

Pensions and savings in a monetary union: an analysis of capital flows

Alain Jousten, University of Liège, CEPR and IZA
Florence Legros, CEPII

Comments welcome at legros@cepii.fr or ajousten@ulg.ac.be

Summary

Most developed countries are undergoing a fundamental demographic transition due to the concurrent tendencies towards lower fertility rates and higher life expectancy. Both the reasons and the consequences of these aging processes are numerous, but they all share the feature that they profoundly impact on the way our societies are organized. While the consequences of a demographic aging process are rather well understood in a closed-economy setting, this clearly does not hold true for more general frameworks.

The previous literature on the impact of demographic aging in an open-economy framework has often focused on the case of different countries facing aging processes at different times and with different intensities (it is the case for Ingénue, 2001, Börsch-Supan et alii, 2001 and Artus, 2001).

The present paper addresses a somewhat different question, namely that of understanding the macroeconomic impact of a universal and simultaneous demographic aging process in an environment characterized by several countries forming a monetary union but conducting an independent pension policy. While we follow the previous authors by abstracting away from an analysis of financial risk, we innovate by considering a demographically homogeneous currency area. More precisely, we study a monetarily integrated zone (free financial flows, identical interest rates, one common currency) composed of two different countries that age at the same speed and at the same date. The demographic shock is modeled to be an exogenous and simultaneous drop in the size of their labor force. The fundamental difference between these two countries is due to their pension schemes: while the first country has a large pay-as-you-go (PAYG) pension scheme, the second one has a pension system that mainly relies on funded pension funds (fully funded schemes FFS), these assumptions being seen as a stylized summary of Eurozone reality.

While it is widely believed that a fully funded system provides insurance against such shocks, we show that this result does not hold true in a general setting. Indeed, for the case of a monetary union with countries similar in their demographic structure but leading different and independent policies with respect to the pensions system, we find that the FFS country will bear part of the burden of the demographic transition. This result hinges on the availability of free capital flows between countries, which allow capital owners to optimally react to variations in factor returns.

Hence, we find some rationale for empirical observations often made. First, the first generation affected by the demographic changes displays a pattern of increased savings when young that clearly helps understanding the higher observed savings rates of PAYG countries in the European Union. Second, the role of pension funds is also highlighted. While individuals do not have the facility of access to the stock market, pension funds do. Hence, they act as market regulators by investing more heavily on the stock market. Doing so, they implicitly decrease the demand for bonds emanating from the FFS country, and hence relax the implicit price pressure on the stock market. Hence, pension funds of the FFS country increase their holdings abroad, again a result empirically observed across the European Union.

Abstract

We analyze the economic impact of a simultaneous aging shock in two countries. The countries are identical in all respects except the financing scheme of their public pension system. While one relies on capitalization, the other one relies on a pay-as-you-go scheme. We show that the two countries react very differently to the demographic shock and its financial implications. Further, we find that the presence or the absence of capital mobility considerably affects the results, both in terms of the size of the burden as in terms of international capital allocation.

Résumé :

La plupart des pays développés font face à une transition démographique due à de faibles taux de fécondité et à l'accroissement des espérances de vie. Les raisons et les conséquences de ce vieillissement démographique sont nombreuses mais elles ont toutes un impact sur la manière dont nos sociétés sont organisées. Alors que les conséquences du vieillissement démographique sont relativement bien documentées dans le cadre d'une économie fermée, il n'en est pas de même dans un cadre plus général.

La littérature sur l'impact du vieillissement démographique dans une économie ouverte porte généralement sur la modélisation de différents pays ou zones n'étant pas en phase démographiquement. Le vieillissement n'y a pas lieu en même temps et l'ampleur de ce vieillissement est différent. C'est par exemple le cas pour Ingénue, 2001, Börsch-Supan et alii, 2001, ou Artus, 2001.

Notre étude a un objectif différent ; il s'agit d'étudier l'impact macroéconomique d'une transition démographique identique et simultanée dans une zone d'union monétaire. Dans cette zone d'union monétaire (libre circulation des capitaux, taux d'intérêt identiques, monnaie unique) que nous imaginons composée de deux pays, les politiques concernant le financement des retraites sont indépendantes et propres à chacun des pays. Comme les études citées ci-dessus nous n'étudions pas spécifiquement le risque financier mais nous nous focalisons sur une zone monétaire démographiquement homogène : les deux pays vieillissent au même rythme et en même temps. Concrètement, le choc démographique est représenté par une diminution de la taille des cohortes actives ; la différence fondamentale entre les deux pays est leur système de retraite. L'un possède un régime de retraite par répartition important alors que le second ne finance les pensions de retraite que par la capitalisation via des fonds de pension. Nous estimons que cette représentation stylisée est une approximation acceptable de la réalité de la zone Euro.

Alors que l'on pense généralement que les régimes préfinancés sont plus à même de résister aux chocs démographiques, nous montrons que ceci n'est pas vrai dans notre cadre. En effet, dans une union monétaire avec des pays qui ont une démographie identique mais des régimes de retraite différents, le pays à régime préfinancé porte une partie du fardeau démographique. Ceci vient largement de la libre circulation des capitaux qui autorise le capital à se localiser là où il est le mieux rémunéré.

