

Estimates of the Effectiveness of Monetary Policy

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Abstract

This paper examines various interest rate rules, including rules derived by solving optimal control problems, for their ability to dampen economic fluctuations caused by random shocks. A tax rate rule is also considered. A multicountry econometric model is used for the experiments. The results differ sharply from those obtained using recent models in which the coefficient on inflation in the nominal interest rate rule must be greater than one in order for the economy to be stable.

1 Introduction

Many of the recent studies examining monetary policy effects have used macroeconomic models with a particular feature. In these models an increase in inflation with the nominal interest rate held constant is expansionary, and the economy is not stable unless the coefficient on inflation in the nominal interest rate rule is

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greater than one.¹ Empirical evidence is presented in Fair (2001b) that suggests that models with this feature, which will be called “modern-view” models, are not good approximations of the economy. In particular, it appears that an increase in inflation is contractionary even when the nominal interest rate is held constant, and so the coefficient on inflation in the interest rate rule does not have to be greater than one for the economy to be stable. The conclusions reached about monetary policy using these models are thus subject to some doubt.

This paper uses the multicountry econometric (MC) model in Fair (1994) to examine monetary policy effects. The MC model has been extensively tested, including tests for rational expectations, and it appears to be a good approximation of the economy.² Section 2 discusses a simple experiment in which the interest rate rule of the Fed is dropped from the model and the U.S. short term nominal interest rate is decreased by one percentage point. It will be seen that although there are substantial real output effects from this change, the effects are much smaller than those in the FRB/US model,³ which is a modern-view model.

Section 3 examines the stabilization features of three interest rate rules for the United States. The first is the estimated rule in the MC model, which has an estimated coefficient on inflation less than one. The second and third rules are the estimated rule modified to have a coefficient on inflation greater than one. It will

¹These models are nicely summarized in Clarida, Galí, and Gertler (1999), Taylor (2000), and Romer (2000). They are used in Clarida, Galí, and Gertler (2000) and in virtually all the papers in Taylor (1999) to examine monetary policy rules.

²See, for example, the tests in Fair (1994) and the tests on the website mentioned in the introductory footnote. The latest version of the model is presented on the website, and it is possible to duplicate the experiment in Section 2 on the website. The MC model is briefly discussed in the appendix to this paper. It is contrasted to modern-view models in Fair (2001b).

³Federal Reserve Board (2000).

be seen that the rules with the larger inflation coefficients lead to a reduction in output and price variability at a cost of an increase in interest rate variability. In general the results are fairly close across the three rules. This conclusion is contrary to what would be obtained using modern-view models, since in these models the economy is not even stable if the inflation coefficient is less than one.

Section 4 then computes optimal rules for particular loss functions. These solutions require a combination of stochastic simulation and solving deterministic optimal control problems, and this is the first time that such solutions have been obtained for a large scale model. The results show that it is easier to lower output variability than price variability. Also, the results using the estimated rule correspond fairly closely to the optimal control results with a high relative weight on inflation in the loss function.

Another feature of the results in Sections 3 and 4 is that considerable variance of the endogenous variables is left using even the best interest rate rule. Section 5 then adds a fiscal policy rule—a tax rate rule—to see how much help it can be to monetary policy in trying to stabilize the economy. The results show that the tax rate rule provides some help. This is also the first time that such a rule has been analyzed using a large scale model.

2 A Decrease in the U.S. Short Term Nominal Interest Rate

To examine the effects of a U.S. interest rate decrease in the MC model, the following experiment was run. The period used is 1994:1–1998:4, 20 quarters.

The first step was to add the estimated (historical) errors to the model and take them to be exogenous. This means that when the model is solved using the actual values of all the exogenous variables, a perfect tracking solution results. The base path for the experiment is thus just the historical path. Then the estimated interest rate rule for the Fed was dropped from the model, and the U.S. short term nominal interest rate (denoted r) was decreased by one percentage point from its historical value for each quarter. The model was then solved. The difference between the predicted value of each variable and each period from this solution and its base (actual) value is the estimated effect of the interest rate change.

Selected results from this experiment are presented in Table 1. Row 3 shows that real GDP increases: the nominal interest rate decrease is expansionary. The peak response is .67 percent after 8 quarters. Row 1 shows the exogenous fall in r of one percentage point, and row 2 shows the response of the long term bond rate to this change. After 12 quarters the bond rate has fallen .80 percentage points. This reflects the properties of the estimated term structure equation for the bond rate, where the bond rate responds to current and past values of r . The unemployment rate is lower (row 4), and the price level is higher (row 5). The peak unemployment response is -.29 percentage points after 8 quarters.

The value of nominal after tax profits (row 6) is higher because of the higher level of real output and higher price level. The nominal value of household capital gains (CG) is larger because of the lower bond rate and higher value of profits. (In the estimated equation for CG , CG depends on the change in the bond rate and the change in the nominal value of profits—see Fair (2001b) for more details.) An increase in CG is an increase in nominal household wealth, and row 8 shows that

