The efficiency of the Taylor rule

A stochastic analysis using the Macsim model

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Abstract

Basically, MacSim is a package teaching the main macroeconomics concepts and aspects, focusing on international trade. Putting together a set of simplified models, representing some of the main countries in the European Union, it describes single-country mechanisms, including some financial elements, and international trade, represented by bilateral flows.

The main behaviors (production factors, prices, external trade…) are estimated for each country, using the same formulation, which follows generally accepted lines. The main originality is to allow alternatives for the formation of interest and exchange rates, and the participation to the European Monetary Union.

This tool can be used to

- Enhance the diagnosis associated with single country shocks, by taking into account the evolution of trade partners, and their retroactions on the local economy.
- Evidence the individual and global consequences of concerted or competitive policies.
- Show how the financial rules can affect the consequences and efficiencies of a given policy.
- And finally, illustrate the consequences of the existence of the European Monetary Union as a whole, and of the participation of a specific country.

Although teaching is the primary goal of the model, the economic consistency of its framework and properties allows it to be used also for more research oriented purposes, such as here.

Recent papers such as (Augier P, Brillet JL, Cette G, Gambini R, The MacSim project, General Presentation, 1999) presented first the global architecture of the system, then its response to demand and supply shocks, according to the choice of the rules (interest rate, exchange rate, EMU participation). In the present paper, after a short but necessary summary of these elements, we shall concentrate on the consequences of the same choices for the uncertainty of model simulations.

In particular, we shall observe if the introduction of the Taylor rule reduces the uncertainty on both real elements (GDP) and prices, and what are the consequences of EMU participation, both on the uncertainty itself and the efficiency of the Taylor rule, managed by a common decision center (the European Central Bank).

For this we shall conduct a series of Monte Carlo simulations, drawing a sample of residuals for demand and supply oriented behavioral equations.

In practice we shall consider 8 cases, with

- Two interest rules : real exogenous rate, Taylor
- Two exchange rate rules : nominal fixed, real fixed (PPP)
- Two EMU configurations : no EMU, the present state.

The main conclusions we shall obtain are :

- The Taylor rule is quite efficient in reducing variability of both GDP and inflation, if uncertainty comes from a real side element. This efficiency is already present at the first period, and grows with the horizon, converging in the long run to a stable value.

- The main reason for the efficiency is not the reduction of short-term variability but the faster convergence of the investment-capital-profit cycle, as a small model can show.

- The PPP option increases the variance in the same proportion for both interest rate assumptions, and the efficiency is maintained.

- For non - shocked countries, the efficiency is increased by the higher share of cyclical variations.

- But the rule is quite inefficient if uncertainty comes from the price side, as this affects the rule directly. It can even have a negative efficiency, in the PPP case which increases price variability by itself.

- EMU participation limits the effect of PPP, for an asymmetric shock. It also makes the Taylor rule more inert, and reduces its immediate role in the shocked country. But its geographically extended effect brings back some of its efficiency through trade (this compensation is a general characteristic of the study).

Part 1 : the model

The present version considers 6 countries :

France, Germany, Italy, Sweden, United Kingdom, Netherlands.

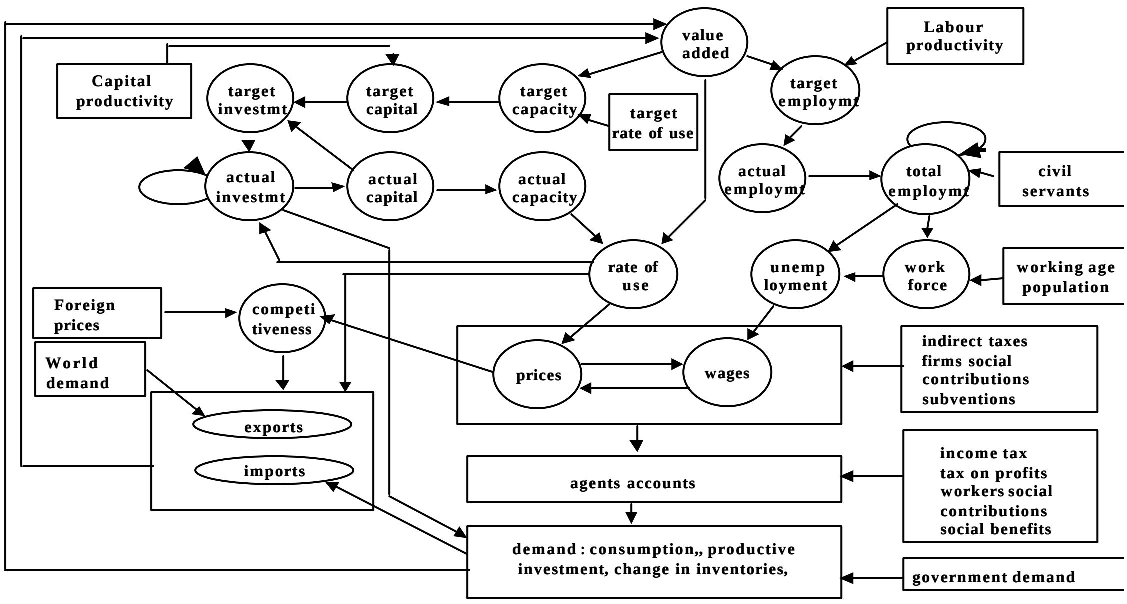
The reasons for this choice are the economic importance of countries for the first four, while the last allows a diversity of cases : Netherlands is a "small" country integrated to the others, Sweden a small country with looser links.

Indeed, our main goal is not to produce a multinational model, but rather to evidence a set of properties and concepts.

The single-country basic model

The single country model uses the structure of the MicroDMS model (Brillet(1997a)) adding a few error correction mechanisms (Brillet(1997b)).

The single country model



We have associated behaviours to the following concepts:

Production factors: investment and employment, unemployment
Prices: wages, value added prices, export and import prices.
Firms: Changes in inventories.
Households: consumption
External trade: exports et imports.
Interest rate: real exogenous value in the base version
Exchange rate: exogenous in the base version

In all we shall estimate 11 equations per country.

When assembling country models, we shall avoid over identifications by computing exports as a sum of individual imports, and the import price index as a weighting of exports prices. Using identities, the associated equations will not be presented.

Productive investment

$$i_t / k_{t-1} = c1.i_{t-1} / k_{t-2} + c2.[(q_t - q_{t-1}) / q_{t-1} + ut_t / 0.85] + c3.tprob_t + c4$$

We suppose here that firms set an investment target, depending on:

- the profits rate, representing both the expected profitability on new investments, and the potential to finance them.
- adapting productive capacity to the expected demand. This takes into account expected future growth, and the present adaptation of capacities to production.
- a strong inertia

The rate of use will ensure in the long run a full adaptation of capacity to production, at a level depending on the profitability of capital.