En outre, nous trouvons quelques justifications théoriques aux observations empiriques. D'une part, dans les pays à système de retraite en répartition, la première génération affectée par la mutation démographique augmente son épargne lors de son activité, ce qui permet d'expliquer les taux d'épargne élevés qui y sont observés ; d'autre part, le rôle des fonds de pension comme régulateurs du marché peut être mis en évidence. Alors que les individus n'ont pas ou peu la faculté d'accéder au marché des actions, les investisseurs institutionnels l'ont. Du coup, les fonds de pension ont la possibilité de réguler le marché des actions durant la période de transition démographique. Durant cette période, en effet, il y a une pression à la hausse sur le marché des obligations due à la hausse de l'épargne individuelle dans le pays en répartition ; pression qui déprime relativement le marché des actions. Les fonds de pension ont alors la faculté d'investir internationalement, comme le montre l'évidence empirique.

Résumé court

Nous analysons ici l'impact d'un vieillissement démographique simultané dans deux pays. Ces pays diffèrent par la manière dont ils financent les pensions de retraite. Alors que l'un a un système qui repose sur la capitalisation, le système du second repose sur la répartition. Nous montrons que les deux pays réagissent différemment au choc démographique et à ses implications financières. Nous montrons également que la liberté ou l'absence de liberté de circulation des capitaux modifie considérablement les résultats, à la fois en terme d'impact du choc démographique et en terme d'allocation internationale des capitaux.

1. Introduction

Most developed countries are undergoing a fundamental demographic transition due to the concurrent tendencies towards lower fertility rates and higher life expectancy. Both the reasons and the consequences of these aging processes are numerous, but they all share the feature that they profoundly impact on the way our societies are organized. While the consequences of a demographic aging process are rather well understood in a closed-economy setting, this clearly does not hold true for more general frameworks.

The previous literature on the impact of demographic aging in an open-economy framework has often focused on the case of different countries facing aging processes at different times and with different intensities. The framework chosen is often one of perfect capital and/or labor mobility between the zones. This is the case for Ingénue (2001), for Börsch-Supan et alii (2001) and Artus (2001), where the latter author presents a theoretical treatment of the special case where the two regions have diametrically opposed demographic cycles. The literature heavily relies upon overlapping generations models in which the world is divided into demographic zones. The international capital flows are substantial between zones, inducing a huge modification in the allocation of property rights across individuals from the different regions (Ingénue, 2001). On the other hand, fluctuations in the returns to capital are limited by the capital flows towards the youngest part of the world (Börsch-Supan and alii, 2001) with the highest returns on capital.

The present paper addresses a somewhat different question, namely that of understanding the macroeconomic impact of a universal and simultaneous demographic aging process in an environment characterized by several countries forming a monetary union but conducting an independent pension policy. While we follow the previous authors by abstracting away from an analysis of financial risk, we innovate by considering a demographically homogeneous currency area. More precisely, we study a monetarily integrated zone (free financial flows, identical interest rates, one common currency) composed of two different countries that age at the same speed and at the same date. The demographic shock is modeled to be an exogenous and simultaneous drop in the size of their labor force. The fundamental difference between these two countries is due to their pension schemes: while the first country has a large pay-as-you-go (PAYG) pension scheme, the second one has a pension system that mainly relies on funded pension funds (fully funded schemes FFS). These assumptions can be seen as a stylized summary of Eurozone reality. The two broad categories of countries exist in the Eurozone, even if, in fact, most of the countries have mixed schemes with two extreme points that are France and Netherlands (table 1).

<<insert table 1>>

Our paper is structured as follows. The next section deals with the long run dynamics of both the countries, first considering the countries as closed economies, second considering them as financially integrated. The third section studies the transition towards a smaller size of the labor force; this short-term analysis focuses on the financial transfers between the countries. In particular, it provides an answer to the following puzzle: the usual economic literacy says that the PAYG countries have a smaller saving rate than the FFS countries when the economies are under-capitalized – i.e. when the interest rate is greater than the wage bill growth rate – while the empirical evidence shows the opposite. The reason for this standard result is that PAYG pension contributions crowd out explicit retirement savings. We find that during the demographic transition period, countries react differently, thus inducing international capital flows. One of the linked problems is the decrease in the demand for equities which results from the pensions funds negative cash flows. This decrease cannot be balanced by the increase in non-pensions savings which are not invested in equities. A way to deal with is to increase the funded schemes contribution rates in order to push up the demand for equities. The fourth section examines the welfare effect of such a policy.

2. The model in the long-run

2.1. Setup

We start by describing the model and presenting some well-known results. We consider a representative agent model where agents live for 2 periods of time. When young, individuals work while they are retired when old. We denote period t wage income w_t , period t individual private savings s_t , the contribution rate to the PAYG pension scheme τ , the compulsory contribution rate to the FFS μ , the replacement rate provided by this pension scheme β , the real interest rate r and the taxes paid by the young and by the old generation in period t u_t and v_t respectively.

The active population N is supposed to increase at a constant rate n :

$$N_t = (1+n)^t N_0 \quad (1)$$

The financial equilibrium condition of the PAYG pension scheme is given by

$$\mathbf{t}w_t N_t = \mathbf{I}w_{t-1} N_{t-1} \quad (2)$$

The production sector produces a single good using a Cobb-Douglas technology (parameter $a \in]0;1[$) with constant returns to scale. The output also serves as numeraire.

$$y_t = N_t^a K_t^{1-a} \quad (3)$$

where K represents total capital in the economy. Assuming a competitive environment and taking first derivatives of this production function, we can write the returns on capital and labor:

$$\begin{cases} r_t = (1-a)k_t^{-a} \\ w_t = ak_t^{1-a} \end{cases} \quad (4)$$

w denoting the real wage and $k = K/L$ representing the degree of capital intensity in the production function.

