How sustainable are old-age pensions in a shrinking population with endogenous labour supply?

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Abstract

In this paper we model an OLG-economy where labour supply is endogenously determined. The main question is whether there is an equilibrium involving an old-age pensions system, partly financed by PAYG and partly by a capital reserve system. Moreover, labour income is assumed to be a non-linear function of individual labour supply, while technological growth depends positively on individual labour effort.

An increasing preference for leisure together with a falling population growth rate will result in a fall of the contribution rates to the PAYG system, a greater accent on the capital reserve system and in a fall of output and average welfare.

Keywords: Ageing, Labour supply, Old-age pensions, Pay-as-you-go.

JEL classification: D91, H55, J14, J22, J26

1 Introduction

A major concern of the developed world is the ageing problem. Birth rates and age-specific mortality rates are falling. The result is that the world is confronted for the first time with the fact that the retired part of populations is becoming quite substantial. The ratio of retired to workers, the dependency ratio, is increasing to 1:3 or even more. It is sometimes thought that this is a transitory phenomenon and that the ratio will tend to normal values again if we reach a new equilibrium. However, it is easy to show that this is only partially true. In a stationary demography the average age is a decreasing function of the birth rate; similarly it is increasing with rising life expectancy. In all probability we have entered into a demographic situation where the population growth rate hovers at zero or even becomes negative, say minus 1 % per year, while life expectancy tends to eighty years or more. Given this new situation the question how the retired have to be supported becomes an urgent question. In most economies the retirement pension is generated from two sources or 'pillars'. The first pillar is a compulsory pension insurance system, based on compulsory savings, where workers save for their old age pension. This is a major source for capital investments. The second pillar is a state pension, which is financed by compulsory contributions from the workers to a pay - as- you - go system (PAYG). The PAYG - system does not generate capital investments. In practice in each country we find a mixture of both systems, but the mix varies widely between countries and within countries between industrial sectors.

In this paper we shall consider the question whether there is a relation between the demographic parameters , i.e. the birth rate, and the probable mix between the two pension systems and what we may expect for the total level of retirement pensions, if the demographic parameters change. This paper builds on Van Praag and Cardoso (2002), but we introduce some important new elements.

In most of the literature it is assumed that labor supply per individual is fixed. In this paper we shall assume that labor supply, that is the number of working hours per week, is determined by the worker. Hence labor supply is assumed to be endogenous. However, this free choice comes at a price. If a worker works less than a full-time working week of 40 hours, but e.g. only for half-time, then the worker will be less productive per hour than when working full-time. Empirical evidence that compensation per hour is lower in part-time jobs than in full-time jobs has been found[1]. Both employers and employees are aware of it. It may be explained by two things. First, a working task requires training and permanent schooling on the job. This implies an investment for the employer in training costs and productive hours ,which are used for learning by the worker. The rentability of such investments decreases if the worker makes shorter hours. It follows that the employer will invest less in the part- time worker than in the full - time worker. A second reason is that a job may be split up in productive hours and in a number of activities and costs, which are comparable to fixed set up costs, e.g. the tools and clothing of a blue-collar worker or the time for meetings and transfer of tasks for a white-collar worker. As the fixed costs are the same, irrespective of the number of hours in employment, the hourly production becomes less for part- time workers. It follows that productivity and productivity wages will depend non -linearly from hours worked. We assume that the worker is aware of this scale effect when determining his/her labor supply. We shall also assume that the growth rate of labor productivity are endogenous. This rate depends on the size of labor - augmenting investments and technological research. For the same reasons as given above we assume that the rate of labor -augmenting innovation is larger if workers work full-time than when they are working part-time. We assume that individual workers are unaware of this relationship and decide about their labor supply as if the growth rate does not depend on the length of the working week.

There is some evidence in practice and in literature that such relationships may exist, but we recognize that the question whether such relations hold is not scientifically beyond debate. Moreover, the socio-political content of this

[1] See, e.g., Lettau (1997)

debate is evident and this makes objective non-prejudiced research on this subject difficult. Finally, we are sharply aware of the fact that in reality there is a strong heterogeneity between individuals, which makes it hard to come to generally valid statements. Nevertheless, we shall ignore such subtleties in line with the abstraction level of this paper . In this paper we postulate a monotonic relation between working hours and productivity, but this choice might be replaced by any other functional specification as well. The major point is that the labor-augmenting growth becomes endogenous as well.

Finally, we observe in most countries that there is a correlation between the birth rate level and labor participation. For females it is well-recognized that they frequently face a choice between (increased) participation or getting children. In modern family life, however, the father is involved in child care as well, which circumstance is an inducement to work time reduction.

In this paper we assume that there are three parties in an Overlapping Generations economy: the workers, the retired, and the state. The workers care for themselves; the retired have their pension savings and some political clout, but as they are an electoral minority their influence is indirect through government; the state cares for the workers *and* the retired. The state has taken over the task to care for the weak (i.e. the retired) from the working citizens, who are less able or unable to do this as individuals. The working citizens pay taxes to the state and the state uses part of it to look after the weak, thus freeing the individual citizens from their altruistic tasks. This is a matter of efficiency. This model implies that the state and the individual workers will have different objectives and hence objective functions to be maximized. We are looking for a Nash - equilibrium between the workers and the state.