Employment

Firms have a target labour productivity, associated with a structural trend (graphs and tests evidence a negative break around 1973). This defines target employment, to which actual employment adapts dynamically.

$$\Delta Log(le_t) = c1.\Delta Log(q_t) + c2.(Log(q_{t-1} / le_{t-1}) - c3.t - c4.(trend\_from\_1973) + c5$$

Unemployment

Changes in employment do not translate fully into unemployment, as an improved labour market will attract previously inactive persons, increasing the work force.

$$\Delta cho_t = c1.\Delta le_t + c2.\Delta pop65_t + c3.(cho_{t-1} - c4.le_{t-1} - c5.pop65_{t-1}) - c6$$

The value added price

$$\Delta Log(pva_t) = c1.\Delta Log(c\sup_t) + c2.ut_t + c3.(Log(c\sup_{t-1} / pva_{t-1}) + c4$$

We shall suppose that firms use the price level to optimize between quantities sold (at a given capacity) and margins on each unit. This introduces a positive link between rate of use and margins. Starting from an optimal combination, changes in external conditions will move both targets in the same direction.

As usual, we shall apply an error-correction mechanism.

The trade prices

Exporters will take into account their own costs or the price of their competitors (in the same currency). The first behaviour will leave margins unchanged, but affect competitiveness. With the second behaviour, the reverse will happen.

In our model, all exporters will apply both behaviours, but favour the first (70% compared to 30%). This is consistent with most estimations.

The wage rate

$$\Delta Log(w_t) = c1.\Delta Log(pc_t) + c2.tcho_t + c4 + c3.Log(c\sup_{t-1} / pva_{t-1})$$

Wages present:

- A dynamic indexation on inflation.
- A role of tensions on the labour market (the unemployment rate).
- An error correction term, ensuring the convergence of the real wage cost to a target, depending on the level of unemployment.

The changes in inventories

Firms try to maintain an inventory level proportional to production. This means changes in inventories will depend on changes in production. Based on the last two years, we get:

$$dstoc_t = c1.\Delta q_t + c2.\Delta q_{t-1}$$

Household consumption

$$\Delta Log(co_t) = c1.\Delta Log(rdm_t / pc_t) + c2.[0.5.\Delta Log(pc_t) + 0.5.\Delta Log(pc_{t-1})]$$
$$+ c3.\Delta Log(tcho_t) + c4.Log[rdm_{t-1} /(pc_{t-1}.co_{t-1})] + c5$$

The formulation (error correcting) combines:

- inertia in adapting to changes in purchasing power per capita
- inflation : financial savings are measured in (future) purchasing power, and must be increased with inflation.
- the fear of unemployment
- a long term unitary elasticity of consumption to revenue.

Imports

Of course, particular attention must be given to this equation.

$$Log(m_t) = c1.Log(df_t.ouv_t) + c2.Log(1 - ut_t) + c3.Log(pim_t / pp_t) + c4$$

Imports are determined by:

- a constant elasticity to domestic demand, which we shall correct by the structural (smoothed) opening of borders. Actually we shall set this elasticity to 1.
- the capacity of local producers to face additional demand
- a comparison between local production and import prices..

Although this equation does not follow the error correction format, it is obviously consistent with a long-term equilibrium.

The capacity variable was seldom significant, but we have deemed its presence necessary, and set the same influence for all countries, to avoid not necessarily justified discrepancies. As to competitiveness, its coefficient is generally significant but rather low.

Integrating the financial variables

We shall now present the way financial elements influence his framework. One will observe that some of the options have been chosen with European Monetary Union in mind.

The options for the interest rate

Actually, the options concern the short-term rate on new borrowings, from which the long-term and average rates are computed in a unique way.

The **short-term rate** for one country can be defined as:

- A nominal exogenous value.
- A real exogenous value.
- A Taylor rule
- A common nominal interest rate with a set of countries (for the EMU), following one of the three previous rules.

For the Taylor rule, the rate will depend on inflation and the output gap, measured by the capacity utilization rate:

TIC = 150 . tx(PC) + 50 . b . (UT-UT*) + c

In short, the rationale for this formulation is the following (Taylor JB (1998)).

The National Bank (for instance, the Federal Reserve of the US) wants to control inflation, or the variability of inflation, or the variability of the couple inflation-growth. If it expects high inflation, it will increase the real interest rate. Symptoms for future inflation are: a present high rate, a high level of output compared to its potential value. In a backward looking framework, this leads to the above formula.

In the absence of potential output, we have introduced the rate of use of capacities. This option can be criticized, as it is constrained by the actual productive process, while potential output depends rather on human capital and resources.

JB Taylor has set the coefficients to the above values, which should represent the behaviour of the FED in the last two decades. However, they do not necessarily apply to other countries.

The **long-term interest rate** is a moving average:

$TIL = a \cdot TIC + (1-c)\, TIL_{-1}$

The **average borrowing rate** is and average of both:

$TI = d\, TIL + (1-d)\, TIC$

The **average rate paid** is a moving average:

$TIM = e\, TIM_{-1} + (1-e)\, TI$

For simulations we shall use the values: c=0.5, d=0.5,e=0.8

The options for the exchange rate

The exchange rate can be defined as:

- Exogenous for a each country
- Following purchasing power parity for the country (based on the consumption price)[1].
- Following uncovered interest rate parity for one country.
- Common to a set of countries, following its average characteristics.

Uncovered interest rate parity will make the exchange rate depend on the interest rate (both real, or both nominal). The rationale is the following:

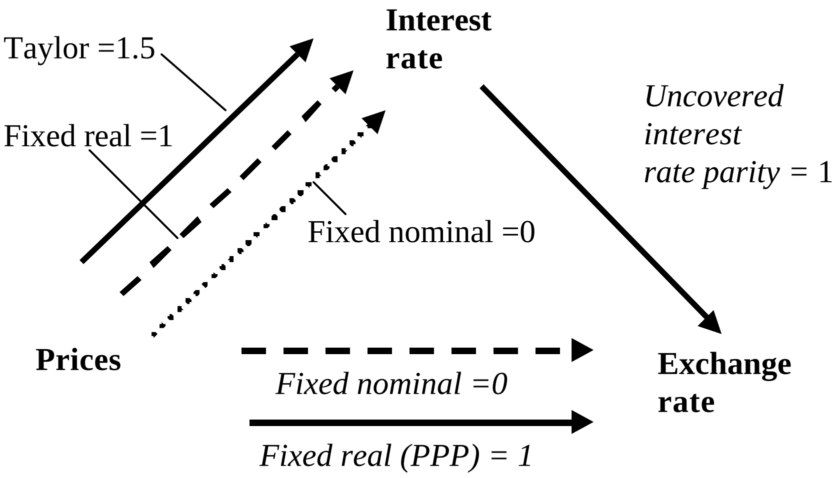
If the agents think that the currency of one country risks losing value in the future, they will ask for a higher interest rate than the international one. So the expected exchange rate will affect the present currency value. If returns are equalized, a one percent expected devaluation will increase the nominal rate by one point. In a backward looking framework, we shall use a simultaneous influence.