The public debt (B) dynamics is given by :

$$B_{t+1} = B_t(1+r_t) + N_t g_t - N_t u_t - N_{t-1} v_t \quad (5)$$

where $g_t = \frac{G_t}{N_t}$ represents public expenditure (G_t) per worker. Similarly expressing the public debt in per

worker terms $b_t = \frac{B_t}{N_t}$, we can rewrite equation (5) as

$$b_{t+1}(1+n) = b_t(1+r_t) + g_t - u_t - \frac{v_t}{1+n} \quad (5')$$

We define the goods and services markets equilibrium (which can also be seen as the equilibrium between the supply and demand of additional savings):

$$(s_t + \mathbf{m}w_t) - \frac{1+r_t}{1+n}(s_{t-1} + \mathbf{m}w_{t-1}) = b_{t+1} + k_{t+1} - (b_t + k_t) \frac{1+r_t}{1+n} \quad (6)$$

The left hand side of equation (6) is the present value of net additional savings that have to cover the public debt increase as well as the capital accumulation. When equation (6) holds at any time period t , we can write the savings and investment equilibrium as

$$N_t [s_t + \mathbf{m}w_t] = K_{t+1} + B_{t+1}$$

or expressed in per worker terms

$$s_t + \mathbf{m}w_t = (1+n)(k_{t+1} + b_{t+1}) \quad (6')$$

We consider two countries. Country 1 is characterized by a compulsory PAYG system with contribution rate τ , but individuals have the ability to choose private savings levels s to complement their retirement income. Country 2 has a compulsory FFS retirement system with contribution rate μ and complementary free private savings s . From (6) and (6') we can already see the motivation for a key result: the savings variable of interest in

these dynamic processes is the total savings rate in the economy ($s + \mathbf{m}$) and not the savings rate separated into the different components. If $s > 0$, a marginal change in the compulsory contribution rate \mathbf{m} to the FFS does not matter, as any increase in \mathbf{m} simply entails an equally valued drop in s , as returns to the two types of savings are totally identical.¹

Finally, to complete the model, we define the utility function as an additively separable log-function with time preference rate \mathbf{r} :

$$U_t = \ln(C_t^y) + \frac{1}{1 + \mathbf{r}} \ln(C_t^o) \quad (7)$$

The representative agents in the economy maximizes the objective function (7) subject to the following resource constraint:

$$\begin{aligned} C_t^y + s_t &= w_t(1 - \mathbf{m} - \mathbf{t}) - u_t \\ C_{t+1}^o &= \mathbf{l}w_t - v_{t+1} + (s_t + \mathbf{m}w_t)(1 + r_t) \end{aligned} \quad (8)$$

where the variable C represents consumption levels, and the superscripts y and o denote the values taken when young and old respectively. Individuals use the currently observed interest rate as their benchmark in the process of determining the amount of savings.²

Taking first-order conditions, we find the following optimality condition with respect to the savings variable:

$$(s_t + \mathbf{m}w_t)(2 + \mathbf{r}) = (1 - \mathbf{t})w_t - \frac{(1 + \mathbf{r})(1 + n)}{1 + r_t} \mathbf{t}w_{t+1} - u_t + v_{t+1} \frac{1 + \mathbf{r}}{1 + r_t} \quad (9)$$

2.2. The long-run

In a steady state characterized by $k_t = k_{t+1} = k$, and thus $w_t = w_{t+1} = w$, condition (9) allows us to make a variety of *ceteris paribus* analysis. First, consider the case of an economy characterized by a lack of capital ($r_t > n$), that occurs as soon as

$$(1-a) k^{-a} > n \quad (10)$$

When changing the parameters of the FFS, for example by decreasing \mathbf{m} the resulting change in the private free savings rate exactly compensate for the lower compulsory savings thus leaving national savings unchanged. Now considering an increase in the PAYG contribution rate τ , we notice that free savings drop. As PAYG contributions do not contribute to national savings, the net effect is a drop in the national savings rate. Notice however that the drop in free savings after an increase in τ is smaller than the corresponding drop in free savings after an increase in \mathbf{m} as the multiplier on the PAYG term is smaller than the one on the FFS term:

$$1 + (1 + \mathbf{r}) \cdot \frac{1 + n}{1 + r} < 2 + \mathbf{r}$$

Equation (9) also tells us that a PAYG country wishing to maintain a constant savings rate after an increase in τ can do so under the condition that the tax on the active population u_t decreases, or alternatively that the tax on the old v_t increases. Further, (9) tells us that it is possible to determine a critical value \mathbf{m}^* such that for all $\mathbf{m} > \mathbf{m}^*$ the corresponding value of s is negative. Note that this is important particularly for studying the optimum in a liquidity-constrained environment.

Further, we can write the public debt process (5') as

$$b(r - n) = u + \frac{v}{1 + n} - g \quad (11)$$

¹ The result does not hold true in a more general setup with different risks attached to different savings instruments, or with a heterogeneous population instead of a representative agent.