Table 1
Effects of a U.S. Interest Rate Decrease
Changes from Base Values

Variable	Quarters Ahead							
	1	2	3	4	8	12	16	20
1. r	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
2. AAA bond rate	-.30	-.34	-.41	-.48	-.68	-.80	-.88	-.93
3. Real GDP	.05	.15	.28	.41	.67	.63	.53	.46
4. Unempl. rate	-.01	-.05	-.10	-.16	-.29	-.27	-.21	-.17
5. Price level	.02	.04	.08	.12	.37	.60	.82	.99
6. Π	.43	1.30	2.35	3.32	5.85	6.33	7.01	7.40
7. CG	81.5	23.0	35.8	32.5	17.6	17.8	11.8	38.0
8. Real wealth	.38	.45	.58	.68	.76	.56	.28	.05
9. Nominal DPI	.01	.05	.11	.18	.39	.45	.45	.44
10. Real DPI	-.03	-.02	-.00	.02	-.02	-.21	-.47	-.66
11. CS	.08	.16	.23	.30	.49	.54	.45	.31
12. CN	.04	.13	.22	.31	.57	.61	.51	.37
13. CD	.07	.25	.44	.61	1.07	.93	.37	-.15
14. IH	-.03	.74	1.64	2.32	3.28	2.38	1.19	.36
15. IK	.09	.30	.60	.93	2.14	2.44	2.46	2.47
16. Japan r	-.12	-.22	-.30	-.37	-.51	-.52	-.48	-.43
17. Germany r	-.20	-.36	-.48	-.58	-.71	-.58	-.35	-.17
18. yen/\$ rate (nominal)	-.32	-.59	-.81	-1.00	-1.54	-1.96	-2.37	-2.81
19. DM/\$ rate (nominal)	-.41	-.71	-.90	-1.04	-1.29	-1.58	-2.13	-2.83
20. Price of imports	.24	.35	.42	.46	.70	1.01	1.55	1.83
21. Real imports	.00	.06	.16	.30	.91	1.19	1.05	.77
22. Price of exports	.04	.08	.12	.17	.42	.65	.88	1.06
23. Real exports	.02	.05	.07	.10	.21	.39	.61	.85
24. Cur. Act.	-.03	-.05	-.06	-.08	-.15	-.20	-.21	-.15

- All variables are for the United States unless otherwise noted.
- Notation: r = nominal short term interest rate, DPI = Disposable Personal Income, Π = Nominal After Tax Profits
 CG = Capital Gains or Losses on Stocks Held by the Household Sector (nominal),
CS = Real Consumption of Services, CN = Real Consumption of Non Durables,
CD = Real Consumption of Durables, IH = Housing Investment,
IK = Real Plant and Equipment Investment,
Cur. Act. = U.S. Nominal Current Account as a percent of Nominal GDP.
- Changes are in percentage points except for Π and CG , which are in billions of dollars.
- Prediction period is 1994.1–1998.4.

real wealth also increases. By quarter 20 real wealth is only slightly larger than the base value. This means that by quarter 20 the negative effect on real wealth from the higher price level has almost completely offset the positive effect from the higher nominal wealth.

Rows 9 and 10 show that although nominal disposal personal income (DPI) increases, real DPI decreases. An important feature of the model is that when interest rates fall, interest payments of the firm and government sectors fall, and this in turn lowers interest income of the household sector. A decrease in household interest income is a decrease in nominal DPI. The household sector is a large creditor, and this interest income effect is fairly large. The increase in nominal DPI is thus less than it otherwise would be, and row 10 shows that the net effect on real DPI is negative. Another factor contributing to the fall in real DPI is that there is a slight fall in the real wage (not shown). Wages lag prices in the model, and the initial response is for the nominal wage rate to increase less than the price level.

Rows 11–14 show that real household expenditures are larger except for a small initial decrease in IH and a decrease in CD in quarter 20. The two positive effects on expenditures are the lower interest rates (a nominal interest rate is an explanatory variable in each of the household expenditure equations⁴) and the higher real wealth. The negative effect is the fall in real DPI. There is an additional negative effect on durable expenditures and housing investment over time, which is an increase in the stocks of durables and housing. Other things being equal, an

⁴Extensive tests of nominal versus real interest rates in these equations have been made, and the results strongly support the use of nominal rates. These tests are discussed in Section 2 in Fair (2001b).

increase in the stock of durables has a negative effect on durable expenditures and an increase in the stock of housing has a negative effect on housing investment. Row 15 shows that real plant and equipment investment rises. This is because of the fall in the bond rate and the rise in real output.

Rows 16–24 pertain to the effect of the rest of the world on the United States and vice versa. Rows 16 and 17 show that the Japanese and German interest rates both decrease. Interest rate rules are estimated for Japan and Germany in the MC model (along with those for many other countries), and the U.S. interest rate is an explanatory variable in each of these equations. This means that the Japanese and German monetary authorities are estimated to respond directly to U.S. monetary policy. Rows 18 and 19 show that the yen and the DM appreciate. (A decrease in a rate is an appreciation of the currency.) This is because there is a fall in the U.S. interest rate relative to the Japanese and German interest rates and because there is an increase in the U.S. price level relative to the Japanese and German price levels (not shown).

The depreciation of the dollar leads to an increase in the U.S. import price level (row 20). This increase is one of the reasons for the increase in the U.S. price level (row 5), since the price of imports has a positive effect on the domestic price level in the U.S. price equation. Even though the price of imports rises relative to the domestic price level, which other things being equal has a negative effect on import demand, the real value of imports rises (row 21). In this case the positive effect from the increase in real output dominates the negative relative price effect.

The rise in the overall U.S. price level leads to a rise in the U.S. export price level (row 22). The real value of U.S. exports rises (row 23), which is due to the

depreciation of the dollar. (The U.S. export price level increases less than the dollar depreciates, and so U.S. export prices in other countries' currencies fall.)

