EMPLOYMENT, LEISURE AND PENSION: INCENTIVES WITH LIMITS

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Abstract

This paper considers two characteristics of a public pension system: the contribution rate and the length of employment. A simple family of optimization models is set up, where the instantaneous utility is a Cobb–Douglas-function of consumption and leisure, furthermore, the life-time utility is a CES-function of the instantaneous utility. We assume that at every instant the individual either works with full capacity or does not work at all. Furthermore, the individuals' parameter values (e.g. of the elasticity of utility with respect to consumption and the expected life-span) differ from the government's ones. First the government calculates its optimal contribution rate and length of employment. That optimal rate is to be paid by the individuals but each can choose how many years he works – for a "proportional" benefit. Since government's actuaries calculate with the average life expectancy, people, correctly expecting to live longer/shorter than average, receive then more/less than they would deserve. This unfairness can only be mitigated by dampened incentives.

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INTRODUCTION

In parallel with the aging of societies, more and more attention is paid to the public pension systems in general and their menacing crises in particular. According to several experts (e.g. Diamond, 1997 and Augusztinovics, 1999), a "parametric reform" is sufficient to preserve the viability of the present systems. Others are convinced that a partial privatization (e.g. World Bank, 1994) or a full privatization (e.g. Kotlikoff, 1997) is necessary.

In this paper we confine our attention only to few, though very important, elements of the public pension system, namely, the *contribution rate* and the *length of employment* (or years of contribution or age at retirement). Assuming full employment and full efficiency, it is evident that the lower the contribution rate or the later the workers retire, the higher the replacement rate, i.e. the ratio of benefit to net earning. Moreover, here we pay much more attention to the length of employment than to the contribution rate.

Although nowadays in most developed countries people live longer and are healthier than some decades ago, they retire earlier. For example, Coile–Gruber (2000) report that 81% of the US male cohort of 62 year-old worked in 1950, and this ratio dropped to 51% by 1995. (Anyway, in the US the normal retirement age is 65 years, while the minimum retirement age is 62 years.)

Of course, everybody agrees that it would be fruitful to stop or even turn around this process. One of the promising measures is to create positive incentives for retiring late and negative incentives against retiring early (e.g. Börsch–Supan, 1998; Samwick, 1998 and Coile–Gruber, 2000). The literature speaks of *actuarial fairness*, when the "expected" discounted life-time pension is equal to the "expected" discounted life-time saving, regardless of the age at retirement. We shall discuss the meaning of expectation later on.

This solution looks very attractive, since everybody can choose between receiving higher benefits for a shorter period or a lower benefit for a longer period without apparently jeopardizing the balance of the system. Since the individual age at death is not known at retirement, it is customary to calculate with the average life-expectancy at the minimum retirement age. But what justifies the assumption of the same life-expectancy at the minimal age of retirement, regardless of the chosen retirement age?

As a reaction, several researchers have recently conjectured that statistically, the longer somebody works, the longer he lives: therefore the allegedly fair system favors people with higher than average life-spans, and punishes those with lower than average life-spans (Simonovits, 1999, 2000; Gruber–Orszag, 1999 and Guegano, 2000). This distortion is probably amplified by another positive correlation: people with higher earnings statistically live longer – and work longer (World Bank, 1994).

In this paper we set up the simplest possible family of models, where everybody is uniformly healthy, the saving/contribution rate or the length of employment are matters of choice. It is assumed that at every instant every individual either works with full capacity or does not work at all. Every individual's specific instantaneous utility is a Cobb–Douglas-function of consumption and leisure; furthermore, the life-time utility is a CES-function of the instantaneous utility (cf. Correira, 1999, Section 3). In Section 1 every individual determines the optimal contribution rate and the optimal length of employment. In Section 2 we introduce a government which sets its own optimal contribution rate and constrains the individual to optimize only his retirement age. (We call the reader's attention to other pension models where a political voting determines the size of the contribution rate (for the latest treatment, Casamatta et al., 2000).) The government and the individuals have different, though unbiased estimates of life-expectancy: *asymmetric information*. While the pension benefit is determined according to the former estimate, the individual's utility function is based on the latter. The derived optimal retirement age is an increasing function of the individual estimate of lifespan. Thus the positive correlation between individual lifespans and retirement ages is established, undermining the fairness of the system. In Section 3, it is shown that to diminish the arising distortions, the incentives have to be dampened. Our analytical results are supported by numerical illustrations. For example, people living five years more, optimally work two years more under undampened incentives and about one year more with dampened incentives. In turn, the imbalances due to incentives are lower and more uniform in the latter case than in the former. In an Appendix, we shall consider the macrobalance under undampened incentives.

To our knowledge, all incentive systems of delayed public retirement use some dampening, via piecewise linear rules: every additional year/month adds a certain percent point to the full benefit. Diamond (2000, p. 11) criticizes this practice: "As workers age, mortality probabilities rise. A linear formula is not a good one. Therefore, to offset a delay of benefits, it is necessary to give larger increases in benefits the older the worker who is delaying the start of benefits." In my opinion, the relevant question is not whether the benefit is linear or not but whether the harmony between efficiency and insurance is assured or not. I admit, however, that for the time being I do not quite know how to define harmony here.

