributions have the form

$$\alpha|\Sigma \sim \mathcal{N}(\bar{\alpha}, \Sigma \otimes \bar{V}) \tag{53}$$

$$\Sigma^{-1} \sim \mathcal{W}(S^{-1*}, \nu^*) \tag{54}$$

where

$$\bar{V} = [V^{-1*} + X'X]^{-1} \tag{55}$$

$$\bar{\alpha} = \bar{V}[V^{-1*}A^{*} + X'X\hat{A}] \tag{56}$$

$$\bar{S} = S + S^{*} + \hat{A}'X'X\hat{A} + A^{*'}V^{-1}\hat{A} - \bar{A}'(V^{-1*} + X'X)\bar{A}' \tag{57}$$

$$\bar{\nu} = T + \nu^{*} \tag{58}$$