Finally, the nominal U.S. current account falls (row 24). The positive effects on the current account are the increase in real exports and the increase in the price of exports. The negative effects are the increase in real imports and the increase in the price of imports. On net the negative effects win, which is primarily due to the increase in the price of imports.

The real value of U.S. GDP in 1998 was about \$9 trillion, and so the results suggest that a one percentage point fall in the nominal interest rate leads to about a \$60 billion increase in real GDP after 8 quarters ($.0067 \times \$9$ trillion). This number is much lower than what one gets in the FRB/US model, where the effect is .6 percent after 4 quarters and 1.7 percent after 8 quarters—Reifschneider, Tetlow, and Williams (1999), Table 3. The effects are even larger after that, and the model eventually blows up if the short term nominal interest rate is held below its base value.⁵ As discussed in Section 1, this is a modern-view feature, where the model is unstable without an inflation coefficient in the interest rate rule greater than one.

3 Stabilization Effectiveness of Three Nominal Interest Rate Rules

The Rules

There is an estimated nominal interest rate rule for each of the major countries in the MC model. Estimated interest rate rules have a long history. The first one is in

⁵Private correspondence with David Reifschneider.

Dewald and Johnson (1963), who regressed the Treasury bill rate on a constant, the Treasury bill rate lagged once, real GNP, the unemployment rate, the balance-of-payments deficit, and the consumer price index. The next example can be found in Christian (1968), followed by many others. In 1978, I added an estimated interest rate rule to my U.S. model—Fair (1978)—and an updated version of this rule is used in this paper.

The main modification that has been made to the 1978 rule is the addition of a dummy variable term to account for the change in Fed operating procedure during the period 1979:4–1982:3 (to be called the “early Volcker” period).⁶ The stated policy of the Fed during this period was that it was focusing more on monetary aggregates than it had done in the past. The estimated interest rate rule already had the lagged growth of the money supply as an explanatory variable, and the change in policy was modeled by adding the lagged growth of the money supply multiplied by a dummy variable as another explanatory variable. The dummy variable is 1 for the 1979:4–1982:3 period and 0 otherwise.

The specification of the rule that is used in this paper is:

$$r = \alpha_1 + \alpha_2 \dot{p} + \alpha_3 u + \alpha_4 \Delta u + \alpha_5 \dot{m}_{-1} + \alpha_6 D1 \times \dot{m}_{-1} + \alpha_7 r_{-1} + \alpha_8 \Delta r_{-1} + \alpha_9 \Delta r_{-2} + \epsilon \quad (1)$$

where r is the three month Treasury bill rate, \dot{p} is the quarterly rate of inflation at an annual rate, u is the unemployment rate, \dot{m} is the quarterly rate of growth of the money supply at an annual rate, and $D1$ equals 1 for 1979:4–1982:3 and 0 otherwise. The results of estimating equation (1) are presented in Table 2.⁷

⁶Paul Volcker was chair of the Fed between 1979:3 and 1987:2, but the period in question is only 1979:4–1982:3.

⁷Note that the three month Treasury bill rate is used for the interest rate. Although in practice the

Table 2
Estimated U.S. Interest Rate Rule
 Dependent Variable is r

	Coef.	t-stat.
constant	.886	5.72
\dot{p}	.066	3.79
u	-.141	-4.52
Δu	-.887	-6.58
\dot{m}_{-1}	.012	2.00
$D1 \times \dot{m}_{-1}$.217	9.53
r_{-1}	.926	47.18
Δr_{-1}	.188	3.26
Δr_{-2}	-.351	-6.74
SE	.475	
R^2	.970	
DW	1.82	
Wald (p-value)	10.59 (.227)	

Estimation period: 1954:1–2001:1 (189 obs.)

Estimation technique: two stage least squares

r = three month Treasury bill rate

\dot{p} = inflation rate

u = unemployment rate

\dot{m} = growth rate of the money supply

$D1 = 1$ for 1979:4–1982:3; 0 otherwise

Wald test: see text

The endogenous variables on the right hand side of equation (1) are inflation and the unemployment rate, and two stage least squares was used to estimate the equation. In the first stage regressions inflation and the unemployment rate are regressed on a set of predetermined variables (the main predetermined variables Fed controls the federal funds rate, the quarterly average of the federal funds rate and the quarterly average of the three month Treasury bill rate are so highly correlated that it makes little difference which rate is used in estimated interest rate rules using quarterly data. The money supply data are taken from the flow of funds accounts.

in the U.S. subset of the MC model). The predicted values from these regressions are then used in the second stage. One can look on the these regressions as those used by the Fed to predict inflation and the unemployment rate, and so it need not be assumed that the Fed has perfect foresight.

Equation (1) is a “leaning against the wind” equation. r is estimated to depend positively on the inflation rate and the lagged growth of the money supply and negatively on the unemployment rate and the change in the unemployment rate. Adjustment and smoothing effects are captured by the lagged values of r . The coefficient on lagged money supply growth is over ten times larger for the early Volcker period than either before or after, which is consistent with the Fed’s stated policy of focusing more on monetary aggregates during this period. This way of accounting for the Fed policy shift does not, of course, capture the richness of the change in behavior, but at least it seems to capture some of the change.