The model presented builds on the general equilibrium two-period overlapping generation model (OLG), first presented in the seminal papers by Samuelson (1958, 1975) and Diamond (1965). Endogenous or elastic labour supply is ignored in most of the subsequent literature dealing with social security[2]. Breyer and Stolte (2001), who present a model of majority voting, is an exception, as they endogenize the labour supply decision . Based on this idea, already presented in Breyer (1994a), the authors found that in case the workers can avoid the payment of taxes by reducing their labour supply, the burden of ageing will penalize more heavily the retired than the young. Casamatta et al. (2001) consider early retirement issues in the context of direct voting and conclude that a PAYG pension system inducing early retirement also generates a higher welfare than one that does not affect retirement decisions.

In a very recent contribution Razin et al. (2002) present an argument why pay-roll taxes may not keep increasing in the scenario of ageing population. In their model attention is paid to human capital investments, where individuals have different talents for learning . Labour supply is assumed to be exogenous, whereas in our model we assume labour supply to be endogenous.

These models are based on static voting, with decisions on the structure of PAYG system being taken once and for all. Cooley and Soares (1999) and Boldrin and Rustichini (2000) introduced a dynamic, repeated-voting game, where voting takes place every period: at the first stage individuals have the option to set-up a PAYG system and in every subsequent period they may vote on the continuation or abandonment of the original system. In both articles it is shown that a PAYG system arises at equilibrium, and once it is in place it will never be dismantled. This implies that the first generation that votes for the installation of the system determines its design for the coming periods. In Boldrin and Rustichini (2000) additional attention is paid to the dynamics of the political equilibrium and its effects on capital accumulation, while Cooley and Soares (1999) work also on measuring the welfare effects of the politico - economic equilibria. In the last contribution endogenous labour supply is endogenised like in our paper.

Despite the wide use of 'the median voter', one may argue that as there

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[2]For comprehensive surveys on the political economy of social security we refer to Verbon (1993), Breyer (1994) and Galasso and Profeta (2002).

are social security programs in democratic *and* non-democratic countries, median voter models are less appropriate to explain the existence of such programs in authoritarian systems or in multi-partisan democracies. We give a contribution by modeling the decision making in the framework of a government that has its own objective function, which comprises the citizen's utility function as one of its arguments. This is the case in representative democracies, but it can describe also the reality in non-democratic countries, where dictators have to avoid public outrage. In this dimension, the work by Meijdam and Verbon (1996) predates this paper as these authors use a similar approach to the policy decision making to study the effects of ageing on the economy.

Some aspects of the ageing problem have been studied in a context of (computable) general equilibrium OLG - models. Examples of this approach are Auerbach and Kotlikoff (1987), Raffelhuschen and Risa (1995), Miles (1999). These models provide a normative analysis in a context of a large number of simultaneously living population cohorts. The model presented in our paper is focusing on a positive rather than on a normative approach.

A considerable literature looks at intergenerational transfers systems with endogenous fertility considering the effect of demography on the (macro) sustainability of the PAYG system . An early bird in this literature is Van Praag and Poeth (1975). It attempts to model a third type of old-age income provision: transfers from children to parents. In this case old age income is dependent on one's offspring . Transfers come associated with altruism from children to parents as in Nishimura and Zhang (1992), therefore children are willing to support their parents. Kolmar (1997) reverses the direction of altruism assuming that parents care for their children's welfare and derives the optimal PAYG-pension formula related to the number of children raised. Cigno (1993) studied intergenerational transfers when any kind of altruism is assumed to be away. Within a three-periods OLG world, it is shown that transfers from selfish middle-age individuals (the only ones with own income) to their parents and offspring can be sustained if people see transfers from the middle-aged individual to the young as loans and from the middle-aged to the old as debt repayments.

We do not include transfers from children to parents as a third pension system as it is relatively unimportant in real-world developed economies. It is replaced by the institute of compulsory membership of a national PAYG-system.

In this paper we distinguish between the capital reserve system and the PAYG - system. We observe that most retired citizens are supported from both sources. The mix differs between countries. The novel contributions of our paper are that we look at the mix and the total level of pensions as the outcomes of a political Nash -equilibrium, with the individual citizens and the govenrment as actors. We allow for endogenous population growth , labour supply , and technological growth.

This paper is organized as follows. First, in section 2, the two period model that will be used is presented. In section 3 the equilibrium is derived. In section 4 numerical results are analysed. In Section 5 we evaluate our results and draw some conclusions.

2 The Model

2.1 The Citizen

We assume that individuals live for two periods. During the first period of life they work and during the second period they are retired. In this economy the worker has two choice variables: savings S_t and labor supply φ_t. In reality labour supply may be varied in a number of ways, e.g., by shorter hours, early retirements, longer holidays, etc. If we replace the 'individual' by the 'household', labour supply may also vary because one or both of the spouses do not work full -time. Then we talk of household supply. In this paper we will not make such differentiations. We scale the variable φ_t such that $\varphi_t \in [0, 1]$. We also assume that the worker takes the full - time wage rate w_t, the interest rate r_t, the growth rate g_t and the pay-roll tax θ_t as exogenously given.