To appreciate the context, note that the present paper examines an *adverse selection* problem (e.g. Arrow, 1963), where people with above average lifespans participate in the scheme disproportionately. This distinguishes it from an otherwise related model like Stock–Wise (1990), which considers the conflict between the retirement incentives of public and private pension systems. We mention two similar problems, *not* discussed here. (i) Diamond–Mirrlees (1986) extended the discussion to the issue that older workers may become *invalid* randomly but some of them only pretend invalidity. Moreover, their government operates a very sophisticated optimal incentive system, which is not actuarially fair, either. On the other hand, these authors assumed common life-expectancy and utility function for every worker. (ii) Philipson–Becker (1998) addressed the question: what happens if higher life-annuity (including health care) implies higher lifespan? In both cases, actuarial fairness also disappears.

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1. OPTIMAL CONTRIBUTION AND LENGTH OF EMPLOYMENT

In this paper a family of models is set up, where the sharp dichotomy between working and being retired is derived from the simultaneous optimization of consumption and leisure, deliberately disregarding the effects of aging. In this Section the simplest case is discussed, where every individual is free to choose how much to save for his old age and how long to work.

For simplicity, we neglect issues of childhood, growth, inflation and the existence of real interest rate. Time is continuous, therefore flows like benefits and contributions are intensities. Expounding the ideas mentioned in the Introduction, we shall make the following assumptions.

A1. An individual starts to work at an age denoted by 0.

A2. An individual expects to live for D years, where D is a positive real number and its value is known to him in advance.

A3. Because of indivisibility, at every instant the individual chooses either *minimal leisure* l_{m} or *maximal leisure* l_{M}, $0 < l_{\mathrm{m}} < l_{\mathrm{M}}$. By normalization, the individual *works* with intensity $l_{\mathrm{M}} - l_{\mathrm{m}} = 1$ or *retires* and has maximal leisure. The *length of employment*, being a positive real, is denoted by R.

A4. Regardless of age, an individual has the same total wage rate (normalized to 1) and the pensioner has a pension benefit, paid from accumulated worker's savings.

A5. Because of our wage concept and zero interest rate, the budget constraint is given by the equality of life-time consumption and life-time earnings:

$$\int_0^D c(t)\,dt = R.$$

A6. In addition to l_{m} and l_{M}, an individual objective function is characterized by two reals: (σ, ν), where $\sigma < 1$ and $0 < \nu < 1$. $1/(1-\sigma)$ is called the *intertemporal elasticity of substitution* and ν is called the *elasticity of utility with respect to consumption.* The instantaneous utility is the product of consumption $c(t)$ to the power ν, and leisure $l(t)$ to the power $1-\nu$. Further, the life-time utility is a CES-function:

$$u = \sigma^{-1}\int_0^R [c^{\nu}(t) l_{\mathrm{m}}^{1-\nu}]^{\sigma}\,dt + \sigma^{-1}\int_R^D [c^{\nu}(t) l_{\mathrm{M}}^{1-\nu}]^{\sigma}\,dt.$$

The individual maximizes his life-time utility function subject to his budget constraint.

We shall comment some of the assumptions.

Ad 2. A most important element of a good life is that the participant insures himself against longevity risk. (One reason, that an individual needs social security with life-annuity, is that it is very expensive to buy private life-annuity.) The expected value of the integral with uncertain upper limit is equal to the value of the integral with certain upper limit ($D = \mathbf{E}\tilde{D}$):

$$\mathbf{E}\int_0^{\tilde{D}} c(t)\,dt = \int_0^{\mathbf{E}\tilde{D}} c(t)\,dt$$

and similarly for the time-additive life-time utility.

Ad 3. Note that in our model *part-time work*, i.e. $l_{\mathrm{m}} < l(t) < l_{\mathrm{M}}$ is not feasible, although without this restriction, a genuine optimal leisure would be a constant $l(t) \equiv l$, fixed between the two bounds all over his life. Of course, the neglected biological factors (which could be expressed by $l_{\rm M}$ declining with age) strongly justify that people work first and retire later.

Ad 4. In general, earnings increase with age and with calendar time. These important facts are also ignored here. For simplicity, we assume away personal income tax (health contribution can be either included or not) but

Ad 5. In this paper we skip the complexity of savings and credits, therefore we neglect the changes within the working and retirement periods. Furthermore, a significant part of the population spends all its current income, thus this assumption is valid for the members of this group. Note that we have excluded unemployment.

At this point we introduce the ratio of minimal and maximal leisure: $\lambda = l_{\rm m}/l_{\rm M}$. Of course, $0 < \lambda < 1$. We shall denote the worker's saving rate by τ, $0 < \tau < 1$.

First we present a simple lemma without proof.

Lemma 1. *a) Under our assumptions, the optimal consumption path is also dichotomous: the worker's consumption is a constant $a = 1 - \tau$ and the pensioner's consumption is another constant b.*

b) The original budget constraint given by an integral reduces to an algebraic equality

$$\tau R = b(D - R), \tag{1}$$

c) and the original life-time utility given by an integral reduces to an algebraic function

$$u = \sigma^{-1}[\lambda^{(1-\nu)\sigma}(1-\tau)^{\nu\sigma}R + b^{\nu\sigma}(D - R)]. \tag{2}$$

In the pension literature τ and $\beta = b/a$ are called *contribution rate* and *replacement rate*, respectively. We shall also need the following relations between various variables:

$$b = \frac{\tau R}{D - R}, \qquad \beta = \frac{\tau R}{(D - R)(1 - \tau)} \quad \text{and} \quad \tau = \frac{\beta(D - R)}{R + \beta(D - R)}. \tag{3}$$

Note that in (3a) the benefit is a strongly nonlinear function of retirement age, as required by Diamond (2000).