The Wald test in Table 2 is of the hypothesis that the coefficients in the rule are the same before the early Volcker period as after (but not including the early Volcker period). The Wald statistic is distributed as χ^2 with (in this case) 8 degrees of freedom. The p-value is .227, and so the hypothesis of stability is not rejected at even the 10 percent level. Other tests of the equation, including other stability tests, are reported in Fair (2001a) and on the website.

The long run coefficient on inflation in Table 2 is .89 [$= .066 / (1 - .926)$], less than one. For the second interest rate rule used in this paper the .066 coefficient was changed to .111, which gives a long run coefficient of 1.5, and for the third rule the coefficient was changed to .185, which gives a long run coefficient of 2.5. None of the other coefficients in the estimated equation were changed for the

second and third rules.

The Stochastic Simulation Procedure

The three interest rate rules are examined using stochastic simulation. The focus in this paper, as in much of the literature, is on variances, not means. The aim of monetary policy is taken to smooth the effects of shocks. In order to examine the ability of monetary policy to do this, one needs an estimate of the likely shocks that monetary policy would need to smooth, and this can be done by means of stochastic simulation. Given an econometric model, shocks can be generated by drawing errors.

There are 361 stochastic equations in the MC model, 194 quarterly and 167 annual. There is an estimated error term for each of these equations for each period. Although the equations do not all have the same estimation period, the period 1976–1998 is common to almost all equations.⁸ There are thus available 23 vectors of annual error terms and 92 vectors of quarterly error terms. These vectors are taken as estimates of the economic shocks, and they are drawn in the manner discussed below. Since these vectors are vectors of the historical shocks, they pick up the historical correlations of the error terms. If, for example, shocks in two consumption equations are highly positively correlated, the error terms in the two equations will tend to be high together or low together.

The period used for the stabilization experiments is 1994:1–1998:4, five years or 20 quarters. Since the concern here is with stabilization around base paths and

⁸For the few equations whose estimation periods began later or ended earlier than the 1976–1998 period, zero errors were used for the missing observations.

not with positions of the base paths themselves, it does not matter much which path is chosen for the base path. The choice here is simply to take as the base path the historical path. As in Section 2, the base path is generated by adding the historical errors to the equations and taking them to be exogenous. For all the stochastic simulations in this paper the historical errors are added to the model and the draws are around these errors.

Each trial for the stochastic simulation is a dynamic deterministic simulation for 1994:1–1998:4 using a particular draw of the error terms. For each of the five years for a given trial an integer is drawn between 1 and 23 with probability $1/23$ for each integer. This draw determines which of the 23 vectors of annual error terms is used for that year. The four vectors of quarterly error terms used are the four that correspond to that year. Each trial is thus based on drawing five integers. The solution of the model for this trial is an estimate of what the world economy would have been like had the particular drawn error terms actually occurred. (Remember that the drawn error terms are on top of the historical error terms for 1994:1–1998:4, which are always used.) The number of trials taken is 100, so 100 world economic outcomes for 1994:1–1998:4 are available for analysis.⁹

The historical errors are added to the interest rate rule, but no errors are drawn for it. Adding the historical errors means that when the model inclusive of the

⁹Another way of drawing error terms would be from an estimated distribution. Let \hat{V} be an estimate of the 365×365 covariance matrix V of the error terms. One could, for example, assume that the error terms are multivariate normal and draw errors from the $N(\hat{\mu}_t, \hat{V})$ distribution, where $\hat{\mu}_t$ is the vector of the historical errors for t . Because of the quarterly-annual difference, \hat{V} would have to be taken to be block diagonal, one quarterly block and one annual block. Even for this matrix, however, there are not enough observations to estimate all the nonzero elements, and so many other zero restrictions would have to be imposed. The advantage of drawing the historical error vectors is that no distributional assumption has to be made and no zero restrictions have to be imposed.

rule is solved with no errors for any equation drawn, a perfect tracking solution results.¹⁰ Not drawing errors for the rule means that the Fed does not behave randomly but simply follows the rule.

Let y_t^j be the predicted value of endogenous variable y for quarter t on trial j , and let y_t^* be the base (actual) value. How best to summarize the 100×20 values of y_t^j ? One possibility for a variability measure is to compute the variability of y_t^j around y_t^* for each t : $(1/J) \sum_{j=1}^J (y_t^j - y_t^*)^2$, where J is the total number of trials.¹¹ The problem with this measure, however, is that there are 20 values per variable, which makes summary difficult. A more useful measure is the following.

Let L^j be:

$$L^j = \frac{1}{T} \sum_{i=1}^T (y_i^j - y_i^*)^2 \quad (2)$$

where T is the length of the simulation period (20). Then the measure is

$$L = \frac{1}{J} \sum_{j=1}^J L^j \quad (3)$$

L is a measure of the deviation of the variable from its base values over the whole period.¹²

¹⁰Each of the three rules used has a different set of historical errors associated with it because the predicted values from the rules differ due to the different inflation coefficients.

¹¹If y_t^* were the estimated mean of y_t , this measure would be the estimated variance of y_t . Given the J values of y_t^j , the estimated mean of y_t is $(1/J) \sum_{j=1}^J y_t^j$, and for a nonlinear model it is not the case that this mean equals y_t^* even as J goes to infinity. As an empirical matter, however, the difference in these two values is quite small for almost all macroeconomic models, and so it is approximately the case that the above measure of variability is the estimated variance.

¹² L is, of course, not an estimated variance. Aside from the fact that for a nonlinear model the mean of y_t is not y_t^* , L^j is an average across a number of quarters or years, and variances are not in general constant across time. L is just a summary measure of variability.