The sets of countries considered can be modified at simulation time. This allows in particular observing the consequences of a change in the composition of the EMU.

Summarizing the influences

The above influences can be summarized as follows:

[1] To avoid under identification of the system, the exchange rate of the Rest of the World is actually fixed, and the other currencies follow purchasing power parity compared to it.



We can observe in particular:

- That combining a Taylor interest rate and Uncovered Interest Rate Parity leads to a 1.5 elasticity of the exchange rate to prices, representing devaluation in real terms. As the exchange rate itself has a highly positive influence on prices (imported inflation) this can lead (and actually will) to exploding properties. But one can question this juxtaposition, as the interpretation of the interest movements leading to UIP is quite different its the determination in the Taylor framework.

- That if the interest rate is fixed in real terms, PPP and UIP should give the same results (they will).

Enhancing the role of interest rates

Having defined a system for interest rates, we had to introduce its influence on real elements. This was done through:

- Investment, with the long term real interest rate
- Consumption, with the short-term real interest rate.

Unfortunately, no estimation really succeeded (which is apparently rather common). We chose coefficients derived from NiGEM, using the same value for all countries.

The above equations become:

$$i_t / k_{t-1} = c1.i_{t-1} / k_{t-2} + c2.[(q_t - q_{t-1}) / q_{t-1} + ut_t / 0.85] + c3.tprob_t + c4$$
$$- 0.006 * (til_t - 100(pc_t - pc_{t-1}) / pc_{t-1}))$$

$$\Delta Log(co_t) = c1.\Delta Log(rdm_t / pc_t) + c2.[0.5.\Delta Log(pc_t) + 0.5.\Delta Log(pc_{t-1})]$$
$$+ c3.\Delta Log(tcho_t) + c4.Log[rdm_{t-1} /(pc_{t-1}.co_{t-1})] + c5$$
$$- 0.001.(tic_t - 100 * (pc_t - pc_{t-1}) / pc_{t-1}))$$

Merging the models

The exchange block

We shall start by establishing a coherent system for trade prices.

We shall suppose that exporters base their price on their costs and the price of the target market:

$$Log(pex_{i,j}) = aLog(pp_i) + (1-a)(Log(pp_j ch_i / ch_j) + bt + c$$

Where i is the exporter and j the client, and ch_i the price of the currency of country i compared to the US dollar.

$$pim_{j,i} = pex_{i,j} ch_j / ch_i$$

$$pim_j = \sum_i m_{j,i} pim_{j,i} \Big/ \sum_i m_{j,i}$$

For utilization rates, we use the same method.

$$utx_j = \sum_i m_{j,i} ut_i \Big/ \sum_i m_{j,i}$$

We can now determine the global imports of country I by introducing the capacity of exporters :

$$Log(m_i) = aLog(df_i ouv_i) + b[Log(ut_i) - 0.5.Log(utx_i)] + dLog(pim_i / pp_i) + e$$

This means that a general decrease in the available capacity of exporters will reduce exports, through a substitution effect. The coefficient 0.5 takes into account the larger associated capacities.

Finally, we separate imports into individual exports. Once again, we shall take into account relative competitiveness, and fluctuations in available capacities, relative to the above average.

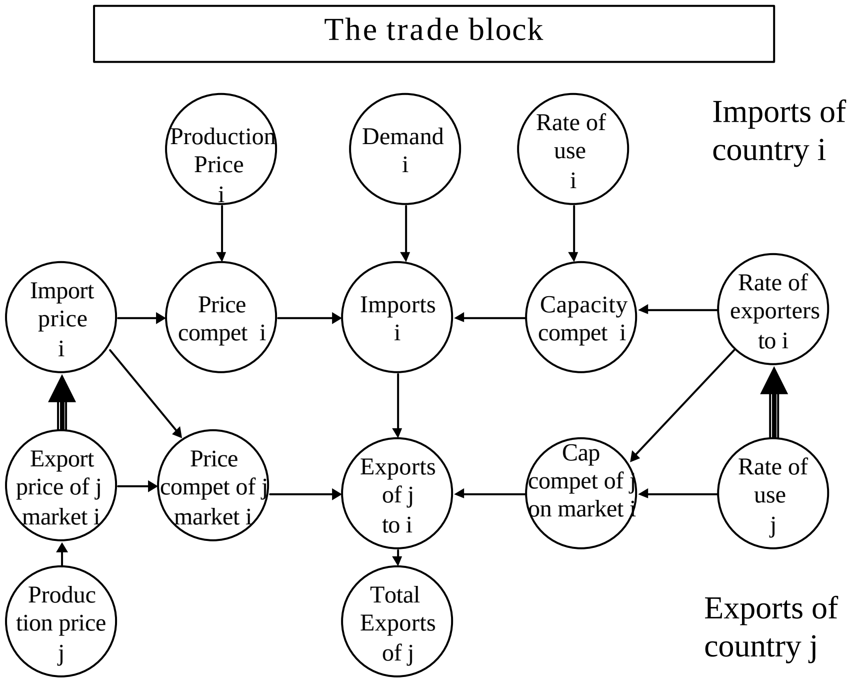
Actually, we shall again use set coefficients:

$$m_{i,j} = m_i . b_{i,j} \left[1 - a(pex_j ch_i / ch_j - pim_i) - b(ut_j - utx_i) \right]$$

which means that (as for the single country models) exporters to one country will increase a « natural » share with competitiveness and available capacity, this time relative to their competitors.

This technique guarantees the identity of the sum of individual exports with its global value, without any correction. The coefficients can be different from one market to another, but not within one market.

The system can be summarized with this graph:



In addition to the above, we have introduced some accounting equations.

<u>The Rest of the World</u>

The Rest of the World will not be associated with a model. Actually, we our goal will only be to give to its trade elements similar properties to the countries we consider. A set of dynamic multipliers will give value added, final demand, local rate of use and local production price. These elements will determine the trade elements, at current and constant prices.