² This assumption is without any consequence on the steady-state results of the model.

with $b_{t+1} = b_t = b$. If there is lack of capital in the economy $r > n$, in order to maintain a stable level of b , there is a need for a public sector budget surplus to compensate the faster growth of interest rates as compared with the rate of growth of the population that has to bear the burden of the debt.

In the steady state, equation (6') tells us that *ceterus paribus* low savings lead to low capital intensities and hence reinforce each other. Further, using (6') and (9), we derive

$$s + \mathbf{m}v = \left(w - \mathbf{t}w \left(1 + \frac{(1+r)(1+n)}{1+r} \right) - u + v \frac{1+r}{1+r} \right) \frac{1}{2+r} = (1+n)(b+k) \quad (12)$$

which tells us that an increase in public debt allows a decrease of taxes on the young, a decrease of PAYG contributions τ , ... Note that r is also endogenously determined. Equations (9) and (12) tell us that all other things being equal, high values of τ have a depressing effect on both k and $s + \mathbf{m}v$.

Summarizing, we find that *ceterus paribus* a FFS country has a higher savings rate (private and national) than a PAYG country. For an under-capitalized country, a decrease in τ increases savings, and hence helps limit the problem of insufficient capital. At the opposite, if $r < n$, an increase in PAYG is helpful to limit the over-accumulation of capital in the economy and also the problem of ever increasing public deficits.

2.3. Monetary union

We now move away from a closed economy setting to one where these two countries are linked together through a monetary union, while keeping social and fiscal policy autonomy. Monetary integration means that the interest rates in both countries are identical as a result of the free movement of capital between the countries. We suppose without any loss of generality that the countries use the same currency.

Rewriting the budget constraints for both the PAYG (country subscripted 1) and the FFS country (country subscripted 2), we find:

$$\begin{cases} C_{1,t}^y = w_{1,t}(1 - \mathbf{t}_1) - s_{1,t} - u_{1,t} \\ C_{1,t}^o = \mathbf{I}_1 w_{1,t} + s_{1,t}(1 + r_t) - v_{1,t} \end{cases}$$

$$\begin{cases} C_{2,t}^y = w_{2,t}(1 - \mathbf{m}_2) - s_{2,t} - u_{2,t} \\ C_{2,t}^o = (s_{2,t} + \mathbf{m}_2 w_{2,t})(1 + r_t) - v_{2,t} \end{cases}$$

We suppose that the population growth rates are identical between countries and that the two countries thus face exactly the same demographic prospects.

$$N_t = N_{2,t} = N_{1,t} = (1+n)^t N_0$$

The equilibrium condition of the PAYG scheme in country 1 reduces to

$$\mathbf{t}_1 w_{1,t} N_t = \mathbf{I}_1 w_{1,t-1} N_{t-1}$$

Each country has its own independent fiscal policy

$$\begin{cases} b_{1,t+1}(1+n) = b_{1,t}(1+r_t) + g_{1,t} - u_{1,t} - \frac{v_{1,t}}{1+n} \\ b_{2,t+1}(1+n) = b_{2,t}(1+r_t) + g_{2,t} - u_{2,t} - \frac{v_{2,t}}{1+n} \end{cases}$$

The equilibrium of the monetary union can be summarized by two identities. First, the interest rate identity (4a') which results from perfect capital mobility implies $K_1 = K_2$

$$\left(\frac{K_{1,t}}{N_t} \right)^{-a} = \left(\frac{K_{2,t}}{N_t} \right)^{-a} = r_t \quad (4a')$$

Using our assumption of identical time paths of the demographic variables, we can also derive a result of no mobility of labor between countries as the marginal products are equalized.

$$w_{1,t} = ak_{1,t}^{1-a} = ak_{2,t}^{1-a} = w_{2,t} \equiv w_t \quad (4'b)$$

Second, the condition (6'') on worldwide savings and investment summarizes goods market equilibrium.

$$s_{1,t} + s_{2,t} + \mathbf{m}_2 w_{2,t} = (1+n)(b_{1,t+1} + b_{2,t+1} + k_{1,t+1} + k_{2,t+1}) \quad (6'')$$

On a per country level, the savings and investment balances are

$$\begin{aligned} s_{1,t} &= (1+n)(b_{1,t+1} + k_{1,t+1}) + f_{1,t+1} & \text{and} & \\ s_{2,t} + \mathbf{m}_2 w_{2,t} &= (1+n)(b_{2,t+1} + k_{2,t+1}) + f_{2,t+1} \end{aligned} \quad (13)$$

where f is the monetary value of the external financial balance with $f_1 + f_2 = 0$.

Assuming identical preferences across the two countries,

$$\begin{aligned} U_{1,t} &= \ln(C_{1,t}^y) + \frac{1}{1+r} \cdot \ln(C_{1,t+1}^o) & \text{and} & \\ U_{2,t} &= \ln(C_{2,t}^y) + \frac{1}{1+r} \cdot \ln(C_{2,t+1}^o) \end{aligned} \quad (7')$$

Taking derivatives, it is straightforward to find an expression for national savings per worker in the two countries.

$$\begin{aligned} s_{1,t}(2+r) &= (1-t_1)w_{1,t} - \frac{(1+r)(1+n)}{(1+r_t)} t_1 w_{1,t+1} - u_{1,t} + v_{1,t+1} \frac{1+r}{1+r_t} & \text{and} & \\ (s_{2,t} + \mathbf{m}_2 w_{2,t})(2+r) &= w_{2,t} - u_{2,t} + v_{2,t+1} \frac{1+r}{1+r_t} \end{aligned} \quad (9')$$