We turn now to the characterization of the individually optimal contribution rate and length of employment.

Theorem 1. *For given and known parameter values $(\sigma, \lambda, \nu, D)$, the individual maximizing (2) under (1) has an optimal replacement ratio (the optimality index will be dropped)*

$$\beta^{\rm o} = \lambda^{(1-\nu)\sigma/(\nu\sigma - 1)}$$

and his optimal length of employment, $R^{\rm o}$ satisfies the quadratic equation

$$q(R) = \sigma^{-1}\mu(1-\beta)R^2 + \mu\beta(\sigma^{-1}+\nu)DR + \beta^{\nu\sigma+1}\nu D^2 = 0, \quad (\mu = \lambda^{(1-\nu)\sigma} - \beta^{\nu\sigma},) \tag{4}$$

assuming that

$$q(D) = \sigma^{-1}\mu + \mu\beta\nu + \beta^{\nu\sigma+1}\nu < 0. \tag{5}$$

Remarks. 1. Note that in our model the optimal replacement ratio is less than 1, because of $0 < \lambda < 1$ and $\sigma < 0$. Note, however, the difference from the familiar problem of optimal consumption. There our elasticity-dependent discount factor $\lambda^{-(1-\nu)\sigma}$ is replaced by a constant $\delta > 1$ (due to more leisure at retirement than at work), and the ratio of the later consumption to the earlier consumption is larger than 1, regardless of the sign of σ.

2. Since the principal coefficient of $q(R)$ is negative and constant coefficient of $q(R)$ is positive, $q(R)$ has in fact a unique positive root, denoted by R^{o}.

3. It is not so easy to see the stringency of (5), because μ and β^{o} depend on σ, λ and ν. In the example given after the proof, for a Cobb–Douglas life-time utility function we shall see that a certain condition has to hold for ν and λ. If we obtain that it is optimal to work until death, we should not forget about our assumption according to which the individual health remains constant until death.

Proof. First we determine the optimal contribution rate for a fixed length of employment, then we optimize the length of employment.

a) Substituting (3a) into (2) yields

$$u(\tau) = \sigma^{-1}\lambda^{(1-\nu)\sigma}(1-\tau)^{\nu\sigma}R + \sigma^{-1}(\tau R)^{\nu\sigma}(D-R)^{1-\nu\sigma}. \tag{6}$$

Taking the derivative of u with respect to τ and making the derivative to zero:

$$u'(\tau) = \lambda^{(1-\nu)\sigma}\nu(1-\tau)^{\nu\sigma-1}(-R) + \nu\tau^{\nu\sigma-1}R^{\nu\sigma}(D-R)^{1-\nu\sigma} = 0.$$

Rearranging:

$$\frac{\tau}{1-\tau} = \lambda^{(1-\nu)\sigma/(\nu\sigma-1)}\frac{D-R}{R}.$$

In view of (3b), the optimal replacement rate is equal to the foregoing power of λ, also yielding the corresponding contribution rate.

b) Now $b(R) = \beta a(R)$ (β is independent of R) and with the optimally chosen contribution rate, by (2), the restricted life-time utility function is equal to

$$u^{\mathrm{o}}(R) = \sigma^{-1}[\lambda^{(1-\nu)\sigma}R + \beta^{\nu\sigma}(D-R)]a^{\nu\sigma}(R) = \sigma^{-1}(\mu R + \beta^{\nu\sigma}D)a^{\nu\sigma}(R). \tag{7}$$

Take the derivative of u^{o} with respect to R:

$$u^{\mathrm{o}\prime}(R) = \sigma^{-1}\mu a^{\nu\sigma}(R) + (\mu R + \beta^{\nu\sigma}D)\nu a^{\nu\sigma-1}(R)a'(R).$$

Take into account an implication of (3c), namely,

$$a(R) = \frac{R}{(1-\beta)R + \beta D} \quad \text{and} \quad a'(R) = \frac{\beta D}{[(1-\beta)R+\beta D]^2} = \frac{\beta D a(R)}{(1-\beta)R^2 + \beta DR}.$$

Substituting back $a'(R)$ into $u^{\mathrm{o}\prime}(R)$ and dropping factor $a^{\nu\sigma}(R)$, we obtain the necessary condition for local optimum:

$$\sigma^{-1}\mu + (\mu R + \beta^{\nu\sigma}D)\nu\frac{\beta D}{(1-\beta)R^2 + \beta DR} = 0.$$

Rearranging yields (4). If (4) is divided by D^2, then another equation obtains for R/D, the coefficients of which only depend on the parameters of the utility function. Since $q(0) > 0$ and $q'(0) > 0$, condition $0 < R^{\mathrm{o}} < D$ is equivalent to $q(D) < 0$, i.e. (5). ∎

For illustration we shall present an example, which is an excluded limiting case.