<u>EMU</u>

In addition to descriptive elements, applying the above rules to the Euro zone calls for the computation of global inflation, and a global rate of use of capacities (for the Taylor rule).

Part 2 : the basic properties

Simulations

We shall now study the properties of the full model. Considering the goal of the present paper, we shall limit ourselves to the elements needed to make more explicit the stochastic properties of the model. We shall also limit the number of graphs to a minimum. However, we shall keep the original graph numbering, to allow an easier comparison.

The base simulation will be conducted on a very long period (200 years) to free the results from any short term fluctuations and also check the presence of a long term equilibrium. shocks will start in 2101. We shall use the following assumptions :

Growth rate of population : 0.3% per year.
Growth rate of labor productivity : 2.4% per year.
Growth rate of prices : 2.4% per year.

In addition to usual assumptions, we need also a definition of the « structural » shares of partner countries in imports and exports of a given country. We shall take them from the NiGEM model (produced by the National Institute for Economic Science and Research, UK).

Table : export market shares by exporting country

Market Exporter	Germany	UK	France	Italy	Netherlands	Sweden	Rest of World
Germany		0,080	0,120	0,075	0,075	0,023	0,627
UK	0,121		0,094	0,047	0,066	0,025	0,647
France	0,168	0,097		0,092	0,045	0,011	0,587
Italy	0,190	0,065	0,131		0,029	0,009	0,576
Netherlands	0,281	0,093	0,105	0,052		0,017	0,452
Sweden	0,133	0,102	0,051	0,034	0,053		0,627
Rest of World	0,227	0,158	0,167	0,230	0,152	0,066	

The shocks in the absence of EMU

We shall limit ourselves to two shocks

- A demand shock, associated with an increase of French Government demand by 0.5 GDP point, staring in 2101 and maintained afterwards.
- A supply shock, associated with a 1 point decrease in the social contributions of French firms.

and five composite rules, combining two options for the interest rate and three for the exchange rate :

- Fixed exchange rate, real interest rate
- Fixed exchange rate, Taylor rule

- PPP, real interest rate
- PPP, Taylor rule

- Uncovered Interest Rate Parity, real interest rate.

We shall not consider the sixth option: Uncovered Interest Rate parity, Taylor rule, which produces a diverging process, and do not seem coherent with each other.

We shall first comment the simplest case (real interest rate, fixed exchange rate) then observe the changes introduced by the other assumptions.

<u>Demand shock in France (basic case)</u>

The results are rather usual, associated with the Keynesian multiplier, with short term tensions, a cycle on capacities and investment, and a long term loss in competitiveness.

Compared to a single country model, foreign growth coming from French imports generates foreign demand and imports. The increase in the multiplier is not negligible (around 20 to 30%).

But the trade ratios for France are of course quite negative, especially in the short run, and positive for other countries, especially Germany (strongest links with France).

Inflation extends to all countries, sometimes after a small initial decrease due to gains in labor productivity. In the long run, the low error correcting coefficients keep inflation increasing for a long period.

<u>The role of the interest and exchange rates</u>

Let us now see how a change of rule affects the model properties.

We shall present graphs for the following variables :

A1 GDP
A3 The rate of use
A4 The value added price
A6 The real interest rate
A8 The export-import ratio at constant prices
A9 The terms of trade
A10 The export-import ratio at current prices.

Basically, before any simulation, we can expect the new assumptions to produce the following effects:

Purchasing Power Parity (PPP) : In our model, as in the general case, French or foreign exporters take predominantly into account their local costs, and less the price in the country to which they export. Facing a higher French inflation, exporters to France do increase their prices, but less than French exporters who try to keep their margins. This means that under a fixed exchange rate assumption, an increase in the local production price does not transmit entirely to local demand prices. It is true that to keep the same global margin ratio, the price moderation of local exporters has to balanced by increasing their local sale price above the average change. But this effect is obviously lower than the inertia of import prices, essentially determined by the margins of exporters to France, which are not affected. The multiplicative effects of the price-wage loop are dampened.

Under an inflationary shock (such as this one) PPP devaluates the currency, by the same amount as local inflation. Measured in local currency, the foreign price increases also. The dampening effect disappears, leading to higher inflation. Both trade prices show the same variation in local currency, and the production and demand prices also. The loss in competitiveness disappears.

Uncovered Interest Rate Parity (UIP) : As stated above, we associate it only with the real interest assumption. In this case, its results should be quite identical to PPP : the direct and full indexation of the exchange rate on inflation transits now through the nominal interest rate, but as both influences are unitary themselves, we should get the same result.

Taylor rule (TR) : Its role is slightly more complex. Inflationary effects lead to an increase of the real interest rate, but we have also to take into account the output gap (or rather the rate of use of capacities in our model). If tensions appear, the real interest rate will increase further. If a shock produces both tensions and inflation, as in our present case, the real interest rate will have two reasons to increase. Such a change will limit growth itself, through investment and consumption, with dynamic effects which we have yet to assess.

And indeed, although our model is rather simple, the above remarks give only a very rough guess of its properties. Observing the actual changes in the main economic variables is needed to check and quantify the above elements. In particular, we have to see how the rules interact, once they are associated with a model. Starting from our basic case, we shall change the exchange rate assumption (to PPP and UIP), then the interest rate (to Taylor), then both.

Changing the exchange rate (to PPP or UIP) : as we expected from the above remarks, PPP creates more inflation (A4). Moreover, prices keep growing steadily. Actually, the level of prices is quite free : intervening through price ratios, by multiplying all prices by a common factor, we would still get a solution of the model.

The only stabilizing effects come from inflation, which appears locally (the real holdings in the consumption equation), and in the long run relationships of the behavioral equations. But in the long run, inflation converges to a long term common value, defined as an assumption.

But now the loss in competitiveness disappears. At constant prices, the loss in trade is much lower (A8) leading to a higher GDP growth (A1) with the same cyclical properties as the basic case (A3). The cycles look stronger in absolute terms, but not in relative ones.

But this means also that the terms of trade do not change. As the competitiveness elasticities are substantially lower than unity, without the appreciation of the basic case (A9), the loss on trade at current prices is much higher (A10).

The interest rate (to the Taylor rule) :

Combined with the basic case, the Taylor rule introduces in the beginning (when tensions and inflation are the highest), a strong increase in the real rate (A6). This change drops sharply, when tensions actually decrease, and converges to zero after a slight upturn, when new tensions appear with the cyclical process (A3).

Here the most interesting observation is the stabilizing property of the Taylor rule. Under the previous assumption we obtained cycles, coming from the strong inertia on investment, which can keep increasing for a while even after the rate of use has gone lower than its target value. Now these cycles almost disappear.