As in the closed economy setting, we find that the presence of a PAYG system in the monetary union lowers savings. In a steady state situation, using (4'b), (6'') and (9') we derive (12') and compare it with its closed economy counterpart (13). The expression indicates that there are international financial flows in order to equalize per capita capital between the economies, while at the same time allowing for different per capita savings levels such as defined by (9').

$$\begin{aligned} s_1 + s_2 + \mathbf{m}_2 w_2 &= \left(2w - t_1 \cdot w \left(1 + \frac{(1+r)(1+n)}{1+r} \right) - u_1 - u_2 + (v_1 + v_2) \cdot \frac{1+r}{1+r} \right) \cdot \frac{1}{2+r} \\ &= (1+n)(b_1 + b_2 + 2k) \end{aligned} \quad (12')$$

For example, take the case of a situation of dynamic efficiency ($r > n$) and a world characterized by an absence of international capital flows. When country 1 decides to increase t , *ceteris paribus*, savings clearly drop as the size of the virtual savings in the PAYG system increases, hence crowding out other forms of real savings. As a result, the external balance of country 2 will increase as country 1 now increases its borrowing abroad so as to permit equalization of wages and interest rates implied by the perfect international factor mobility. This pushes up the external financial balance. Hence, intuitively, in the PAYG country, the level of per capita capital would be higher in the monetary union than in the closed economy setting. This in turn implies a lower interest rate in country 1 under the regime of monetary union. The inverse obviously holds true for the FFS country. Surprisingly however, the general conjecture that PAYG countries should be characterized by lower private savings rates than FFS countries does not seem to hold true in the real world. Often cited examples are some countries in Western Europe such as France and Belgium where private savings rates are noticeably higher than in some countries that do rely much more heavily on FFS.

The above *ceteris paribus* analysis is obviously only a very partial one, as the other tax variables of the model also impact on the savings level. Further, as the FFS country does not have the intergenerational link through the pension system, it might want to use the public debt process to achieve a certain degree of intergenerational redistribution. Such a different debt policy is not without consequence on the international financial markets, as can easily be read off equations (7).

3. Short-run analyses of a demographic transition

One explanation for the lower private savings rates in FFS countries as compared with PAYG ones runs as follows. Consider a demographic shock where the size of the active population is subject to a one-time drop in the population from a level of N in period $T-1$ to a level of $M < N$ for all periods starting from T , and this in both countries. We assume that all variables are at their steady-state level in both countries. We further assume that the PAYG country keeps a constant contribution rate τ . Further, all fiscal variables are also supposed to remain unchanged for generation $T-1$. This is an assumption of "grandfathering" of implicit entitlements. We successively analyze the impact on savings, wages and interest rates in a closed economy and in a monetarily integrated one.

The generation born in $T-1$ is the first one to learn about the aging process and it is also the first one to be confronted with the consequences of the demographic changes. While young, members of this birth cohort still earn a wage which is equal to the steady state level ($w_{T-1} = w$) as the capital stock entering the production process was decided upon by generation $T-2$ that was still unaware of the imminent shock. In the second period of their life, individuals born in $T-1$ face the full consequences of the shock as the PAYG replacement rate is affected by the demographic changes through the balanced-budget condition of the PAYG system:

$$M \cdot \tau \cdot w_T = N \mathbf{I}_{T-1} \cdot w_{T-1}$$

Further, factor returns in period T are also affected by the demographic changes as

$$\begin{cases} w_T = a \left(\frac{K_T}{M} \right)^{1-a} \\ r_T = (1-a) \left(\frac{K_T}{M} \right)^{-a} \end{cases} \quad (4'')$$

Adapting the individual's budget constraint accordingly, we derive the closed economy savings rate of individuals belonging to generation $T-1$ as given by expression (9'').

$$(2 + \mathbf{r})(s_{T-1} + \mathbf{m}w) = w(1 - \tau) - w \frac{1 + \mathbf{r}}{1 + r_{T-1}} \mathbf{I}_{T-1} - u_{T-1} + \frac{1 + \mathbf{r}}{1 + r_{T-1}} v_T \quad (9'')$$

Equation (9'') indicates that the FFS country has an unchanged savings rate of people born in $T-1$ in the presence of the demographic shock. The reason is that none of the parameters affecting these individuals has changed, as - for example - the contribution rate τ as well as the replacement rates \mathbf{I} and \mathbf{I}_{T-1} are all equal to zero. We find a different and less trivial result for the PAYG country when casually comparing with equation (9). To determine the impact of the demographic shock on generation $T-1$, we first have to consider the value of \mathbf{I}_{T-1} . While the direct effect of the reduction in the active population obviously has a negative impact on \mathbf{I}_{T-1} , the indirect effect through the increased level of w_T works in the opposite direction. It is straightforward to show that for our production function, the direct effect dominates at any given level of savings, and hence that \mathbf{I}_{T-1} is smaller than the steady state value of \mathbf{I} . Hence, we establish that individuals born in the PAYG country in $T-1$ react to the shock by increasing their savings levels, and this in anticipation of a reduction of pensions replacement rates and levels.