A worker's consumption equals

$$C_t^t = \{(1-\theta_t)w_t - S_t)\}h(\varphi_t)A_t \tag{1}$$

The superscript stands for the birth rate and the subscript for calendar time. We assume that each worker stands for A_t labour efficiency units. If he/she works for a fraction φ_t, his/her gross wage will be $wh(\varphi_t)A$. The share φ_t can either be seen as the number of years spent working until (early) retirement or as the number of working hours per week. The function $h(\varphi_t)$ expresses the (non-linear) relation between labour effort and labour income, with $h'(.) > 0$ and $h''(.) > 0$. The variable θ_t is the fraction of wage w_t, which is paid as contribution to the PAYG system. Savings amount to S_t per labour efficiency unit supplied. In the second period of life the individual will be retired and the consumption will be:

$$C_{t+1}^t = \{P_{t+1} + (1+r_{t+1})S_t\}h(\varphi_t)A_t \tag{2}$$

where P_{t+1} represents the PAYG - pension and $(1+r_{t+1})S_t$ represents the pension from the funded pillar, both per unit of effective labour supplied by the individual in the first period. The interest rate is denoted by r_{t+1}. We assume for the (working) individual a separable lifetime utility function

$$U^t = U_1(C_t^t, 1-\varphi_t) + \rho U_2(C_{t+1}^t) \tag{3}$$

where ρ is the individual time preference discount factor. The working period utility depends on consumption C and leisure $(1-\varphi_t)$, while the retirement period utility depends on consumption only.

In order to allow for further numerical treatment we shall assume that the consumption utility of an individual is specified as a function of the Constant Relative Risk Aversion (CRRA)-type, that is, $U(C) = C^{(1-\gamma)}/(1-\gamma)$. In line with most empirical estimates we assume that the relative risk aversion parameter γ is larger than one ($\gamma > 1$). This implies that the utility value is negative but increasing in C. If we assume that $U_1 = U_c(C).U_{le}(1-\varphi)$ where the first factor is consumption utility and the second factor is leisure utility, it follows that for U_{le} positive and monotonically increasing in leisure an increase in leisure would lead to a decrease of overall utility U_1. It implies that we have to adapt our specification of the leisure part to account for $\gamma > 1$. We assume the specific form: $u(1-\varphi_t) = \frac{1}{(1-\varphi_t)^{\varepsilon}}$, where $\varepsilon > 0$. Lifetime utility is

$$U^t = \frac{(C_t^t)^{1-\gamma}}{1-\gamma}\frac{1}{(1-\varphi_t)^{\varepsilon}} + \rho\frac{(C_{t+1}^t)^{1-\gamma}}{1-\gamma} \tag{4}$$

The utility function then becomes after substitution of (1) and (2) in (4):

$$U^t = \frac{(\{(1-\theta_t)w_t - S_t)\}h(\varphi_t)A_t)^{1-\gamma}}{(1-\gamma)(1-\varphi_t)^{\varepsilon}} + \rho\frac{(\{P_{t+1} + (1+r_{t+1})S_t\}h(\varphi_t)A_t)^{1-\gamma}}{1-\gamma} \tag{5}$$

2.2 Demography

We assume that population L_t grows at the rate n_t per period, $L_{t+1} = (1 + n_t)L_t$. It is assumed, based on the observed trends in the last century and on demographic family- planning- surveys, that the longer working weeks go hand in hand with smaller families. Thus increased working efforts will reduce population growth. The equation of population growth is specified as

$$n = \tilde{n} - e\varphi_t^{\chi}$$

The idea is that there is a maximum birth rate $\tilde{n}$ which is reduced with a heavier work burden. Typically this reduction will be non - linear. We assume $e, \chi \geq 0$. There is a maximum growth rate $\tilde{n}$ and a minimum rate $\tilde{n} - e$,which may be negative.

2.3 Technology

For the sake of simplicity we consider a one -commodity closed economy, where the commodity can be either used as capital or as consumption good.

Consider a closed economy, consequently the current aggregate capital stock is the sum of savings of all old individuals: $K_{t+1} = S_t h(\varphi_t) A_t L_t$. we assume a Cobb-Douglas production function, which yields : $Y_t = K_t^{\alpha}(A_t h(\varphi_t) L_t)^{1-\alpha}$ and $y_t = k_t^{\alpha}$, with $k_t = (\frac{K_t}{A_t h(\varphi_t) L_t})$, where y and k stand for the production and the capital per employed efficiency unit. In the case of competitive markets the interest and wage rates per offered unit of effective labour equal the marginal productivity of capital and labour, respectively. We get as usual:

$$r_t = \frac{dy}{dk} = \alpha \ k_t^{\alpha-1}$$

$$w_t = y_t - r_t k_t = (1-\alpha) \ k_t^{\alpha}$$

In this paper labor input is endogenous. First it depends on the hour supply φ_t but it depends on the quality of labor as well. Each individual embodies a total number of A_t labour efficiency units. Now we assume that $A_t = A(\varphi_t)$, that is, labor quality is partly determined by the number of hours supplied. For example, an industrial worker who works only one morning per week will be not very productive because he lacks on -the - job training, experience and routine. If the worker would have to work for seven days a week, he would become so tired that his hours are not very productive either. We cannot exclude that $A(\varphi_t)$ will first increase and later on decrease due to fatigue. In short we assume a non-linear relationship $A(\varphi_t)$. In this paper we assume $A(\varphi_t)$ increasing over the relevant range and hence that we have only to account for the increasing part of this relationship. We shall assume that

$$h(\varphi_t) A_t = {\varphi_t}^{\lambda} A(1) \tag{6}$$

where $A(1)$ stands for the labour efficiency units of a complete/full job history and where $\lambda \geq 1$. In the case of $\lambda = 1$ there are no scale effects. We shall assume that the worker himself does take this micro-relation into account when choosing his labor time φ_t.