To explain this, we have to consider the respective evolutions of both supply and demand, synthesized by the rate of use of capacities (A3). We can see that in the beginning, and especially in the two first periods, if capacities increase less, demand (consumption and investment) and production increase even less, leading to a lower disequilibrium. But in the medium term the effect of tensions on investment is now countered by the higher real interest rate, and this reduction is of course cumulative. Capacities adapt slower, which means the rate of use will take more time in converging, and more important will not overshoot as much as in the previous case. The cycles will be quite dampened.

In the medium long run inflation reaches a limit, and what is left is the change in the rate of use, which cycles much less, as more tensions bring less growth and less tensions. The rest of the equilibrium follows.

This economic explanation is confirmed by a more mathematical one, with details in the annex. A simple 6 equations model concentrating on the production-investment-capacity dynamics gives almost the same properties as the full model, regarding the period and the convergence of the cyclical process (but not the same multiplier). In the near future, we shall try to apply this technique to the full model itself, the only problem being the size of the dynamic Jacobian (6 times the number of dynamic variables) .

Both assumptions

Another interesting observation is that the assumptions are not orthogonal, or that combining both changes in rules does not represent the sum of individual ones. Indeed the PPP assumption brings more inflation and more tensions (in the absence of a loss in competitiveness). Moreover, the inflationary effect is now permanent. The arguments in the Taylor formulation will grow, and the real interest rate hike will be both higher and maintained (A6). While retaining its stabilization properties, the Taylor rule will now give a much lower GDP increase, still higher however than all fixed exchange rate assumptions.

Supply shock in France (basic case)

We observe the usual shocks associated with a single-country model : the increase in margins and capital profitability leads to disinflation and spontaneous creation of productive capacity, both leading to an improvement of production, mainly through external trade. Exports improve significantly, and imports grow much less than local demand.

The low intensity of the price correction slows disinflation, later reduced by the tensions on the labor market and the growth of the real wage rate. But the strong incentives to invest (demand and profitability) lead to a slight overshooting of productive capacity.

The initial increase in demand, the slow speed of disinflation, and the tensions on capacities, lead in the short run to a slight degradation of the real trade balance. It improves later, but the large loss on the terms of trade keeps the evolution of the current balance negative, despite a substantial improvement in the long run.

These elements could have been obtained with a plain French model. We observe indeed a minimal reaction from other countries. The reduction of contributions lowers French inflation, but increases its demand. Our partners lose shares, on both the international and French markets (although they have less capacity problems). But the French market itself expands.

In terms of prices, the imported disinflation compensates the increased local tensions.

In the short run, the Rest of the World, with its large capacities, profits the most from the shock. When capacities adapt and prices keep decreasing, the results are inverted.

The role of the interest and exchange rates

We shall present graphs for the following variables :

B1 GDP
B3 The rate of use
B4 The value added price
B6 The real interest rate
B8 The export-import ratio at constant prices
B9 The terms of trade
B10 The export-import ratio at current prices.

We have to remember that this shock is only supply-side in the short and medium run as firms try to stabilize their margins ratios, and will return the reduction of contributions to wage earners (as gross wages).

As most of the effects of this shock come through inflation and supply side mechanisms, one can expect that the choice rules will greatly affect the results.

Let us start again with **PPP** alone. The results are quite clear. As long as the shock produces deflation (B4) GDP shows a lower improvement (B1) through external trade (B8). Indeed, the real export import ratio never actually improves. And deflation itself (B4), looser than in the previous case, and facing a lower GDP growth, reaches a much higher level.

The consequences from using the **Taylor rule** are more mixed : a higher rate of use and deflation have opposed influences. Its consequences depend on the period, and the dynamic changes of these two elements.

It is interesting to consider the consequences for the other countries, as they are not subject to the initial shock. As we shall see in Part 5, if the effects are small, the main influences come through real effects. This makes the Taylor rule more efficient than in France.

Combining **both changes**, the Taylor rule is subject with PPP to a lower inflation, and the real interest rate decreases now substantially (B6). This brings higher GDP growth, but also). However, this brings the highest loss on trade (B8).

Part 3 : Introducing EMU

We shall now present how the introduction of the Euro affects our results. We shall first apply the present composition of EMU to our 6 country system. This means that we shall leave Great Britain and Sweden out.

We shall reconsider our five options, supposing now that both the exchange rate and the interest rate are common to the zone. The first change is institutional. The second corresponds to the most logical option. However it makes the context of our study somewhat questionable: the countries will have different inflation rates, and the ones with the highest inflation will have the lowest real rates, and benefit from them. And no currency movement can correct this effect.

<u>Demand shock in France</u>

We shall limit ourselves to Graph C1

Let us first observe that even in the basic case, a common interest rate limits the role of the higher French inflation, and the real French rate actually decreases, which favors France over its partners (for which it increases, of course)! Compared to graph A1, GDP growth is higher, at least initially.

We see also that the cyclical properties remain. They might be even accentuated, as now an increase in demand makes the interest rate favor investment. A shock produced in any of the countries would bring similar cycles, in terms of periodicity and amplitude.

In the long run, the price change stabilizes, the inflation effect disappears, and we get back to the non-EMU case.

Coming to the options, we see that **PPP** has much less impact than before on French elements: the exchange rate remains fixed compared to its EMU partners, and changes much less than French inflation compared to others. France loses competitiveness to all countries, almost as much as with a real exchange rate. But as there is more inflation, the higher reduction of the real interest rate enhances GDP growth.

This is a clear consequence of EMU : PPP no longer compensates inflation differentials within EMU, and their external influence is smoothed for the country concerned.

The adverse effects of the **Taylor** rule are also limited for France, as its uses EMU elements. But it increases the real interest rate for its partners, and the lower growth of their GDP affects France in turn. Its stabilizing properties remain, although at a lower level. We see here the importance of having an integrated model.

The consequences of combining **both rules** almost add up, due to the small effects of PPP.

<u>Supply shock in France</u>

We shall limit ourselves to Graph D1.

Now the basic case produces a little less GDP growth than the non-EMU case, as the real interest rate increases in France, at least in the beginning.

Actually, we observe a convergence of all assumptions, to the real exchange rate case (at least when we are facing a pure supply shock). Without EMU, the main source of difference was the disappearance of the competitiveness gains, now practically maintained. And the Taylor rule takes now into account the changes in all partner countries, in particular their inflation.

Finally, a general observation: the consequences for EMU of an asymmetric shock are not too different from the non-EMU case: at EMU level, averaging EMU variables (exchange rate, interest rate) before applying them, or leaving them play individually and averaging their consequences, does not change much to the diagnosis.