Further we assume that the governments are forced to keep the debt level B_T at a given level due to Maastricht-style criteria, the national savings identity

$$K_T + B_T = \sum_{T-1}^{N \cdot (s_{T-1} + \mathbf{m}w_{T-1})}$$

implies that an increase in national savings in period $T-1$ induces an increase in the level of capital in the PAYG country in period T , while not having any effect in the FFS country.

For the generation born in T, the problem is more complex in both countries. No matter which pension system they opted for, their choices are now affected by the change in the size of the active population. The individuals' budget constraints remain unchanged in their form, while the PAYG system's balanced budget constraint reduces to $\mathbf{t} \cdot w_{T+1} = \mathbf{I}_T w_T$.

The individual saving rate is given by:

$$(s_T + \mathbf{m}w_T)(2 + \mathbf{r}) = w_T(1 - \mathbf{t}) - \frac{1 + \mathbf{r}}{1 + r_T} w_T \mathbf{I}_T - u_T + v_{T+1} \frac{1 + \mathbf{r}}{1 + r_T} \quad (9''')$$

We first consider the case of the FFS country in the closed economy. As savings levels of the generation born in T-1 were not affected by the exogenous shock and given our assumption of constancy of the debt level B_T , higher per capita capital levels induce a higher wage $w_T > w$ and a lower interest rates $r_T < r$. Their direct impact on savings of individuals born in T is an increase of total savings per person. Notice however that the equation determining public debt dynamics $B_{T+1} = B_T(1 + r_T) + Mg_T - Mu_T - Nv_T$ requires an endogenous change in taxes (u_T or v_T) while keeping expenditures (g_T) constant (or vice versa). Depending on the parameter values, the endogenous tax changes can either reinforce or offset the previously discussed effects on private individual savings. Most probably, u has to be increased (with $r_T < r$ and $N-M > r_T - r$). This offsets the previously discussed effect and decreases the individual saving rate. This can be explained by the smaller and better-off working population (with higher wages) that is able to pay the higher taxes.

Now looking at total national savings and their utilization in period T+1, it is important to remember that the number of savers also affects the national savings level.

$$K_{T+1} + B_{T+1} = \sum_{M \cdot (s_T + \mathbf{m}w_T)} \quad , \text{ in T+1.}$$

Except maybe for extreme scenarios with respect to the values of the tax variables in the savings equation (9'''), it is easy to show that total savings in the FFS country decrease as a consequence of the drop in the active population. This in turn implies that the capital stock K_{T+1} available in period T+1 decreases, thus pushing up returns to saving in the future period.

In the PAYG country, the situation is slightly more complicated as the increase in k_T is more pronounced than in the FFS country because of the endogenous change in the total savings of the generation born in T-1. Hence, the effect on both wages and return to capital are stronger, and thus too the reaction due to the government budget constraint. However, there is an additional effect on private individual savings due to the PAYG pension system as summarized by the second term on the right hand side of equation (9'''). This latter effect is a priori difficult to sign, thus making the effect on individual savings highly uncertain. However, the effect on total savings is again heavily affected by the decline in the active population.

It is perhaps possible to have an idea about the change in the individual saving rate when $r_{T-1} \gg r_T$, $\mathbf{r} \gg r_T$ and u_T is increased. We further assume that the value of second period taxation v_{T+1} is smaller than $\mathbf{I}_T w$.³ In this case, the combined effect of the second and fourth members of (9''') would have a negative effect on the saving rate and most probably these effects would offset the positive effect of w_T . This negative effect can be explained by the fact that – if the preference for the present is very high – the depressive effect of the contribution payment on the wage in T is strengthened by the high preference for present. In this case, total savings in T and per capita capital in T+1 are declining together with the size of the active population (the population remains stable between T and T+1). The resulting decline of wages from w_T to w_{T+1} is dominated by the strongly depressed interest rate r_T thus giving the expected effect.

In the opposite scenario where the preference rate for the present is very weak as compared to the interest rate and the tax rate u_T is close to constant, we see that the positive effect on individual savings rates is dominant, and hence individual savings might increase slightly. In this latter case, total national savings will likely still be shrinking due to the massive decline in the active population.

Next, we consider the effects of a demographic shock in the framework of a monetary union. We qualitatively discuss the features of a shock when the countries are already monetarily integrated in the steady-state situation

³This constraint is rather plausible in the real world given the close institutional (though not budgetary) links between the social insurance board and the government.

in T-1, which clearly is the most relevant scenario.⁴ When subjecting these economies to the demographic shock, the external credit of the PAYG country in period T-1 increases as it has a relative abundance of capital. Given the strong similarity in the economic structure of the countries, there is an incentive for savers of the PAYG country to invest their money in the FFS country until there is an equalization of returns to capital all across the union. The common interest rate that is paid all across the union is higher than that of the PAYG country in the closed economy setting, while the contrary holds true for the FFS country. Our model thus describes one possible theoretical model supporting the empirical evidence that PAYG countries are characterized by higher savings rates. Saving more is an optimal strategy for the first generation affected by a demographic transition in the PAYG country, and international capital mobility allows the capital resources to find their most profitable employment. Notice that rather complex general equilibrium and distributional issues might arise between countries and between generations. For example, as a consequence of the increased capital flows out of the PAYG country, period T workers in that country will find their wages reduced with respect to those paid in a closed economy setting. The opposite holds true for workers in the FFS country. Hence, people born in the PAYG country in period T-1 will react because of a lower expectation of retirement benefits (lower replacement rate) by readjusting their savings levels upwards because of expectations of lower retirement income. Thus the international capital reallocation is not without distributional consequences on an international scale.