We assume technological growth g_t in the sense that $A_{t+1}(1) = (1 + g_t)A_t(1)$. But also here we shall assume that g_t is endogenous, that is, $g_t = g(\varphi_t)$. If labor supply is low, it is less profitable and there are fewer opportunities to enhance the quality of labor due to learning-by-doing than when participation is high. We specify $g_t = \tilde{g} * \varphi_t^{\eta}$ $(\eta \geq 0)$[3]. The maximum growth rate is denoted by $\widetilde{g}$.

We see this as a macro-relation of which the individual worker is unconscious. It does not affect his labour supply.

The total growth rate of the labour force in terms of efficiency units is denoted by υ_t, and we have $\upsilon_t \cong g_t + n_t$.

2.4 The state and the PAYG social security system

Decisions on the size of the PAYG system can be modelled in at least two ways: one is to assume a direct voting process, where the decision will depend on the 'median voter'. However, if the political spectre is not one - dimensional, it is frequently impossible to define a 'median voter'. The median-voter - solution is irrelevant in non-democratic or multi-party systems. The other way, which shall be followed here, is to consider the state, represented by its government, as a separate agent in the economy. This is obviously the everyday practice in most countries. The rationale for the existence of a state is that it can do things which citizens want but cannot do themselves or only at much higher costs. This holds especially for the production of collective goods, merit goods, and all kinds of basic collective insurances like for old-age, disability, defense in case of war, and natural disasters. This makes it also possible that individual citizens have not to bother

[3] When $\eta = 0$ and $\lambda = 1$ we are back into the situation of 'pure' exogenous growth.

about such tasks. Individuals do not have to care for their anonymous fellow - citizens, because the state will care for them if necessary. The state becomes a defender of the weak. In the simplified world, which we consider, the weak are thought to be identical with the retired (generation $t-1$). They have no power to ask for contributions from the workers (generation t) and mainly they dispose only of a minority of the votes. These activities are financed by levying a tax θ_t.

The government takes into account the interests of both the currently living workers and the retired. So government policies are influenced by both living generations. It behaves as if it is maximizing a *composite utility function* (W), a weighted average of the utilities of the different generations. It will be called the government objective function (GOF). An additive form for the GOF is assumed:

$$W = U^t(C_t^t, 1-\varphi_t, C_{t+1}^t) + \delta(n_t)U^{t-1}(C_t^{t-1}) \tag{7}$$

The first term in (7) represents the lifetime utility function of the present workers. The last term stands for the utility of the currently retired. The weight $\delta(n_t)$ reflects the relative weight that is assigned to the retired, being 1 the weight of the young generation. The weight distribution $(1, \delta(n_t))$ may depend on the ratio between the number of retired and workers and on the over or under-representation of the interests of the retired by the government, then it may become $\delta_t = \frac{\tilde{\delta}}{1+n_t}$. For the baseline scenario the value $\tilde{\delta} = 1$ will be considered. If the population does not accept the government's behavior, that is its GOF, there will be discontent. If that discontent is large enough, the government or at least its GOF will be changed, either by democratic means or by revolution. We will assume in this paper that the GOF reflects the expectations of the citizens as to what the government should stand for.

We notice that the government has only one instrument in this model, the tax rate θ_t, by which it can influence the present and future consumption of the workers C_t^t, C_{t+1}^t and the consumption of the presently retired C_t^{t-1}. We assume that the workers when making their decisions on saving and labour supply do this under the hypothesis that the current tax rate will be left unchanged in the future.

We notice that it looks as if the government has a finite horizon of only one period ahead. However, we may extend this to a more - period or even infinite horizon. We might suggest the following extended government objective function

$$W = U^t(C_t^t, 1-\varphi_t, C_{t+1}^t) + \delta(n_t)U^{t-1}(C_t^{t-1}) + \zeta U^{t+1}(C_{t+1}^{t+1}, 1-\varphi_{t+1}, C_{t+2}^{t+1}) \tag{8}$$

where we assume in this example that the government is sensitive to the interests of the first-next unborn generation. We notice that in a dynamic equilibrium consumption will increase by the growth rate $g(n)$ and hence utility by $g(n)^{1-\gamma}$. It follows that the above expression can be rewritten in terms of the original GOF. The generalization is rather easy for a longer time horizon. Although we will not take this more -period extension explicitly into account, it follows that such an interpretation is possible. This extension is certainly relevant in a normative approach, however it is hard to believe that a usual government is so far -sighted. For a positive analysis, which aims at a description of what 'is' and not what 'ought' the extension will not be taken into account[4].

The government collects contributions from the workers and pays all the revenue as pensions to the elderly, so an equality between expenditures and tax revenues holds. Thus the PAYG pension is determined by the budget constraint:

$$P_t * A_{t-1} * L_{t-1} * h(\varphi_{t-1}) = \theta_t * w_t * A_t * L_t * h(\varphi_t)$$

[4]Moreover, that concern about future generations well-being is more likely to arise from the side of the altruistic individuals towards their offspring than directly from a government itself.

and we find

$$P_t = \theta_t w_t (1 + \upsilon_t) \frac{h(\varphi_t)}{h(\varphi_{t-1})} \tag{9}$$

The rules of behaviour are now obvious. The workers maximize their utility by deciding about their labour supply φ_t and their savings S_t. The state maximizes the GOF with respect to θ_t. Moreover, a Nash behaviour is assumed: both the individual and the government take the decisions of the other part as given. So the intricated relations between contribution rates, savings and labour supply are not taken into account by the agents as we may doubt whether in fact real decision-makers know these relationships.