Part 4 : The stochastic properties, no EMU

In the previous studies, we have observed how of the choice of rule for the interest rate and the exchange rate affected model properties. In particular, we saw that the Taylor rule smoothed the consequences of shocks, particularly demand oriented ones, without necessarily reducing their efficiency on the whole. This can be seen as confirming that the application of this rule by Central Banks achieves its goal.

But of course one could argue that the goal of the Central Bank is not to smooth the dynamic consequences of shocks, but rather their variability, concerning both production (GDP) and inflation.

It is probably already clear that both properties are linked. If the consequences of a permanent shock stabilize fast, it means that a one time shock, such as those generated by random terms in an estimated equation, will see their influence decrease quickly after the first period. The variance introduced by residuals will come mostly from their immediate effect, and the contribution of the lagged residuals will be small.

This is of course all the truest if the equation in which the residual appears has fast converging dynamics. In the case of an error correction formula, it means that the error-correcting coefficient should be large.

In this last part, we shall observe if these ideas are confirmed in practice. For that we shall select a set of cases in the above study, and produce in each case a series of random simulations, by drawing random residuals for some estimated equations. By using several cases we shall observe if the properties we observe can be considered as general.

In practice, for the time being, we shall consider only two estimated equations:

- For demand oriented properties, household consumption.
- For supply oriented properties, the value added price index.

We shall consider the 4 cases:

- No EMU, fixed exchange rate
- No EMU, purchasing power parity
- EMU, fixed exchange rate
- EMU, purchasing power parity

And in each case observe if and in which degree the introduction of the Taylor rule reduces the standard error of GDP and inflation, compared to exogenous real interest rates.

The simulations will be conducted on the period 2000 - 2150, with shocks in 2101-2150, a horizon at which we can assume the distance to the steady state path is small enough not to interfere. In each case, the size of the simulations sample will be at least 500.

The residuals will be drawn assuming a normal distribution, using the standard error of the estimation (0.76 for the value added price, 0.95 for household consumption). However, we have checked that we obtained very similar results using a bootstrap technique. We only seem to converge a little slower to the actual standard errors (probably because the bootstrap sample is small), and the technique for drawing the residuals is a little more time-consuming.

To improve the comparison between the interest rules, we shall not redraw the sample when changing that option.

For each of the 4 cases, we shall present the evolutions of the standard errors of GDP and inflation :

according to the rule
according to the country

in both tables and graphs.

To make our presentation clearer, the standard errors will be measured in relative terms, for both variables. Technically, this means that we shall use the logarithms of GDP and inflation to compute the squares and averages of the formula. With such a method, a value of 0.81 for GDP for instance will mean that the standard error on GDP represents 0.9% of its value.

Of course, this part of the study could have been conducted more accurately if we had estimated the equations using co integration techniques: first evidencing the long-term relationship, then estimating the dynamics. With our small sample, this option was questionable. And the results would not have been too different, probably. What we are trying to evidence is the fact that a Taylor formulation reduces the standard error by a wide margin, not the precise size of this reduction.

Case A : residual on consumption , no EMU, fixed exchange rate.

The graphs will present:

Aa : the standard errors of GDP in France, according to the rule
Ab : the standard errors of inflation in France, according to the rule
Ac : the ratio of standard errors (Taylor / Real rate).
Ad : the ratio of GDP standard errors for the 6 countries.
Ae : the ratio of inflation standard errors for the 6 countries

Graphs Aa and Ab present the evolution of standard errors, for GDP and the inflation rate.

In the first period, the Taylor option already reduces the standard error of GDP. Actually, this should only be associated to the lower increase in GDP (see Graph A1 in the previous part), coming from the higher real interest rate. This should not be counted as a true improvement.

In the following periods, for both interest rate options, the standard errors of GDP and inflation grow, converging apparently after 20 or 30 periods. For the real interest rate option, the increase is about 2.3 times, which is surprisingly consistent with the gap created in the long run by the error correction process in the consumption equation, equal to

(c1-1)/c4 =(.386-1)/.264=2.32.

Of course this is only indicative, as other elements come into play, and we are considering GDP and not consumption.

As to the long-term value, a standard error of 0.90 on GDP is of the same order as the error of the consumption equation (.97 standard error). Again, GDP is not consumption, and other elements intervene.

What is striking is that for the Taylor rule the increase is much slower, while on average (Graph A1) the evolution of the multiplier was quite similar, which means that we have now a true improvement.

This leads to a very important reduction in the long term standard error, as shown by the ratios in Graph Ac: around 40% for inflation, and 60% for GDP. For any intermediary period, the ratios are of course higher, but still represent a sizable improvement. Of course, their evolution is also much smoother, as we have used the same residuals for both options, and the sign and magnitude of that residual have closely related influences on the error, even with different rules.

As we have explained above, this result was to be expected, but maybe not at such a high level. For GDP, Graph A1 (again) showed that under the real interest rate assumption the shock led to sizable cycles. What is true for a maintained shock applies also to a one-time shock. At a given period the standard error sums the effect of a series of independent cyclical variations. While under the Taylor assumption, these cyclical effects are almost eliminated. What justifies the increase in the standard error, in addition to the first period effect, is the dynamics of the downward trend (due to the loss of competitiveness through inflation).

For inflation, Graph A4 showed that inflation (not the relative variation of the deflator itself, but its derivative) showed the same cycles as GDP and the other real elements, at an even higher level relative to the average variation. Again, this is due to the large sensitivity of prices to cyclical elements, like the rate of use. And this explains that the reduction in standard error is larger for prices than for quantities.

Moreover, we observe here an additional reason for the efficiency of the Taylor rule: smoothing GDP and limiting the output gap, combined with a limited variability of inflation, reduces in turn the variability of the interest rate. In summary, the efficiency is auto sustained.

If we consider the other countries, we can see (Graphs Ad and Ae) that the reduction is quite higher concerning GDP, and generally higher for inflation. This again was to be expected, for several reasons. First, a large share of the French GDP increase comes from the deterministic increase in exogenous demand. At the same time, the improvement of foreign GDP come mainly from French imports, which are affected not only by French demand but also (almost at the same level) by the French level of unused capacities, which follows strong cycles, as shown in graph A11. And again, the larger the share of the cyclical component, the more efficient the Taylor rule. Finally, in these countries, the share of investment in the GDP increase is high, and also affected by the cycles. This is also true, at a lesser degree, for the role of inflation.

Actually, for the Taylor rule, the standard error of GDP for countries other than France does not increase much after two periods (the error correction on consumption explains the two period wait).

<u>Case B : residual on consumption , no EMU, Purchasing power parity.</u>

We shall use the same graphs, this time with the letter B.