Example 1. Cobb–Douglas life-time utility function (cf. Section VII of Diamond–Mirrlees, 1984). Let the instantaneous utility function be $\nu \log c + (1-\nu)\log l$, corresponding to the limiting case $\sigma = 0$.

a) Then $\beta^{\rm o} = 1$ and $a(R) = b(R) = R/D$. If we had obtained a realistic $\beta^{\rm o} < 1$ rather than unity, perhaps the whole struggling with CES functions would have been superfluous. The application of Cobb–Douglas instantaneous utility function, however, proved to be very handy.

b) It would be quite difficult to take the limit at the optimization of length of employment, therefore we repeat the main steps of the second stage in this special case.

$$u^{\rm o}(R) = D\nu \log \frac{R}{D} + (1-\nu)R\log\lambda + (1-\nu)D\log l_{\rm M} \to \max, \quad \text{subject to} \ (1);$$

$$u^{\rm o\prime}(R) = \nu\frac{D}{R} + (1-\nu)\log\lambda = 0,$$

i.e. the optimal length of employment is given by

$$R^{\rm o} = \frac{\nu}{(1-\nu)\log(1/\lambda)}D,$$

assuming that $R^{\rm o} < D$, i.e. $[\nu/(1-\nu)]\log(1/\lambda) < 1$. This bound is always effective for ν, e.g. for $\lambda = 1/e$, $\nu < 1/2$. ∎

We shall portray Theorem 1 with a series of numerical examples in Table 1. We only fix the lifespan: $D = 50$ years. At the interpretation of the result, do not forget our practice of calculating ages from the time of entering work, say from 20 years, i.e. $D = 50$ reads as 'seventy' years.

Table 1. *Parameters and optima*

		Optimal	
Min/max ratio λ	Elasticity for consumption ν	Length of employment $R^{\rm o}$	Replacement ratio $\beta^{\rm o}$
0.2	0.3	9.7	0.245
	0.4	20.8	0.342
	0.5	40.5	0.447
0.3	0.3	16.1	0.349
	0.4	32.5	0.448
0.4	0.3	24.4	0.449
	0.4	47.5	0.543
0.5	0.3	36.0	0.545

It is obvious that the optima are very sensitive to the parameter values. In the first row, the ratio of maximal and minimal leisure and the elasticity of utility with respect to consumption are very low: 0.2 and 0.3, respectively. Then the optimal length of employment and replacement ratio are ridiculously low: less than 10 years and 1/4, respectively. An increase in the elasticity itself raise both optima steeply: to 40.5

years and 44.5%, respectively. The raising of the min/max ratio has a similar effect, the lengths of the blocks become shorter and shorter, because the optima cease to be internal – and this case is excluded.

As a side remark, note that a rather similar model was developed for the optimal duration of migration by Stark et al. (1997).

2. OPTIMAL LENGTH OF EMPLOYMENT, GIVEN CONTRIBUTION

In this Section we shall move to a more realistic model with a government which sets the mandatory contribution rate but the individuals can still determine how long to work: *constrained choice*.

Obviously, to have a relevant model we must assume that the government and the individuals have different parameter values for σ, λ, ν and D. Probably, the government does not know the individuals' parameter values.

The government has its own σ^*, ν^* λ^* and D^* and it determines its 'own' optimal replacement ratio β^*, contribution rate τ^* and length of employment R^*. The individuals' parameters are denoted as before and their decisions are distinguished by a tilde.

We only emphasize the modified assumptions.

A$\tilde{1}$. Every individual starts to work at the same age denoted by 0.

A$\tilde{2}$. Every worker has an unbiased expectation of his lifespan D.

A$\tilde{3}$. The minimal leisure l_{m} and maximal leisure l_{M} and their ratio λ may vary with individuals but the normalization $l_{\mathrm{M}} - l_{\mathrm{m}} = 1$ should hold.

A$\tilde{4}$. Regardless of age, every worker has the same total wage rate (normalized to 1), and he contributes a constant share τ^* to social security.

A$\tilde{5}$. The government announces that if somebody works R years, he will obtain a life-time per-period benefit

$$(\tilde{3}) \qquad \tilde{b}(R) = \frac{\tau^* R}{D^* - R}, \qquad 0 < R < D^*$$

derived from constraint (1) with the average lifespan D^*, τ^* is determined by A$\tilde{6}$. We speak of *undampened incentives* in this case.

A$\tilde{6}$. The government and the individuals maximize their own utility functions.

We shall again comment our (modified) assumptions.

Ad $\tilde{1}$. Individuals participate in learning and child care in different extent, therefore the lengths of their employment differ, even if the individuals retire at the same age. This important fact is neglected here.

Ad $\tilde{2}$. Assume that an individual *expects* to live for $D = \mathbf{E}\tilde{D}$ years. The government takes the average of the individual expectations: D^*, this is also unbiased but at an aggregate level. The difference between expectations leads to *asymmetric information.*

In the bulk of the literature, it is assumed that the individuals and the government have the same unbiased expectations: $\tilde{D} = D^*$ or more generally, there is no statistical correlation between the age at retirement and age at death. As already emphasized, these assumptions are replaced in this paper. In the simplest case, the individuals know their lifespans: $\tilde{D} = D$, also varying with the individuals.

Ad $\tilde{3}$. The dispersion of individual elasticities is much less interesting than that of the lifespans. Nevertheless, if individuals only differed in lifespans (and the government knew the common values of the utility parameters), then in principle, the government would be able to infer the individual lifespans from the individual optima and could manipulate the rules accordingly.

Ad $\tilde{4}$. It would be interesting to consider heterogeneity of earnings, because most pension formulas are not strictly earnings related. Moreover, there is a strong correlation between life-time earnings and lifespans, mentioned already in the Introduction. However, we omit these complications.

Ad $\tilde{5}$. Having introduced mandatory savings, our wage concept definitely includes both the employee and the employer pension contributions.