The Results

The results for this section are presented in the first four rows in Table 3. Values of L are presented for real GDP (Y), the level of the private nonfarm deflator (P), the percentage change in P (\dot{P}), the unemployment rate (U), and r . The following discussion will focus on Y , P , and r . The results for U are similar to those for Y , and no further discussion is needed about U . The results for \dot{P} are generally similar to those for P , although the differences in L across rules are larger for P than for \dot{P} . All the experiments for the MC model use the same error draws. This considerably lessens stochastic simulation error across experiments.

L for Y falls from 3.75 for no rule (r exogenous) to 2.73 for the estimated rule, and L for P falls from 3.76 to 2.77. Both output and price variability are thus lowered considerably by the estimated rule. Note that this is contrary to what one would get using a modern-view model, where a rule like the estimated rule would be destabilizing because the long run inflation coefficient is less than one.

Rows 3 and 4 in Table 3 show that the rules with the larger inflation coefficients lower the variability of P at a cost of raising the variability of r . (The variability of Y is little affected by the use of the different inflation coefficients.) Which rule one thinks is best depends on the weights one attaches to P and r variability, and it is at least a close call between the estimated rule and the modified rule (1.5). The main point here, however, is that the modified rules are not noticeably better. In the modern-view models they obviously would be.

Table 3
Variability Estimates: Values of L

MC Model					
	Y	P	\dot{P}	U	r
1. No rule	3.75	3.76	2.05	0.87	0.00
2. Estimated rule	2.73	2.77	1.83	0.63	0.96
3. Modified rule (1.5)	2.73	2.61	1.80	0.63	1.12
4. Modified rule (2.5)	2.74	2.38	1.74	0.63	1.46
5. 2. with tax rule	2.29	2.63	1.80	0.57	0.83
US Model					
6. No rule	4.50	3.65	2.24	0.92	0.00
7. Estimated rule	3.43	2.98	2.07	0.67	0.95
8. Optimal (0.5)	2.82	3.85	2.23	0.66	1.64
9. Optimal (1.5)	3.06	3.45	2.14	0.69	1.25
10. Optimal (2.5)	3.25	3.30	2.09	0.72	1.03

Y = real GDP

P = private nonfarm deflator

\dot{P} = percentage change in the private nonfarm deflator

U = unemployment rate

r = three month Treasury bill rate

Simulation period = 1994:1–1998:4

Number of trials = 100

Modified rule (1.5) = estimated rule with long run
inflation coefficient = 1.5

Modified rule (2.5) = estimated rule with long run
inflation coefficient = 2.5

Optimal (0.5) = loss function weight on inflation of 0.5

Optimal (1.5) = loss function weight on inflation of 1.5

Optimal (2.5) = loss function weight on inflation of 2.5

4 Optimal Control

The Procedure

Most of the literature on examining rules has not been concerned with deriving rules by solving optimal control problems,¹³ but optimal control techniques are obvious ones to use in this context. The following procedure has been applied to the U.S. subset of the MC model. This subset, which will be called the “US model,” is discussed in the appendix.

The US model is completely quarterly, and quarterly historical errors for the 1976:1–1998:4 period (92 quarters) were used for the draws. Each vector of quarterly errors had a probability of 1/92 of being drawn. Not counting the estimated interest rate rule, there are 29 estimated equations in the US model plus the export (*EX*) and price of imports (*PIM*) equations discussed in the appendix.

The optimal control methodology requires that a loss function be postulated for the Fed. For the basic loss function used here the Fed is assumed to weight output and inflation deviations equally and to care about interest rate fluctuations. In particular, the loss for quarter t is assumed to be:

$$H_t = 0.5 * 100[(Y - Y^*)/Y^*]^2 + 0.5 * 100(\dot{P} - \dot{P}^*)^2 + \alpha(\Delta r_t - \Delta r_t^*)^2 + 1.0/(r_t - 0.999) + 1.0/(16.001 - r_t) \quad (4)$$

where Y is real GDP, \dot{P} is the percentage change in the private nonfarm deflator, and * denotes a base value. The last two terms in (4) insure that the optimal values of r will be between 1.0 and 16.0. The value of α was chosen by experimentation

¹³Exceptions are Feldstein and Stock (1993), Fair and Howrey (1996), and Rudebusch (1999).

to yield an optimal solution with a value of L for r not much larger than the value that results when the estimated rule is used. The value chosen was 9.0.

Assume that the control period of interest is 1 through T , where in this paper 1 is 1994:1 and T is 1998:4. Although this is the control period of interest, in order not to have to assume that life ends in T , the control problem should be thought of as one of minimizing the expected value of $\sum_{t=1}^{T+n} H_t$, where n is chosen to be large enough to avoid unusual end-of-horizon effects near T . The overall control problem should thus be thought of as choosing values of r that minimize the expected value of $\sum_{t=1}^{T+n} H_t$ subject to the model used.

If the model used is linear and the loss function quadratic, it is possible to derive analytically optimal feedback equations for the control variables.¹⁴ In general, however, optimal feedback equations cannot be derived for nonlinear models or for loss functions with nonlinear constraints on the instruments, and a numerical procedure must be used. The following procedure was used for the results in this paper. It is based on a sequence of solutions of deterministic control problems, one sequence per trial.