The teachings are very similar to the previous case. For France, the ratios have the same profile, and the same long-term value. The same is true for other countries also, except perhaps Sweden. However we can see that:

- The standard errors on GDP are a higher for France, which should be attributed to the higher sensitivity of GDP to the residual, in the absence of a loss in competitiveness.

- They also converge must faster. This is due to the quasi absence of trend in the multiplier effects (see Graph A1), while with a fixed exchange rate, the growing loss in price competitiveness led to a sustained decrease. Subsequently, the dynamic effects of a single period shock are much lower.

- The standard errors on inflation are quite higher, and converge also slower. This is again coherent with the previous study. In Graph A4, we see that with PPP a maintained shock on demand introduces a growing inflationary effect; while a fixed exchange rate reduces inflation with time, but at a slower pace. Again, this means that a single period shock will have a higher influence on inflation.

- Concerning the efficiency of the Taylor rule, it is maintained, at about the same level for GDP. Graph A1 shows that with PPP the change in GDP is higher, but the cycles are also more pronounced, making their elimination as efficient. For inflation the improvement is even higher.

Case C : residual on prices , no EMU, fixed exchange rate.

The results are presented in the Graph series C. We should first note that the comparison with the deterministic shock is less valid here, as the wage shock became demand oriented with time, the error correction on wages being quite strong for France. Here the residuals apply to prices, which are much more inert, and the shock is pure supply oriented.

Once this is said, we can see that the conclusions are quite different from the previous cases. Let us start with the fixed interest rate.

For France, the standard error of inflation is almost constant, around 1.0 (slightly higher than the 0.76 standard error of prices). On the contrary, the standard error of GDP rises very fast, but is also rather fast to stabilize, after around 10 periods.

This is not very difficult to explain. In our model and for France especially, the error price correction is quite slow, and the dynamic effects on inflation of a single period shock are comparatively very low. But this means that the change on the price **level** will be maintained, leading to large evolutions of GDP through trade elements. Even if the residuals are independent, the standard error of GDP will increase regularly for a long time.

We can see also that the Taylor assumption give only a very small improvement on inflation, and almost none on GDP (the standard error even increases at first)

This is also quite logical. Now a given residual will lead to opposed changes in the rate of use and inflation, and the introduction of the Taylor rule itself will have limited effects. It is only in the beginning, when only first period deflationary effects are considered, that the Taylor rule brings more growth and increases the standard error (as we could see in Graph B1).

For the other countries, the situation is quite different. Of course the standard error is much lower, both on GDP (French demand increases, but competition with French exporters is harder) and especially on prices (French changes in inflation are exported, but at a lower level, and balanced by a relative increase of GDP through exports). But on the whole, a negative French residual will bring a negligible change in inflation, and a small but significant increase of GDP and the rate of use. We find ourselves in the situation of the small model in Annex 1, where the Taylor rule stabilized the role of the shocks and reduced the standard error of GDP with time. Graph Cd shows clearly that the Taylor rule plays its stabilizing role, but only for the other countries.

It is clear also that if we consider the uncertainty on prices in all countries (we could repeat the experiment with residuals in all price equations), the Taylor rule would stabilize GDP variations in all countries also.

Case D : residual on prices , no EMU, purchasing power parity

The results are presented in Graphs D.

Again, the conclusions are changed.

For France, the freedom left to the decrease of prices leads the standard error of inflation up, to an almost constant value around 1.4 (let us recall the standard error is 0.76). As with the previous case, the error price correction is quite slow, and most of the consequences of a single period shock are obtained in the first period.

However, we can see that the Taylor rule reduces a little the standard error of inflation.

But for GDP, the diagnosis is completely new. Now the Taylor rule actually increases the standard error of French GDP!

This surprising result is actually easy to explain, as for all the previous ones. If prices show a negative residual, the over indexation of the Taylor rule reduces the real interest and adds to the other production improving effects. For France, the Taylor rule increases the standard error by about 50%. The standard error of inflation itself does not change, but its uncertainty has a higher effect on GDP.

For the other countries, the dynamics are similar to the previous case. They follow in part the higher French inflation change, and the higher variation of French GDP and demand affects their exports.

For the Taylor rule, the increase of the French GDP standard error translates to the other countries at first, then the efficiency observe in case C takes over gradually.

Part 5 : The stochastic properties, EMU

We shall now consider that France, Germany, Italy and Netherlands form a monetary Union.
These countries will share a common exchange rate, and also a common nominal interest rate. When the rule uses inflation or an output gap, it will be computed at EMU level, by weighting by local capacities in the first case, and local consumption in the second.

Case E : residual on consumption , EMU, fixed exchange rate.

Let us start by studying the consequences of EMU formation, by comparing results to case A, using the real interest rate assumption.

First we observe a slight increase in GDP standard error, due of course to the dubious effect we had observed earlier: the interest rate being determined at EMU level, it changes less than French inflation, and the real interest rate will decrease with demand. In turn, this increases also the variability of inflation.

For other countries, the situation is much less clear: a positive shock on French demand will bring more exports, but the local real interest rates will increase.

For France, the efficiency of the Taylor rule is lower as expected, as its elements are determined at EMU level, and show a lower variability. However, this is partly balanced by the EMU-led increases in the real interest rate of other countries, and the reduction in their own GDP variability. This is also true for prices.

It is interesting to observe that participation to EMU and the subsequent loss in independence limits the local efficiency of rules, but exporting its effects brings back part of this loss through external trade.

For the other countries, the situation is even less clear.

<u>Case F : residual on consumption , EMU, Purchasing power parity.</u>

As with a deterministic shock, the consequences are very close to the previous case, especially for France. The exchange rate variability is much lower, and the standard error on GDP is not really changed. This is not exactly true for inflation, where the PPP assumption decreases slightly the error.

For the other countries, the changes are very small compared to case E.

As to the Taylor rule, the evolution of its efficiency is very logical. Compared to the PPP non-EMU case, the variability of GDP does not change significantly, but that of inflation is much closer to the fixed exchange rate Case A. Compared to the previous case, it increases, essentially with the reduction of GDP increase. But the gain is lower than in the non-EMU case.

<u>Case G : residual on prices , EMU, fixed exchange rate</u>

Let us compare the results to case B. The main observation is linked to the deterministic case : the interest rate being determined at EMU level, a positive shock on prices makes it increase less than French inflation, bringing the real rate down and lowering the loss on GDP. As to inflation, its variation keeps a similar level, as expected.

As to the Taylor rule, in the above case, it produces a higher increase of the interest rate, limiting the dampening effect, at least in the beginning. But in the long run, the price variation is reduced by the error correcting mechanism, and inflation is reversed. The Taylor rule regains a very limited efficiency.