In reality there is a sharp distinction between the *entry pension* given at the first period (generally year) of retirement and the *pension in progress* calculated from the benefits of the previous period. A further complication is that the entry pension is generally based only on a subperiod of the earning stage and earnings are only partially taken into account. A third complexity is the following: in addition to own-right old-age pension, important role is played by disability and survivor's pensions. The significant deviation between male and female life-expectancies are deliberately neglected in the pension formulas. It is also assumed that only a negligible part of the population dies during the employment (definitely not the case in Hungary) and life-expectancies are calculated at an age close the usual retirement age. Finally we underline the artificial limitation: nobody can retire after passing the expected lifespan D^*, because he would get negative benefit by ($\tilde{3}$).

Ad $\tilde{6}$. One reason that mandatory pension systems exist is that the government wants to prevent shortsighted individuals from not saving for their old age. If the individual parameter values were much different from the government's, then very short employment length could occur (e.g. in the first row in Table 1), transforming the pension benefit into unemployment aid.

Of course, our model does not apply to such hectic pension systems as the Hungarian system was in the last decade. It has often occurred that somebody worked longer than it was necessary and by now he still receives much lower benefit than if he had retired earlier and enjoyed the properly indexed pension in progress.

We can now formulate

Theorem 2. *If the government sets the contribution rate* τ^* *which is its own optimum corresponding to parameters* $(\sigma^*, \lambda^*, \nu^*, D^*)$ *and if the individual parameter values* $(\sigma, \lambda, \nu, D)$ *do not deviate from the government values too much, then the individual's optimal length of employment* $(\tilde{R})$ *under benefit rule* ($\tilde{3}$) *satisfies*

$$(\tilde{8}) \qquad \tilde{p}(R) = \sigma^{-1}\lambda^{(1-\nu)\sigma}(1-\tau^*)^{\nu\sigma} + \left[\nu\frac{(D-R)D^*}{(D^*-R)R} - \sigma^{-1}\right]\left(\frac{\tau^* R}{D^*-R}\right)^{\nu\sigma} = 0,$$

assuming that

$$(\tilde{9}) \qquad \tilde{p}(D) < 0.$$

Proof. Since the government's optimal contribution rate is generally not optimal for the individuals, we have to return from (7) to the counterpart of (6). But now we have to optimize with respect to R from the beginning rather than first optimizing along τ:

$$\tilde{u}(R) = \sigma^{-1}\lambda^{(1-\nu)\sigma}(1-\tau^*)^{\nu\sigma}R + \sigma^{-1}\tilde{b}^{\nu\sigma}(R)(D-R).$$

Take the derivative of $\tilde{u}$ with respect to R and equalize it to zero:

$$(\tilde{10}) \qquad \tilde{u}'(R) = \sigma^{-1}\lambda^{(1-\nu)\sigma}(1-\tau^*)^{\nu\sigma} + \nu\tilde{b}^{\nu\sigma-1}(R)\tilde{b}'(R)(D-R) - \sigma^{-1}\tilde{b}^{\nu\sigma}(R) = 0.$$

Taking into account $(\tilde{3})$, we obtain

$$(\tilde{11}) \qquad \tilde{b}'(R) = \frac{\tau^* D^*}{(D^*-R)^2} = \tilde{b}(R)\frac{D^*}{(D^*-R)R}.$$

Substituting $(\tilde{11})$ into $(\tilde{10})$, yields $(\tilde{8})$. According to our assumption, the two parameter vectors do not deviate too much from each other, thus by continuity, $(\tilde{9})$ guarantees the existence of a local optimum. ∎

$(\tilde{11})$ provides the percentage change in the 'fair' benefit: for example, with $R =$ 35 year, $\tilde{b}'(R)/\tilde{b}(R) = 50/(15 \cdot 35) = 0.095$/year is quite close to the US reward 7% percent/year.

To evaluate actuarial fairness, we need the concept of *per period net contributions*: $\tilde{z} = \tau^* - \tilde{b}(\tilde{R})(D-\tilde{R})/\tilde{R}$.

We separately formulate a

Corollary. *a) The longer an individual is expected to live, the longer he works. b) If the individual has the same utility function as the government has, but his lifespan is longer/shorter than the government estimation, then his optimal length of employment is longer/shorter than the government optimum, and his life-time net contribution is negative/positive. c) If the individual's lifespan is equal to the average, but his elasticity for consumption is higher/lower than the government's, then his optimal length of employment is longer/shorter than the government's optimum but the per period net contribution is equal to zero.*

Proof. a) In equation $(\tilde{8})$ D only appears at a single place where it is multiplied by a positive quantity, i.e. for given R, $\tilde{q}$ is an increasing function of D. Thus for $D_1 > D_2$, $\tilde{q}_1 > \tilde{q}_2$, i.e. $\tilde{R}_1 > \tilde{R}_2$.

b) For $D > D^*$ a) yields $\tilde{q} > q^*$, therefore $\tilde{R} > R^*$. Since this individual lives longer than the average, he also receives the benefit – calculated for the average lifespan – longer than the average person figuring in the government calculation. ∎

We proceed with the analytical illustration.