If there exists a stationary equilibrium, there have to be values S, φ, θ which are constant over time such that the first - order- conditions for both parties are satisfied. Moreover, there are three side - conditions which have to be fulfilled. Those deal with the endogeneity of the population growth rate n, the growth rate g and the level of the efficiency unit as a function of φ.

2.5 The first - order conditions

The first-order-condition for the individual problem equation (5) with respect to leisure gives

$$\frac{\partial U^t}{\partial \varphi_t} = (C_t^t)^{-\gamma} \frac{1}{(1-\varphi_t)^{\varepsilon}} \frac{C_t^t}{h} .h' + \varepsilon (C_t^t)^{1-\gamma} . \frac{1}{(1-\gamma)(1-\varphi_t)^{\varepsilon+1}} + \rho \{ (C_{t+1}^t)^{-\gamma} \frac{C_{t+1}^t}{h} .h' \} = 0 \tag{10}$$

As $h = \varphi^{\lambda}$ we notice $\frac{h'}{h} = \frac{\lambda}{\varphi}$

$$\{ \frac{C_t^t}{C_{t+1}^t} \}^{1-\gamma} = \frac{-\rho\lambda(1-\varphi_t)^{\varepsilon+1}}{(1-\varphi_t)\lambda + \frac{\varepsilon\varphi_t}{1-\gamma}} \tag{11}$$

We get for the first-order-condition with respect to savings:

$$\frac{\partial U^t}{\partial S_t} = (C_t^t)^{-\gamma} * (-A_t h(\varphi_t)) * \frac{1}{(1-\varphi_t)^{\varepsilon}} + \rho(C_{t+1}^t)^{-\gamma}(1+r_{t+1})A_t h(\varphi_t) = 0 \tag{12}$$

and after further simplification:

$$\frac{C_t^t}{C_{t+1}^t} = (\rho(1+r_{t+1})(1-\varphi_t)^{\varepsilon})^{-\frac{1}{\gamma}} \tag{13}$$

The government takes the decisions of the individuals as given, and then the first-order-condition for government optimizing behaviour is given by:

$$\begin{aligned} \frac{\partial W}{\partial \theta_t} &= (C_t^t)^{-\gamma}(-w_t A_t h(\varphi_t)) * \frac{1}{(1-\varphi_t)^{\varepsilon}} + \\ &+\rho(C_{t+1}^t)^{-\gamma}(w_{t+1}(1+\upsilon_{t+1})A_t h(\varphi_{t+1})) + \\ &+\delta(C_t^{t-1})^{-\gamma}(w_t(1+\upsilon_t)A_{t-1}h(\varphi_t)) = 0 \end{aligned} \tag{14}$$

after simplification we get:

$$\begin{aligned} &-(C_t^t)^{-\gamma}(w_t h(\varphi_t))\frac{1}{(1-\varphi)^{\varepsilon}} + \rho(C_{t+1}^t)^{-\gamma}(w_{t+1}(1+\upsilon_{t+1})h(\varphi_{t+1})) + \\ &+\delta(C_t^{t-1})^{-\gamma}(1+\upsilon_t)\frac{1}{(1+g_t)}w_t h(\varphi_t) = 0 \end{aligned} \tag{15}$$

or

$$\begin{aligned} &-(C_t^t)^{-\gamma}(w_t h(\varphi_t))\frac{1}{(1-\varphi)^{\varepsilon}} + \rho(C_{t+1}^t)^{-\gamma}(w_{t+1}(1+\upsilon_{t+1})h(\varphi_{t+1})) + \\ &+\delta(C_t^{t-1})^{-\gamma}(1+n_t)w_t h(\varphi_t) = 0 \end{aligned} \tag{16}$$

We observe that factor prices w_t and r_t are also influenced by θ_t.

3 The Equilibrium

In the equilibrium, as usual, we assume that the variables per effective labour unit φ_t, S_t and θ_t, and consequently w_t and r_t are constant over time. The only sources of growth are the population growth n_t and the technological progress g_t. Variables without time index represent equilibrium values.

The solution to the three first-order-conditions of our model, two for the individual and one for the government behaviour, will give the solution for the variables of our model. We will go back to the first-order-conditions of the individual problem. For leisure, from equation (11) we get:

$$\{\frac{C_t^t}{C_{t+1}^t}\}^{1-\gamma} = \frac{-\rho\lambda(1-\varphi)^{\varepsilon+1}}{(1-\varphi)\lambda + \frac{\varepsilon\varphi}{1-\gamma}}$$

or

$$\{\frac{C_t^t}{C_{t+1}^t}\}^{1-\gamma} = \frac{-\rho(1-\varphi)^{\varepsilon+1}}{1-\varphi(1+\frac{\varepsilon}{\lambda(\gamma-1)})} \tag{17}$$

For savings, from equation (13) we find

$$(\frac{C_t^t}{C_{t+1}^t})^{-\gamma} = \rho(1+r)(1-\varphi)^{\varepsilon} \tag{18}$$