Our discussion also shows, that the largest benefit of a monetary union is to allow a smooth transition from one demographic regime to another one by allowing capital to reallocate itself internationally according to its highest marginal productivity. This benefit is most obvious in period T-1, where the largest difference between countries exist. Interpreting the current situation in Europe as being the start of the transition period, our model would tend to indicate that the benefits of the monetary union do not necessarily increase over time.

The following tables summarize our findings.

PAYG country; closed economy

Period	Macroeconomic savings	Population	Per capita capital (from the previous period savings) : k	Interest rate, wages
T-1	↑	N		
T	↓	$M < N$	↑	$r \downarrow ; w \uparrow$
T+1		$M = M$	↓	$r \uparrow ; w \downarrow$

FFS country; closed economy

Period	Macroeconomic savings	Population	Per capita capital (from the previous period savings) : k	Interest rate, wages
T-1	=	N		
T	↓	$M < N$	↑	$r \downarrow ; w \uparrow$
T+1		$M = M$	↓	$r \uparrow ; w \downarrow$

Monetary union

Period	Capital flows	Comparison with closed economy	
		Interest rate r	Wages w
T-1			
T	PAYG → FFS	increase in the PAYG country	decrease in the PAYG country
T+1	-	Id	id

Of course, our results of large financial flows in period T-1 crucially hinge on the assumption that the PAYG country does not pursue an anti-cyclical fiscal policy by means of which it would increase the public debt sufficiently in period T-1 so as to absorb the additional private savings hence leaving the capital stock largely

⁴ A second possible scenario consists of a monetary integration that occurs simultaneously with demographic shock. Some of the discussed results also apply to this different setting.

unchanged.⁵ This finding that an adequate fiscal policy can undo the effects of a change in the PAYG variables is not new by any standard. However, while our assumption of a self-imposed budgetary discipline might not be an optimal economic policy, it still approximates the example of the European Union rather strongly.

4. Impact on financial markets: asset allocation

Next we consider the question of which impact a demographic shock is likely to have on financial markets in a monetary union made up of a PAYG and a FFS country. We focus on a setting where there only exist two types of financial instruments, one-period government bonds paying a return r_t with certainty, and stocks paying a random return R_t . Uncertainty on the stock return stems from an uncertainty on the productivity of the capital in the economy.⁶ We summarize this uncertainty by defining the production function as

$$y_t = ak_t^{1-a} + k_t \mathbf{e}$$

where $\mathbf{e}_t \approx \mathfrak{N}(0, \mathbf{S}^2)$

We assume that individuals' free savings are under the form of the safe asset, while compulsory savings in pension funds are entirely invested under the form of stocks. The assumption though clearly extreme, is a rather good approximation of reality. In France, 65% of private life-insurance savings were done under the form of bonds, while only 25% took the form of stocks. Further, a lot of the private saving is done through low risk and low return savings accounts. Pension funds on the other hand heavily rely on stock market investments, given their generally much longer time horizon as well as their much better access to tools and instruments allowing a good diversification of risks in their portfolio. Further, table 2 shows the allocation of pension investments in the Netherlands that clearly displays a bias towards investments in equity at the ages considered.

Consumers in the PAYG country 1 face the following budget constraints in their individual optimization

$$\begin{cases} C_{1,t}^y + s_{1,t} = w_{1,t}(1 - \mathbf{t}) - u_1 \\ C_{1,t+1}^o = \mathbf{I}_t w_{1,t} - v_{1,t+1} + s_{1,t}(1 + r_t) \end{cases}$$

while their counterparts in the FFS country use the current period distribution of returns in their optimization and hence respect

$$\begin{cases} C_{2,t}^y + s_{2,t} = w_{2,t}(1 - \mathbf{m}) - u_{2,t} \\ C_{2,t+1}^o = -v_{2,t+1} + s_{2,t}(1 + r_t) + \mathbf{m}w_{2,t}(1 + R_t) \end{cases}$$

The only form of uncertainty affecting behavior in the two economies is the random nature of the return R_{t+1} on capital. The PAYG system in country 1 faces the usual balanced budget constraint $\mathbf{I}_t = \mathbf{t} \cdot \frac{w_{t+1} N_{t+1}}{w_t N_t}$.

Integrating an expected utility term in the utility function, we have $U_i = \ln(C_{i,t}^y) + \frac{1}{1 + \mathbf{r}} E \ln(C_{i,t+1}^o)$.

Applying a Taylor expansion and integrating the budget constraints, we write the objective functions as

$$U_1 \approx \ln[w_t(1 - \mathbf{t}) - u_{1,t} - s_{1,t}] + \frac{1}{1 + \mathbf{r}} \ln[s_{1,t}(1 + r_t) - v_{1,t} + \mathbf{I}_t w_t] \quad (16)$$

$$U_2 \approx \ln[w_t(1 - \mathbf{m}) - u_{2,t} - s_{2,t}] + \frac{1}{1 + \mathbf{r}} \ln[s_{2,t}(1 + r_t) - v_{2,t} + \mathbf{m}w_t(1 + \bar{R}_t)]$$

$$- \frac{1}{1 + \mathbf{r}} \frac{(\mathbf{m}w_t)^2}{\bar{C}_{2,t+1}^o} \frac{1}{2} E(R_t - \bar{R}_t)^2 \quad (17)$$

⁵ The same holds true for a so-called buffer fund, i.e. a partial element of capitalization in a PAYG pension system.