Example 1. Continuation. Let us go shortly through the general calculations in the special case of Cobb–Douglas life-time utility function:

$$\tilde{u}(R) = \nu R\log(1-\tau^*) + (1-\nu)R\log l_{\mathrm{m}} + \nu(D-R)\log\tilde{b}(R) + (1-\nu)(D-R)\log_{\mathrm{M}},$$

$$\tilde{u}'(R) = \nu\log(1-\tau^*) + (1-\nu)\log l_{\mathrm{m}} - \nu\log\tilde{b}(R) + \nu(D-R)[\log\tilde{b}(R)]' - (1-\nu)\log_{\mathrm{M}},$$

where

$$[\log \tilde{b}(R)]' = \frac{1}{R} + \frac{1}{D^* - R}, \quad \text{i.e.} \quad \nu(D-R)[\log \tilde{b}(R)]' = \nu \frac{(D-R)D^*}{(D^*-R)R},$$

$$\log(1-\tau^*) = \log R^* - \log D^* \quad \text{and} \quad \log \tau^* = \log(D^* - R^*) - \log D^*.$$

Substituting into the necessary condition for local optimum and making the computations:

$$\nu \log R^* + (1-\nu)\log\lambda - \nu \log \tilde{R} - \nu \frac{(D-\tilde{R})D^*}{(D^*-\tilde{R})\tilde{R}} = 0.$$

Comparing with the original Example 1, it can be seen that even in the special case of $D^* = D$, in general $R^* \neq \tilde{R}$. It is remarkable that even the Cobb–Douglas example lacks the explicit solution for the constrained optimum. ∎

Computer simulation is by now inevitable. For illustration, we chose $\lambda^* = 0.4$; $\nu^* = 0.35$; $D^* = 50$ year for the government, yielding $\tau^* = 0.183$ (σ remains –2). The individual values of the parameters ν and D are dispersed around these values.

Table 2. *Optima at undampened incentives*

		Optimal		
Individual lifespan D	Elasticity for consumption ν	Length of employment $\tilde{R}$	Replacement ratio $\tilde{\beta}$	Per-period net contribution $\tilde{z}$
45	0.30	29.1	0.310	0.044
	0.35	32.5	0.413	0.052
	0.40	34.9	0.516	0.060
50	0.30	31.0	0.365	0
	0.35	34.5	0.496	0
	0.40	37.0	0.633	0
55	0.30	32.9	0.431	–0.054
	0.35	36.5	0.605	–0.068
	0.40	39.0	0.792	–0.083

It is evident that the optimal length of employment is very sensitive to the individual lifespan and the elasticity. For example, if the lifespan increases by 5 years, then the optimal length of employment increases by about 2 years (cf. Corollary b). If the elasticity increases by 0.05, then the corresponding value jumps about 3 years. We underline that except for the middle third, where the individual and government lifespans are the same, the per period net contributions $\tilde{z}$ are not zero: people with shorter/longer than average lifespan contribute more/less than needed (cf. Corollary b and see first and third parts of Table 2, respectively). The absolute value of deviation increases in both directions with the increase of the elasticities.

3. DAMPENED INCENTIVES

We have seen in Section 2 that the so-called actuarially fair system is far from being fair. As is known from the economics of information, to find a compromise between efficiency and insurance the government have to dampen the incentives.

In practice, the government can dampen the incentives. Of course, the weaker the incentives, the stronger the insurance. Similar results can be obtained from the analysis of a *progressive* US or Hungarian formula where higher earnings and longer employment yield marginally less benefit.

For our purposes, it is worth citing the various German schedules. A weak reward/premium system was introduced in Germany in 1972. It was planned to be replaced in the coming years only by a partially 'corrected' system. (The new Social-Democrat–Green government immediately suspended this measure in 1998.) These are presented in Table 3 based on a figure in Börsch-Supan (1998) together with the 'fair' solution.

Table 3. *Flexible retirement: Germany*

Age (years)	60	63	65	67	70
Benefit -1972	100	100	100	105	105
Benefit- 2004	80	90	100	110	130
'Fair'	72	85	100	120	160

We speak now of *dampened incentives* if 1. the benefit is an increasing function of the length of employment and 2. this function lies lower/higher than the so-called actuarially fair benefit function for people working longer/shorter than the government optimum:

$$b(R) < \tilde{b}(R) \quad \text{if} \quad R > R^* \quad \text{and} \qquad b(R) > \tilde{b}(R) \quad \text{if} \quad R < R^*.$$

We shall modify our assumption A$\tilde{5}$ as follows.

A$\hat{5}$. The government uses a dampened incentive system with a benefit function $b(R)$.

Modifying Theorem 2, we have

Theorem 3. *If the government uses a dampened incentive system with a benefit function $b(R)$, then the optimal length of employment ($\hat{R}$) of an individual satisfies*

$$(\hat{8}) \qquad p(R) = \sigma^{-1}\lambda^{(1-\nu)\sigma}(1-\tau^*)^{\nu\sigma} + \nu b^{\nu\sigma-1}(R)b'(R)(D-R) - \sigma^{-1}b^{\nu\sigma}(R) = 0.$$

Proof. In the proof of Theorem 2 replace $\tilde{b}$ by b, etc. ∎

For illustrations, it will be helpful to discuss two special dampened incentive systems, displayed with the undampened one in Figure 1.