The first-order condition for government behaviour (16) becomes after some algebra:

$$-(C_t^t)^{-\gamma}(w\varphi)\frac{1}{(1-\varphi)^{\varepsilon}} + \rho(C_{t+1}^t)^{-\gamma}w\varphi(1+\upsilon) + \delta(C_t^{t-1})^{-\gamma}w\varphi(1+n) = 0 \tag{19}$$

Simplifying and dividing by $(C_{t+1}^t)^{-\gamma}$,while taking into account that in equilibrium $C_{t+1}^t = (1+g)C_t^{t-1}$ we get

$$-(\frac{C_t^t}{C_{t+1}^t})^{-\gamma}\frac{1}{(1-\varphi)^{\varepsilon}} + \rho(1+\upsilon) + \delta(1+g)^{\gamma}(1+n) = 0 \tag{20}$$

3.1 Leisure

Combining the first order conditions for leisure (17) and the government (20) we get the equation

$$-(\frac{\rho(1-\varphi)^{\varepsilon+1}}{\varphi(1+\frac{\varepsilon}{\gamma-1})-1})^{\frac{-\gamma}{1-\gamma}}\frac{1}{(1-\varphi)^{\varepsilon}} + \rho(1+\upsilon) + \delta(1+g)^{\gamma}(1+n) = 0 \tag{21}$$

After some simplification we get the first-order condition for leisure:

$$-(\frac{\rho}{\varphi(1+\frac{\varepsilon}{\lambda(\gamma-1)})-1})^{\frac{\gamma}{\gamma-1}}(1-\varphi)^{\frac{\gamma+\varepsilon}{\gamma-1}} + \rho(1+\upsilon) + \delta(1+g)^{\gamma}(1+n) = 0 \tag{22}$$

The solution to this reduced form equation gives the value for the optimal labor supply φ in equilibrium. We notice that this is an equation in one unknown φ as δ, n and g and consequently υ are functions of φ as well. Having determined the endogenous φ we may roll back the system and solve for the other unknowns. Unfortunately there is no closed -form -solution for this equation, so it turns out that we have to use numerical methods . The outcomes of numerical solutions will be presented below in section (4).

3.2 Contribution rate

The equilibrium contribution rate is found from equation (20). We get:

$$(\frac{C_{t+1}^t}{C_t^t})^{-\gamma} = \frac{1}{(1-\varphi)^{\varepsilon}} * \frac{1}{(\rho(1+\upsilon)+\delta(1+g)^{\gamma}(1+n))} \tag{23}$$

$$\left(\frac{(1+r)S+P}{(1-\theta)w-S}\right)^{-\gamma} = \frac{1}{(1-\varphi)^{\varepsilon}} * \frac{1}{(\rho(1+\upsilon)+\delta(1+g)^{\gamma}(1+n))} \tag{24}$$

After some straightforward algebra we obtain an explicit solution for θ:

$$\theta = \frac{(1-\frac{S}{w})H-(1+r)\frac{S}{w}}{1+\upsilon+H} \tag{25}$$

$$\text{with } H = ((\rho(1+\upsilon)+\delta(1+g)^{\gamma}(1+n))(1-\varphi)^{\varepsilon})^{\frac{1}{\gamma}}$$

3.3 Interest rate and Savings

Combining the first-order-conditions for the government with the individual's savings problem, equations (20) and (18) we get:

$$(\rho(1+\upsilon)+\delta(1+g)^{\gamma}(1+n))(1-\varphi)^{\varepsilon} = \rho(1+r)(1-\varphi)^{\varepsilon}$$

$$\rho(1+\upsilon)+\delta(1+g)^{\gamma}(1+n) = \rho(1+r)$$

yielding

$$r = \upsilon + \frac{\delta}{\rho}(1+g)^{\gamma}(1+n) \tag{26}$$

The interest rate is a function of the growth rate of this economy υ plus a term that depends on the political weight of the old δ. In case the government does not take into account the utility of the old, i.e. $\delta = 0$, we find the traditional golden rule solution of capital accumulation. If δ increases, it is hard to say what will be the effect on r, because n, g and υ depend on δ as well. However, the numerical explorations will provide some insight in the sensitivity of the solution with respect to parameter values.

We are now able to derive the optimal savings per labour effective unit in equilibrium. Using a Cobb-Douglas production function we have that $r = \alpha k^{\alpha-1}$ in a competitive equilibrium. It follows that, in equilibrium, the capital stock becomes, $k = (\frac{r}{\alpha})^{\frac{1}{\alpha-1}}$. Savings are then

$$S = (1+\upsilon)(\frac{\upsilon + \frac{\delta}{\rho}(1+g)^{\gamma}(1+n)}{\alpha})^{\frac{1}{\alpha-1}} \quad (27)$$

4 Numerical Explorations.

The model outlined above hangs on the solution of the non-linear equation (22). If we want to analyse the dependency between the various parameters and the outcome variables, we have to take recourse to numerical simulations, where we vary the parameter values. We will now present numerical results of our model.

The basic parameters of the model are:

a. population parameters $\tilde{n}, e$ and χ, where $\tilde{n}$ stands for maximum population growth per period , $(\tilde{n} - e)$ stands for minimum growth and χ stands for the elasticity of growth with respect to the length of the working week (w.w.);

b. the technological parameters $\alpha, \widetilde{g}, \eta$ and λ ,where α stands for the production elasticity of capital in the C.D. production function, $\widetilde{g}$ for maximum productivity growth, η for the elasticity of productivity growth with respect to w.w. and λ for the wage elasticity with respect to the w.w;

c. the individual parameters γ, ε and ρ , where the first two describe the effect of consumption and leisure on the utility and where ρ stands for the subjective time discount rate;

d. the political parameter δ ,which reflects the weight assigned to the weak part of the population.