For the other countries, the variance of GDP is also lower, with French demand. And as the shock is actually rather demand-oriented, the Taylor rule will keep its efficiency, at a lower lever however.

<u>Case H : residual on prices , EMU, purchasing power parity</u>

In the absence of EMU, the PPP assumption led to much higher inflationary effects in the whole system, but especially for France, which made the Taylor rule quite inefficient for that country.

Here the results are mixed: for France, the exchange rate keeps close to the fixed rate assumption, and the growth of prices is very limited compared to the corresponding case G (see Graph Hb). The standard error of GDP is very similar, but the influence of the Taylor rule is a little higher, in both ways (Graph Ha).

For the other countries, the results are quite similar.

Annex 1 : a small model illustrating the real effects of the Taylor rule

We shall now explain, on a very crude example, the role of the real element of the Taylor rule in our type of model.

Let us consider the following model, measured in real terms, without external trade

$$(1)\, ut_t = q_t /(k_t\, pk_t)$$
$$(2)\, i_t / k_t = 0.65 i_{t-1} / k_{t-1} + 0.10[(q_t - q_{t-1})/q_{t-1} + (ut_t - ut^*)/ut^*] + d$$
$$+\, a(ut_t - ut^*)$$
$$(3)\, c_t = r_t q_t (1 - b(ut_t - ut^*))$$
$$(4)\, k_t = k_{t-1}(1 - dd_t) + i_{t-1}$$
$$(5)\, q_t = c_t + i_t + g_t$$

with the endogenous variables :

c : household consumption
i : productive investment
k : capital
q : GDP
ut : rate of use of capacities

the exogenous variables :

dd : depreciation rate
g : government consumption
pk : productivity of capital
r : ratio of consumption to GDP
ut* : target rate of use

In the Taylor rule, we have only considered the role of the output gap, which we have represented (this can be questioned) by the rate of use of capacities.

So we shall use the parameters :

a : sensitivity of investment to the rate of use
b : sensitivity of consumption to the rate of use

If we compute the dynamics of this model by differentiating its equations numerically,

$$\Delta y_t / y_t = A\Delta y_t / y_t + B\Delta y_{t-1} / y_{t-1} + C\Delta x_t / x_t$$

where A, B and C are relative Jacobians supposed constant with time,
we get

$$\Delta y_t / y_t = (I - A)^{-1} (B\Delta y_{t-1} / y_{t-1} + C\Delta x_t / x_t)$$

To describe the dynamic process, we must now compute the eigenvalues of $(I - A)^{-1} B$, a 6x6 matrix.

But of course, independently from the values of a and b, we should get only two non-zero eigenvalues, as there are only two equations with lagged elements (and three such elements).

Starting with a=b=0, we get two complex eigenvalues, with

Modulus : .977 period : 18.8

To compare these results with our graphs, we must consider that they are associated with a maintained shock, while the matrix describes only to the consequences of a single past shock, and does not take into account its present continuation. This means we must consider in the graphs the fluctuations around the converging process, with zero mean. In this case the cumulative sum of cyclical processes of zero mean has the same modulus and period as the individual elements.

Under these conditions, our computations are quite consistent with the graphs, especially regarding the period. As to the modulus, the initial shock after 18.8 periods should decrease by a factor of 0.646, which seems slightly higher than what we observe on the graph.

Now if we put the value of a to 0.03, consistent with our Taylor rule (coefficients of -.06 in the investment equation, and .5 in the Taylor rule itself) we get :

Modulus : .872 period : 20.6

This is again consistent with our observations : the period is slightly longer, and the convergence is much increased. After 18.8 periods (let alone 20.6) the factor is now 0.076.

Considering b (the role of the interest rate in consumption) we get only a small increase in the period, and in the speed of convergence.

In conclusion : the dynamics are almost fully explained by the real elements of capital formation, including the real interest rate.

List of variables

We shall limit the list to variable used in the above equations.

- Endogenous variables are in bold.
- The base year is 1990 in all cases.
- All operations on goods are at constant 1990 prices if not stated otherwise.
- All transfers are at current prices.
- All variables are in base units (pounds, francs, persons...)

Variable	**definition**
CHO	Unemployment
CIE	Net interests paid by Firms
CO	Household consumption
COMPM	Imports price competitiveness (import price / local production price)
COMPX	Exports price competitiveness (export price / foreign production price)
CSUP	Unitary wage cost
DF	Final local demand
DFG	Final demand of Administrations
DSTOC	Change in inventories
I	Productive investment
K	Productive capital
LE	Firms employment
LT	Total employment
M	Imports
Mi	French imports from country i
MARG	Margins of firms
MONN	Exchange rate (value of the local currency)
MVAL	Imports at current prices
OUV	Independent trade growth
PC	Consumption price index
PEX	Export price index
PEXi	Price index of exports to country i
PIBM	Market Gross Domestic Product
PIBMVAL	Market Gross Domestic Product at current prices
PIM	Import price index
POPAC	Work force (employed + unemployed)
POP65	Population in age of working
PP	Production price index
PPIB	GDP price index
PPX	Weighted production price of client countries
PROD	APPPrent labour productivity
PRODK	Structural capital productivity
PROF	Profits of Firms
PSOC	Social benefits
PU	Local demand price index

PVA	Value added price index
Q	Value added
QVAL	Value added at current prices
RDM	Households disposable income
RDR	Household disposable income in purchasing power
SOLCO	Trade balance
TAII	Rate of Other Indirect taxes (than VAT)
TCHO	Unemployment rate
TCSE	Rate of social contributions by Firms
TCSS	Rate of social contributions by wage earners
TCVAL	Export - import ratio at current prices
TCVOL	Export - import ratio at constant prices
TDEC	Depreciation rate of capital
TECH	Terms of trade (export - import price ratio)
TEMPS	Time (0 in 1980)
TI	Current interest rate on new loans
TIC	Short term interest rate on new loans
TICR	Real short term interest rate on new loans
TIL	Long term interest rate on new loans
TILR	Real long term interest rate on new loans
TIM	Average interest rate on current debts
TIS	Rate of the tax on profits
TMARG	Margins ratio
TPRO	Profits ratio
TPROB	Ratio of margins to capital at replacement cost
TSUB	Rate of subsidies (to value added at current prices)
TTV	VAT rate
TVA	Value Added Tax
UT	Rate of use of capacities
UTX	Rate of use of capacities for exporters to France
W	Annual wage rate
X	Exports
Xi	Exports to country i in 1990 francs
XVAL	Exports at current prices

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