(Figure 1)

Example 2. Let us specify the *logarithmically dampened benefit function* with factor α, $0 < \alpha < 1$, where the modified benefit is the product of two powers: the government optimum $b^* = \tilde{b}(R^*)$ to the power $(1-\alpha)$ and the 'fair' benefit taken to power α. In formula:

$$b(R) = b^{*1-\alpha}\left(\frac{\tau^* R}{D^* - R}\right)^{\alpha}, \qquad 0 < R_{\mathrm{m}} \le R \le R_{\mathrm{M}} < D^*,$$

where R_{m} and R_{M} are the minimal and the maximal age for retirement, respectively.

$$\hat{b}'(R) = b^{*1-\alpha}[\tilde{b}^{\alpha}(R)]' = b^{*1-\alpha}\alpha\tilde{b}^{\alpha-1}(R)\tilde{b}'(R) = \alpha b(R)\frac{\tilde{b}'(R)}{\tilde{b}(R)}.$$

Thus the optimal length of employment $(\hat{R})$ of an individual, following $(\hat{3})$, satisfies

$$p(R) = \sigma^{-1}\lambda^{(1-\nu)\sigma}(1-\tau^*)^{\nu\sigma} + \left[\alpha\nu\frac{(D-R)D^*}{(D^*-R)R} - \sigma^{-1}\right] b^{*(1-\alpha)\nu\sigma}\left(\frac{\tau^* R}{D^* - R}\right)^{\alpha\nu\sigma} = 0.$$

Choosing α well, the conflict between incentives and insurance can be weakened. This is demonstrated in Table 4 with $\alpha = 0.8$. We introduce notation $\hat{z} = \tau^* - b(\hat{R})(D - \hat{R})/\hat{R}$.

Table 4. *Optima at a logarithmically dampened incentive*

		O p t i m a l		
Individual lifespan D	Elasticity for consumption ν	Length of employment $\hat{R}$	Replacement ratio $\hat{\beta}$	Per period net contribution $\hat{z}$
45	0.30	26.5	0.289	0.018
	0.35	30.8	0.381	0.038
	0.40	33.9	0.475	0.055
50	0.30	28.5	0.328	–0.020
	0.35	32.8	0.440	–0.006
	0.40	36.1	0.561	0.005
55	0.30	30.4	0.372	–0.064
	0.35	35.0	0.515	–0.058
	0.40	38.3	0.677	–0.059

Due to the dampening, the deviations of the per period net contributions have diminished: in the first row, 4.4% drops to 1.8%, while in the last row –8.3% 'increases' to –5.9%. At the same time, the balance is also destroyed for individuals having average lifespans, even if the arising imbalances are not very large. Unfortunately, even such a modest dampening sharply reduces the work effort and the replacement rate of everybody but especially the less diligent: people in the first row work for 26.5 years rather than 29.1 years and have to accept a replacement rate 28.9% instead of the previous 31.5%. Even for the hardest working and longest living people, the optimal length of employment drops by 0.7 years and their relative benefits sunk from 79.2% to 67.7%. ∎

Now we turn to another dampened incentive system which is quite widespread in practice.

Example 3. Linear benefit function with a dampening factor $\alpha = 0.8$: $b(R) = b^* + \alpha\tilde{b}^{*\prime}(R - R^*)$, where $b^{*\prime} = \tilde{b}'(R^*)$. Note that low enough R may imply negative benefits, calling for a lower bound on length of employment.

By now it is enough to present the new series of calculations without giving the formulas.

Table 5. *Optima at the linear benefit function*

	O p t i m a l			
Individual lifespan	Elasticity for consumption	Length of employment	Replacement ratio	Per period net contribution
D	ν	$\hat{R}$	$\hat{\beta}$	$\hat{z}$
45	0.30	30.6	0.351	0.047
	0.35	32.4	0.418	0.049
	0.40	34.1	0.482	0.057
50	0.30	31.6	0.391	–0.003
	0.35	33.6	0.464	–0.002
	0.40	35.4	0.532	0.004
55	0.30	32.6	0.428	–0.057
	0.35	34.8	0.507	–0.059
	0.40	36.7	0.579	–0.053

We do not repeat the in-depth analysis of the previous cases, but mention that even the fine details of the incentive mechanism have a profound influence of the outcomes.∎

It is easy to see that no benefit function can assure zero individual balance in general. Indeed, consider two persons with the same optimal employment length but different lifespans and utility functions. These two individuals receive the same monthly benefit for the same total contribution but they enjoy the benefit stream for different periods.

It is an open question what a good benefit function is and how a good macro balance is assured by it. We just outline a possible solution.

Let $n > 1$ be an integer and let distinguish n types of individuals in the population with lifespan D_k, probability p_k, $k = 1, \ldots, n$. Let $b(R)$ be a possible benefit function. Individual's of type k optimal length of employment is $\hat{R}_k$, his per period net contribution is $\hat{z}_k = \tau^* - \tau^*(D_k - \hat{R}_k)/(D^* - \hat{R}_k)$.

Let the *welfare functional*, depending on the benefit function b, be an increasing function of the individual utilities. To avoid simple sums, we use again CES-functions, now with a negative power $\varepsilon < 0$. Thus we limit the substitution among individual utilities. Since $u_k < 0$, we have to apply the following transformation: $U[b] = \sum_k p_k|u_k(R_k)|^\varepsilon$; now we do not need the coefficient ε^{-1}. The *aggregated net contribution* is $Z[b] = \sum_k p_k z_k$. The we have to find a benefit function b^{o}, for which the welfare functional is maximal: $U[b^{\text{o}}] \geq U[b]$, assuming that all functions considered assure zero balance: $Z[b^{\text{o}}] = Z[b] = 0$.

If one considers continuous rather than discrete distributions of types, then one gets a degenerate isoperimetric calculus of variation problem, where both the objective and the constraint functions have to be determined numerically.