We should be aware that we deal with a period length of a generation of say about 35 years. It follows that a maximum population growth rate $\tilde{n}$ of 0.3 per period is equivalent to about 0.7% per year. Similarly the parameters e, $\tilde{g}$, and ρ have to be annualized. We will start from a baseline scenario and look what changes in the equilibrium if one parameter is allowed to vary. We will assume as our baseline scenario an exogenous maximum population growth rate $\tilde{n}$ of 0.7% per year, an exogenous maximal technological growth rate $\tilde{g}$ of 3.2% per year, and a subjective time discount rate of about 1.5% per year. The values for the growth rate reflect the observed ones in several real economies, the time preference is in line with Miles (1999) and further references presented there. All the other parameters of the model are independent of the period length considered. For the relative risk aversion γ a value of 2 is taken. Capital share in the Cobb-Douglas production function is set at α= 20%. For the political weight of the old δ we take $\frac{\tilde{\delta}}{1+n}$ (starting with $\tilde{\delta} = 1$ in the baseline scenario). The relative preference of leisure vs. consumption, expressed by ε is fixed at 0.5. For η and λ, we assume 0.8 and 1.4. For the impact of the labour effort on demographic growth, we take $e = 0.2$ and χ =0.8. This baseline scenario is chosen partly because empirical estimates are known, e.g. for α. For the other part they have been chosen by calibration, as we looked for realistic outcomes.

Later we will look at the sensitivity of the outcomes of our model to changes in some of these parameters, the first of them, the population growth rate, to see how different demographic scenarios will influence the design of the pensions systems.

To summarize, our baseline scenario becomes:

$\widetilde{n}$	e	χ	ε	ρ	γ	$\widetilde{g}$	λ	η	α	δ
0.3	0.2	0.8	0.5	0.6	2.0	2.0	1.4	0.8	0.2	$\frac{1}{1+n}$

The value $\tilde{\delta} = 1$ is taken to start this baseline scenario.

We are interested in the resulting values for the endogenous variables. Those variables are:

n the population growth rate
npa the population growth rate (p.a.)
g the technological growth rate
gpa the technological growth rate (p.a.)
φ the length of the working week
θ the contribution rate
Y Gross income
υpa the growth rate of the economy
rpa the interest rate (p.a.)
k/y capital output ratio (p.a.)
S/w Gross savings rate
FR Funding ratio: $\frac{(1+r)S}{(1+r)S+(1+\upsilon)\theta w}$
$BenR$ Benefit ratio: $\frac{C_t^{t-1}}{C_t^t}$
$AvgW$ Total welfare: $U_t^t + \frac{1}{1+n}U_t^{t-1}$

Most of the variables do not need further explanation. The capital - output ratio is the ratio of a timeless variable and one which is calculated per period. It has to be annualized by multiplying it by 35. The funding ratio measures the mix between the PAYG - and the capital- reserve- system. The benefit ratio is C_t^{t-1}/C_t^t. Total welfare stands for the average utility of the workers and the retired, weighted by their population shares.

Table 1. Outcomes for the baseline scenario.

npa	0.5%
gpa	1.9%
φ	0.38
θ	19%
Y	0.144
υpa	2.4%
rpa	4%
k/y	2.42
$\frac{S}{w}$	0.199
FR	63.8%
$BenR$	66.7%
$AvgU$	-10.84

The resulting outcomes are a population growth of 0.5 % and a productivity growth of 1.9% per year. The labour participation ratio φ becomes 0.38. This means that people will work somewhat above one third of their total time available, which would be 24 hours a day. The social security tax is 19 % of labour income. The total growth rate becomes 2.4 % and the rate of interest 4 %. The capital output ratio is 2.42 and the savings ratio 20 %. The resulting funding ratio is about 64 %. This means that about two thirds of the retirement income is coming from own savings while the remainder stems from the pay - as -you - go system. The benefit ratio is here 67%.

The really interesting question is now how the outcomes change when we change the baseline parameters in an interval around the baseline values. The ranges of values considered are presented in Table 2. The outcomes are almost always monotonic functions of the input parameters. Although those relationships do not exhibit constant elasticity, the average elasticities over the relevant ranges are indicative of those relationships. The values of the elasticities of the impacts of changes in the endogenous variables as functions of changes in the exogenous parameters of the model are presented in Table 3.

Table 2: The ranges of the parameters values

	Lower bound	Upper bound	Increment
$\widetilde{n}$	-0.1 (-0.003)	0.4 (0.01)	0.05
e	0	1.0	0.1
χ	0	1.0	0.1
ε	0.1	1.1	0.1
ρ	0.35 (0.009)	0.85 (0.018)	0.05
$\widetilde{g}$	0 (0)	2.5 (0.037)	0.25
λ	1.0	2.0	0.1
η	0	2.0	0.2
α	0.14	0.34	0.02
δ	0.5	1.5	0.1

In parenthesis the corresponding annualised values are presented

Table 3A: Impact elasticities of the demographic and individual preferences parameters