Recall what a trial for the stochastic simulation is. A trial is a set of draws of 20 vectors of error terms, one vector per quarter. Given this set, the model is solved dynamically for the 20 quarters using an interest rate rule (or no rule). This entire procedure is then repeated 100 times (the chosen number of trials), at which time the summary statistics are computed. As will now be discussed, each trial for the optimal control procedure requires that 20 deterministic control problems be solved, and so with 100 trials, 2,000 solutions are required.

¹⁴See, for example, Chow (1981).

For purposes of solving the control problems, the Fed is assumed to know the model (its structure and coefficient estimates) and the exogenous variables, both past and future. The Fed is assumed *not* to know the future values of any endogenous variable or any error draw when solving the control problems.¹⁵ The Fed is assumed to know the error draws for the first quarter for each solution. This is consistent with the use of the above rules, where the error draws for the quarter are used when solving the model with the rule.

The procedure for solving the overall control problem is as follows.

1. Draw a vector of errors for quarter 1, and add these errors to the equations. Take the errors for quarters 2 through k to be their historical values (no draws), where k is defined shortly. Choose values of r for quarters 1 through k that minimize $\sum_{t=1}^k H_t$ subject to the model as just described. This is just a deterministic optimal control problem, which can be solved, for example, by the method in Fair (1974).¹⁶ Let r_1^* denote the optimal value of r for quarter 1 that results from this solution. The value of k should be chosen to be large enough so that making it larger has a negligible effect on r_1^* . (This value can be chosen ahead of time by experimentation.) r_1^* is a value that the Fed could have computed at the beginning of quarter 1 (assuming the model and exogenous variables were known) having knowledge of the error draws for quarter 1, but not for future quarters.
2. Record the solution values from the model for quarter 1 using r_1^* and the error draws. These solution values are what the model estimates would have occurred in quarter 1 had the Fed chosen r_1^* and had the error terms been as drawn.

¹⁵The main exogenous variables in the US model are fiscal policy variables. The other exogenous variables are either unimportant or easy to forecast. Remember that since the base is the perfect tracking solution, the historical errors are always added to the model.

¹⁶This method sets up the problem as an unconstrained nonlinear optimization problem and uses an optimization algorithm like DFP to find the optimum. Almost all the computer time for the overall procedure in this section is spent solving these optimization problems. The total computer time taken to solve the 2,000 optimization problems is about 16 hours on a Dell Pro 200, which uses an early Pentium chip, purchased in May 1996. On newer computers the time would obviously be much less.

3. Repeat steps 1 and 2 for the control problem beginning in quarter 2, then for the control problem beginning in quarter 3, and so on through the control problem beginning in quarter T . For an arbitrary beginning quarter s , use the solution values of all endogenous variables for quarters $s - 1$ and back, as well as the values of r_{s-1}^* and back.
4. Steps 1 through 3 constitute one trial, i.e., one set of T drawn vectors of errors. Do these steps again for another set of T drawn vectors. Keep doing this until the specified number of trials has been completed.

The solution values of the endogenous variables carried along for a given trial from quarter to quarter in the above procedure are estimates of what the economy would have been like had the Fed chosen r_1^*, \dots, r_T^* and the error terms been as drawn.¹⁷

By “optimal rule” in this paper is meant the entire procedure just discussed. There is obviously no analytic rule computed, just a numerical value of r^* for each period.

The Results

The results are presented in rows 6–10 in Table 3. The experiments in these rows use the same error draws to lessen stochastic simulation error across experiments, although these error draws are different from those used for the experiments in rows 1–5. Rows 6 and 7 are equivalent to rows 1 and 2: no rule and estimated

¹⁷The optimal control procedure just outlined differs somewhat for the procedure used in Fair and Howrey (1996, pp. 178-179). In Fair and Howrey (1996) the Fed is assumed not to know the exogenous variable values, but instead to use estimated autoregressive equations to predict these values for the current and future quarters. Also, the Fed is assumed not to know the error draws for the current quarter when solving its problem. In addition, stochastic simulation is not done. Instead, the error terms are set to zero (instead of to their historical values), the target values are taken to be the historical means (instead of the actual values), and the (one) trial uses for the error draws for a given quarter the actual errors for that quarter.

rule, respectively. The same pattern holds for both the MC model and US model results, namely that the estimated rule substantially lowers the variability of both Y and P .

Row 8 presents the results for the optimal solution with equal weights on output and inflation in the loss function. In this case the variability of Y is lowered substantially (much larger than for the estimated rule), but the variability of P is increased. The variability of r is also noticeably larger than for the estimated rule, with a value of L of 1.64 versus 0.95. Row 9 presents results when inflation is weighted three times as much as output (the coefficient on inflation in equation (4) is 1.5 instead of 0.5). In this case the variability of Y is not lowered quite as much, as would be expected. The variability of P is lowered relative to the no rule case, but not relative to the estimated rule one. The value of L for r is down to 1.25. Finally, row 10 presents results when inflation is weighted five times as much as output (coefficient of 2.5). Comparing row 10 to row 9, the variability of Y is larger, the variability of P smaller, and the variability of r smaller, all as expected.

The optimal control results show that it is easier for monetary policy to lower output variability than price variability. Some insight into this result can be gleaned from a property of the U.S. price equation, which is that the price level responds only modestly to demand (a common feature of most estimated price equations). Since the interest rate primarily¹⁸ affects the price level through its effects on demand, the price level responds only modestly to interest rate changes. The

¹⁸The interest rate also affects the price level through its effect on the value of the dollar and thus on the price of imports.

interest rate is thus better at changing output than at changing the price level for the same size change in the interest rate.