4. CONCLUSIONS

In this paper we have examined a family of models, where the government sets the optimal contribution rate and the corresponding benefit proportional to the length of employment according to its own preferences and the individual can only decide how long to work at this constraint. As is known from the *principal–agent* literature, the conflict between efficiency and insurance cannot be eliminated but only mitigated. Our numerical examples amply demonstrate the power of over-rewarding and the limits of dampening.

Further analysis is needed to clarify the sensitivity of our results to the assumptions. We believe that our simplifying assumptions (e.g. time- and age-invariant earnings, zero interest rate, etc.) do not change the essence of our observations. It is probably much more problematical that we have made drastic simplifications in describing the pension system: the original weakness of the connection between contributions and benefits, the resulting avoidance of contributions and differing tax treatment of earnings and benefits.

APPENDIX: MACROBALANCE AT UNDAMPENED INCENTIVES

At the end of Section 2, discussing Table 2 we have mentioned that on balance, the government loses money on this business. In this Appendix we would like to make this statement precise.

Even the table mentioned above demonstrates that if the society consists of individuals with short lifespan and high consumption elasticity (row 3) and those with long lifespan and low consumption elasticity (row 7), then the balance is 0.006, i.e. positive. The reason is that here the short-lived individual works more than the long-lived one, turning the sign of the balance to the opposite. In the sequel we are looking for assumptions which eliminate such a case. We do not need explicit optimization, thus we omit tilde from the length of employment. Let us start with the simplest case.

Example A1. There are two types: short and long-lived with lifespans D_1, D_2, $D_1 < D_2$, with employment lengths R_1, R_2, $R_1 < R_2$, respectively. Their shares in the population are p and $1-p$, $0 < p < 1$, respectively. Therefore the average life expectancy is $D^* = pD_1 + (1-p)D_2$. The so-called fair benefit is given by $\tilde{b}_k = \tau^* R_k/(D^* - R_k)$, $k = 1,\ 2$. A simple calculation yields the per period net contributions: $\tilde{z}_k = \tau^* - \tau^*(D_k - R_k)/(D^* - R_k) = \tau^*(D^* - D_k)/(D^* - R_k)$.

We shall show that the aggregate balance is negative: $\tilde{Z} = p\tilde{z}_1 + (1-p)z_2 < 0$. Indeed, dropping the common multiplier τ^*,

$$\tilde{Z} \approx p\frac{(1-p)(D_2 - D_1)}{D^* - R_2} + (1-p)\frac{p(D_1 - D_2)}{D^* - R_2} = \frac{p(1-p)(D_1 - D_2)(R_2 - R_1)}{(D^* - R_1)(D^* - R_2)}.$$

Because of our inequalities, $\tilde{Z} < 0$. ∎

To generalize Example A1, we continue the analysis started at the end of Section 3. Let $\tilde{b}_k = \tau^* R_k/(D^* - R_k)$ be the fair benefit of type k. Assume that those people who expect to live longer also work longer. By proper indexation of the types:

$$\text{(A.1)} \qquad D_1 < D_2 < \cdots < D_{n-1} < D_n \quad \text{and} \quad R_1 < R_2 < \cdots < R_{n-1} < R_n.$$

Theorem A1. *Under the monotonicity assumption (A.1) the macrobalance of undampened incentives is negative:* $\tilde{Z} = \sum_k p_k \tilde{z}_k < 0$.

Remark. If $R_1 = R_2 = \cdots = R_{n-1} = R_n$, then $\tilde{Z} = 0$.

Proof. At first we prove the Remark. For identical length of employment, the joint denominator $D^* - R_1$ can be factored out, yielding $Z \approx \sum_k p_k(D^* - D_k) = 0$.

We prove the theorem by mathematical induction. The statement has been proved for $n = 2$ in Example A1. Let $n \geq 3$. If there is an integer k, for which $D_k = D^*$, then the balance of this group is zero, i.e. it can be dropped. We can assume that for every k, $D_k \neq D^*$. Because of symmetry, we can assume that $D_k > D^*$ for at least two ks: e.g. for $n-1$ and n. Simple computation demonstrates that by decreasing R_n to R_{n-1} the balance increases. But the Remark implies that then the two groups can be integrated. Our inductive assumption then guarantees that the balance remains negative. ∎

Note, however, that in Table 2 different lengths of employment correspond to given lifespans. What can we say in such a case? Assume that for any given lifespan, there are m types of individuals with different elasticities of consumption and indexed by $l = 1, \ldots, m$: their length of employment is equal to $R_{k,l}$, their share is $r_{k,l}$. Assume for any type l, that individuals with higher lifespans work longer. With suitable indexation,

(A.2) $$D_k < D_{k+1} \quad \text{and} \quad R_{k,l} < R_{k+1,l}, \qquad k = 1, \ldots, n-1, \qquad l = 1, \ldots, m.$$

We shall also need the stochastic independence of lifespans and elasticities: there exist marginal distributions $\{p_k\}_{k=1}^n$ and $\{q_l\}_{l=1}^m$, such that

(A.3) $$r_{k,l} \equiv p_k q_l.$$

Corollary. *Under the monotonicity and independence assumptions (A.2)–(A.3) the macrobalance of undampened incentives is negative:* $\tilde{Z} = \sum_k \sum_l p_k q_l \tilde{z}_{k,l} < 0$.

Proof. Fixing l, Theorem A1 applies for the subsums. ∎

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