	$\tilde{n}$	e	χ	ε	ρ
n	1.736	−2.778	0.289	0.094	0.202
g	0.010	−0.015	0.003	−0.234	−0.466
φ	0.013	−0.019	0.004	−0.292	−0.582
θ	−0.160	0.216	−0.049	−0.453	−0.608
Y	0	0.004	0	−0.644	−1.145
vpa	0.290	−0.382	0.081	−0.164	−0.226
rpa	0.078	−0.121	0.023	−0.229	−0.752
k/y	−0.137	0.207	−0.041	0.397	1.365
S/w	0.016	−0.026	0.005	0.306	1.180
FR	0.046	−0.062	0.014	0.177	0.324
$BenR$	0.017	−0.023	0.005	−0.057	0.259
$AvgW$	0.099	−0.164	0.029	−0.452	−0.549

Table 3B: Impact elasticities of the technological and political parameters

	$\tilde{g}$	λ	η	α	$\tilde{\delta}$
n	-0.065	-0.064	0.073	0	-0.084
g	1.154	0.144	-1.397	0	0.192
φ	0.200	0.180	-0.234	0	0.239
θ	0.586	-0.465	-1.383	-6.65	1.294
Y	0.825	-0.840	-0.141	-1.066	0.396
vpa	0.562	0.070	-0.697	0	0.093
rpa	0.559	-0.273	-0.676	0	0.480
k/y	-0.912	0.484	1.049	1.000	-0.866
S/w	-0.544	0.542	0.644	1.323	-0.789
FR	-0.281	0.205	0.366	1.123	-0.465
$BenR$	-0.086	1.120	0.099	0	0.304
AvW	0.606	-1.123	-0.119	-0.866	0.149

It is obviously impossible to comment on all 120 elasticities. So let us concentrate on the most important values. The first three parameters $\widetilde{n}$, e , and χ have their expected effects on the population growth rate n. Moreover, more fertility implies a lower tax rate and a decrease of the capital-output ratio. The other variables are hardly affected.

The individual parameters ε and ρ have strong effects. Those effects have always the same sign, but the time discount effect is absolutely much larger. Actually, this similarity may be explained as individuals with a strong preference for leisure will (in this model) derive much satisfaction from retirement as it offers 100 % leisure. When the preference for leisure ε or ρ increases,we find that the population growth rate mildly increases, but that other outcomes, especially the labour supply, the PAYG tax rate, the effective technical growth rate, national product and average welfare fall rather strongly. Savings, the capital intensity and the funding ratio are seen to increase.

Now let us look at the technological parameters. Our model predicts that an increase in the maximum technological growth $\tilde{g}$ will lead to an increment in all the growth rates. Nevertheless, the capital intensity and the savings rate will fall considerably. It appears that most of the growth will trickle down in an enlarged PAYG - system. Labour participation will grow. Inversely, if the possibilities for growth, as reflected by the upper bound $\tilde{g}$ diminish, we find that the labor participation will tend to less full time jobs, to a greater dependency on own savings and a reduction of social security, thus to less welfare on the average. This may be the situation at the end of a technical innovation wave (or the end of a Kondratiew cycle), where the upper limit of growth seems to reduce. The higher λ, the more wage is reduced when working part - time. It follows that labour supply rises and the population growth rate n shrinks. In the case that jobs require more skills and it becomes more difficult to combine two half -time jobs to one full -time we may expect a λ, which is considerably higher than one, the situation of ideal 'putty'- jobs. It seems that in western economies, where unskilled piece work is diminishing and hence λ is increasing, we see this phenomenon coming up, although countered by an increase in leisure preferences in some Western countries. A change in η, the sensitivity of technological growth with respect to longer hours, has similar effects. An increase in the capital intensity α will lead to a relatively more funded pension system, with an accompanying reduction of θ and an increase in the savings rate. Although the system mix changes the total benefit ratio remains constant, when α varies.

Finally, let us look at the effect of δ. If the retired generation becomes more influential, that is the state becomes more a countervailing power to the workers, it results in a huge increase of the PAYG -system with a simultaneous but lesser reduction of the capital - reserve system. Labour supply φ will increase as well. As a consequence fertility will be slightly negatively affected. The output level and growth and interest rates will increase. The savings ratio and the capital - output ratio fall considerably. Not unexpectedly the benefit ratio will increase and the average welfare will increase as well.

5 Conclusion

In this paper we developed an OLG - model where labour supply is endogenously determined in the model. The same holds for the population growth rate and the mix of the old - age pension system between PAYG and funded pillar. It is evident that this model is a very stylized version of reality. It is a two - period model with a homogeneous population. Some of the parameters of this model seem even difficult to estimate in practice. Another possible point of critique of this paper is that we look for an equilibrium path, while in practice we can only observe a tendency to an equilibrium path. Nevertheless, this exercise in comparative statics does tell us something about the current situation and the tendencies which may be expected in reality in the near future, as these results hold for a broad range of parameters values. The increasing weight on leisure together with the falling population growth rate will result in a fall in the contribution rates to the PAYG system, a greater accent on the capital reserve system, a lowering of the benefit ratio, and a fall in average welfare.

The most interesting question is whether we can find long-run trends in the parameters which are observed in reality. In our view they are related with a change in time use. Stronger preferences for leisure will lead to reduced labour effort. Consequently, the wage basis from which taxes are collected shrinks. Moreover, it is a fact that fertility is falling as well. So the demographic trends are also adverse to the maintenance of extensive pensions systems running on an unfunded basis. It seems to us that this is the situation which describes the trends of the evolution of the near future and the model presented in this paper accounts for these trends.

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