It is interesting to compare rows 7 and 10. Even though row 10 uses a loss function with five times the weight on inflation than on output, the variability of P is still slightly greater than it is in row 7, which uses the estimated rule. Assuming that the estimated rule is a reasonable approximation of Fed behavior, row 7 versus row 10 is thus indirect evidence that the Fed puts more weight on inflation than on output.

The fact that rows 7 and 10 are fairly similar also suggests that the Fed as reflected in the estimated rule is close to behaving optimally for loss functions that place a much higher weight on inflation than on output.

5 Adding a Tax Rate Rule

Turning back to the MC model, it is clear in Table 3 that considerable overall variability is left in rows 2–4. In this section a tax rate rule is analyzed to see how much help it can be to monetary policy in stabilizing the economy. The idea is that a particular tax rate or set of rates would be automatically adjusted each quarter as a function of the state of the economy. Congress would vote on the parameters of the tax rate rule as it was voting on the general budget plan, and the tax rate or set of rates would then become an added automatic stabilizer.

Consider, for example, the federal gasoline tax rate. If the short run demand for gasoline is fairly price inelastic, a change in the after-tax price at the pump will have only a small effect on the number of gallons purchased. In this case a change

in the gasoline tax rate is like a change in after-tax income. Another possibility would be a national sales tax if such a tax existed. If the sales tax were broad enough, a change in the sales tax rate would also be like a change in after-tax income.

For the results in this paper a constructed federal indirect business tax (IBT) rate based on data from the national income and product accounts is used for the tax rate rule. In practice a specific tax rate or rates, such as the gasoline tax rate, would have to be used, and this would be decided by the political process. The constructed tax rate for quarter t , denoted τ_t , is the ratio of overall federal indirect business taxes to total consumption expenditures. In the regular version of the model τ_t is taken to be exogenous.

The following equation is used for the tax rate rule:

$$\begin{aligned} \tau_t = \tau_t^* + 0.125[.5((Y_{t-1} - Y_{t-1}^*)/Y_{t-1}^*) + .5((Y_{t-2} - Y_{t-2}^*)/Y_{t-2}^*)] \\ + 0.125 * [.5(\dot{P}_{t-1} - \dot{P}_{t-1}^*) + .5(\dot{P}_{t-2} - \dot{P}_{t-2}^*)] \end{aligned} \quad (5)$$

where, as before, Y denotes real GDP and \dot{P} denotes the percentage change in a private nonfarm price deflator. It is not realistic to have tax rates respond contemporaneously to the economy, and so lags have been used in (5). Lags of both one and two quarters have been used to smooth tax rate changes somewhat. The rule says that the tax rate exceeds its base value as output and the inflation rate exceed their base values.

Results using this rule along with the estimated interest rate rule are reported in row 5 in Table 3. The use of the rule lowers L for Y from 2.73 when only the estimated interest rate rule is used to 2.29 when both rules are used. The respective numbers for P are 2.77 and 2.63. The tax rate rule is thus of considerable help

in lowering output variability and a modest help in lowering price variability. The variability of r falls slightly when the tax rate rule is added, since there is less for monetary policy to do when fiscal policy is helping.

6 Conclusion

In modern-view models an experiment like that in Table 1, where the nominal short term interest rate is decreased, would lead to the economy blowing up. Also, the use of a rule like the estimated rule in Table 3, where the long run coefficient on inflation is less than one, would not be stabilizing. The analysis of monetary policy can thus be very sensitive to the macro model used. If modern-view models are not good approximations of the economy relative to the MC model, then the monetary policy implications from these models are not trustworthy since they differ so much from those in Tables 1 and 3.

The main conclusions about monetary policy using the MC model are the following:

1. The estimated rule explaining Fed behavior substantially reduces output and price variability. Similar results are obtained when the long run inflation coefficient is made greater than one, although a long run coefficient of 2.5 leads to a large interest rate variability.
2. A tax rate rule is a noticeable help to monetary policy in its stabilization effort.
3. The optimal control results show that it is easier to lower output variability than price variability for the same size change in the interest rate. This is because the price level responds only moderately to interest rate changes.
4. The results using the estimated rule correspond fairly closely to the optimal control results with a high relative weight on inflation in the loss function.

This is indirect evidence that the Fed puts a high relative weight on inflation in its loss function and that for this loss function its behavior is close to optimal.

5. Even when both the estimated interest rate rule and the tax rate rule are used, the values of L in Table 3 are not close to zero. Monetary policy even with the help of a fiscal policy rule cannot come close to eliminating the effects of typical historical shocks. In this sense Fed power is quite limited.

Appendix

The MC Model

The MC model in Fair (1994) is used for the results in this paper. An updated version of this model has been used for the present work, and this version is presented on the website mentioned in the introductory footnote. There are 38 countries in the MC model for which stochastic equations are estimated.¹⁹ There are 31 stochastic equations for the United States and up to 15 each for the other countries. The total number of stochastic equations is 361, and the total number of estimated coefficients is 1671. In addition, there are 1030 estimated trade share equations. The total number of endogenous and exogenous variables, not counting the trade shares, is about 5000. Trade share data were collected for 59 countries, and so the trade share matrix is 59×59 .²⁰

The estimation periods begin in 1954 for the United States and as soon after 1960 as data permit for the other countries. They end between 1996 and 2000. The estimation technique is two stage least squares except when there are too few observations to make the technique practical, where ordinary least squares is used. The estimation accounts for possible serial correlation of the error terms. The variables used for the first stage regressors for a country are the main predetermined

¹⁹The 38 countries are the United States, Canada, Japan, Austria, France, Germany, Italy, the Netherlands, Switzerland, the United Kingdom, Finland, Australia, South Africa, Korea, Belgium, Denmark, Norway, Sweden, Greece, Ireland, Portugal, Spain, New Zealand, Saudi Arabia, Venezuela, Colombia, Jordan, Syria, India, Malaysia, Pakistan, the Philippines, Thailand, China, Argentina, Chile, Mexico, and Peru.

²⁰The 21 other countries that fill out the trade share matrix are Brazil, Turkey, Poland, Russia, Ukraine, Egypt, Israel, Kenya, Bangladesh, Hong Kong, Singapore, Vietnam, Nigeria, Algeria, Indonesia, Iran, Iraq, Kuwait, Libya, the United Arab Emirates, and an all other category.

variables in the model for the country. A list of these variables is available from the website.²¹

There is a mixture of quarterly and annual data in the MC model. Quarterly equations are estimated for 14 countries (the first 14 in footnote 19), and annual equations are estimated for the remaining 24. However, all the trade share equations are quarterly. There are quarterly data on all the variables that feed into the trade share equations, namely the exchange rate, the local currency price of exports, and the total value of imports per country. When the model is solved, the predicted annual values of these variables for the annual countries are converted to predicted quarterly values using a simple distribution assumption. The quarterly predicted values from the trade share equations are converted to annual values by summation or averaging when this is needed.

Since the MC model is discussed in detail in Fair (1994) and on the website, it will not be discussed in detail here. The key properties of the model that are relevant for present purposes are the effects of interest rates on the economy, and these properties will now be outlined.

The main U.S. short term interest rate in the model (r) is the three month Treasury bill rate, which the Fed is assumed to control. A change in r affects the U.S. economy in the following ways:

1. Long term interest rates depend on current and lagged values of r .
2. An interest rate appears as an explanatory variable in each of the household expenditure equations (three consumption and one housing investment) and

²¹Some of the equations in the model are changed beginning in 1999 to incorporate the EMU. Beginning in 1999, the exchange rate equations of the individual EMU countries are replaced with one exchange rate equation, and the individual interest rate rules are replaced with one rule. These changes are not relevant for this paper because the simulation period ends in 1998.

the plant and equipment investment equation, all with negative coefficient estimates.

3. An interest rate has a negative effect on capital gains (CG) in the CG equation, and changes in CG change household wealth. Household wealth is an explanatory variable in the consumption and housing investment equations with positive coefficient estimates.
4. Interest payments of firms and the government—and thus interest income of households—change when interest rates change, and household interest income appears in the household expenditure equations through a disposable income variable, which has a positive effect in these equations.
5. A change in r leads to a change in the value of the dollar vis-a-vis the other major currencies through exchange rate equations—an increase in r leads to an appreciation of the dollar and a decrease leads to a depreciation. A change in the value of the dollar leads to a change in U.S. import prices, which then results in a change U.S. domestic prices through an import price variable in the domestic price equation. The change in the value of the dollar also leads to a change in the demand for U.S. exports through the trade share equations, and it leads to a change in U.S. import demand through an import price variable in the U.S. import equation.
6. r appears as an explanatory variable in some of the other countries' interest rate rules, and so foreign interest rates in part follow U.S. rates.

The net effects of, say, a decrease in r on U.S. output and the price level are positive. Output increases because there is an increase in the demand for U.S. domestically produced goods, and the price level increases because of the increase in demand and the depreciation of the dollar.

The US Model

The optimal control procedure is too costly in terms of computer time to be able to be used for the MC model, and for this work the U.S. subset of the model was

used. The “US model” used for the optimal control results is exactly the same as the model for the United States in the MC model except for the treatment of U.S. exports (EX) and the U.S. price of imports (PIM). These two variables change when r changes—primarily because the value of the dollar changes—and the effects of r on EX and PIM were approximated in the following way.

First, $\log EX_t - \alpha_1 r_t$ was regressed on a constant, t , $\log EX_{t-1}$, $\log EX_{t-2}$, $\log EX_{t-3}$, and $\log EX_{t-4}$, and $\log PIM_t - \alpha_2 r_t$ was regressed on a constant, t , $\log PIM_{t-1}$, $\log PIM_{t-2}$, $\log PIM_{t-3}$, and $\log PIM_{t-4}$. Second, these two equations were added to the US model for particular values of α_1 and α_2 , and the experiment in Table 1 was run (an exogenous decrease in r of one percentage point). This was done many times for different values of α_1 and α_2 , and the final values chosen (-.0006 and -.0009 respectively) were ones that led to results that were similar to those in Table 1. Third, the experiment in Table 3 using the estimated rule was run for the US model with the two equations added and with the estimated errors from these equations being used in the drawing of the errors. When an error for the EX equation was drawn, it was multiplied by β_1 , and when an error for the PIM equation was drawn, it was multiplied by β_2 . The experiment was run many times for different values of β_1 and β_2 , and the final values chosen were ones that led to results similar to those in row 2 of Table 3. The values were $\beta_1 = .40$ and $\beta_2 = .75$. The results using these values are in row 7 of Table 3. The chosen values of α_1 , α_2 , β_1 , and β_2 were then used for all the experiments in rows 6–10.

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