⁶ This production function implies a complete insurance of workers against income fluctuations due to productivity shocks.

where variables with an upper bar represent the mean values of the given variable. Notice that as before, wages and interest rates are equalized between countries because of the perfect international capital mobility. Hence, we have a single value of r and a single value of w per period.

Rearranging the first order condition of (16) with respect to the decision variable $s_{1,t}$, we find an optimality condition

$$C_t^y = \frac{1+r}{1+r_t} \bar{C}_{t+1}^o \quad (18)$$

that determines a savings relation similar to (9').

We conclude that the uncertainty of capital productivity has no effect on wages, replacement ratios and thus consumption and savings. However, a drop of the active population from a level N in period $T-1$ to a level $M(<N)$ in period T still has the same positive effect on savings in $T-1$.

In the FFS country, the maximization of (17) with respect to the private savings variables $s_{2,t}$ gives us the following first order conditions

$$\frac{\partial U_2}{\partial s_2} = -\frac{1}{C_t^y} + \frac{1+r_{1,t}}{1+r} \frac{1}{\bar{C}_{t+1}^o} + \frac{1+r_t}{1+r} \frac{D_2}{\bar{C}_{t+1}^o} = 0 \quad (19)$$

where $D_2 = \frac{(\mathbf{m}w_{2,t})^2}{\bar{C}_{t+1}^o} E(R_t - \bar{R}_t)^2 = \frac{(\mathbf{m}w_{2,t})^2}{\bar{C}_{t+1}^o} \mathbf{S}^2$, where \mathbf{S}^2 represents the effect of uncertainty.

Condition (19) describes optimal behavior with respect to individual savings in the FFS country subject to a given level of \mathbf{u} . Notice that the projected consumption pathway $C_t^y = \frac{1+r}{1+r_t} \frac{\bar{C}_{t+1}^o}{1+D_2}$ from young to old looks

similar to the one we observe for the PAYG country. The only difference is the additional term in D_2 due to the impact of uncertainty on the allocation of consumption between periods. While individuals in the PAYG country are shielded against uncertainty, the individuals in the FFS country face uncertainty through the risk involved in the institutional investments of the public pension fund.

The reaction in period $T-1$ of individuals in the FFS country to a drop of the active population from a level N in period $T-1$ to a level $M(<N)$ in period T is less trivial than the one in the PAYG country. For an unchanged level of the contribution rate \mathbf{m} to the compulsory fully funded system, individuals in the FFS country still have an unchanged savings behavior. However, because all national governments are forced to keep the total of bonds outstanding unchanged under a Maastricht-style criteria ($B_{j,t} = B_j$, " j,t "), the combined demand for these bonds originating from individuals in the PAYG and the FFS has changed with respect to the previous period. Further, at this stage, the total amount of capital available to companies stays constant, as the savings of the pension funds have not changed.

This situation does obviously not represent an equilibrium as there is an implicit pressure towards higher relative prices of bonds with respect to stocks. However, such an outright relative loss of value of the institutional pension assets is not in the interest of the fund, as it would produce large fluctuations in the pension income of the elderly in country 2. As a stylization of this asset market pressure, we assume that the pension fund of country 2 rationally intervenes as a sort of regulator or social planner on the financial markets to keep the interest rate r_T constant or expressed a little differently to restore the quantity equilibrium in the bond market. The pension fund operates through adjustments of the compulsory contribution level \mathbf{m}_1 such that the demand for bonds in country 2 decreases in line with the variation in demand for fixed-income assets emanating from individuals in country 1, whose pension system cannot react. Mathematically, this is equivalent to having the pension fund set the decision variable \mathbf{m}_1 such that the level of private savings as implicitly defined by equation (19) exactly corresponds to the value that insures the respect of the bond market clearing condition

$$s_{2,T-1} = \frac{1}{N_{T-1}} (B_1 + B_2) - N_{T-1} s_{1,T-1} \quad (20)$$

Intuitively, the pension fund will thus adjust its holdings of the risky asset if individuals in country 1 increase their savings volume as a result of a demographic shrinking process. The relative risk exposure of individuals

belonging to the generation T-1 in country 2 will thus increase. The clear winners in the process are individuals in country 1 which take advantage of the higher capital stock at home, and hence of higher wages for their workers. The results are less clear-cut for individuals of the FFS country. Their additional risk intake will to some degree be compensated by the higher returns on risky rather than on riskless assets. However, they will suffer a negative income effect as the increase in total productive capital and the decrease in the size of the labor force both lower the returns R_T . However, the decrease in the relative price of stocks is smaller than the one the individuals would have had to face if the fund had not adjusted its contribution rate and increases its asset holdings at home and abroad. The intuitive reason is that by readjusting, the fund is able to bridge some of the pricing pressure by using the additional savings to generate additional value added in the production sector.

An interesting and important remark relates to the Maastricht-style criterion that the economies have to face. The baseline assumption we have taken relies heavily on the situation the member states of the European Union currently face. Equation (20) shows that the dynamics of the savings and asset allocation problems would be fundamentally different if the limitation on the debt levels B_i were to be expressed in per capita terms b_i ($i=1,2$).

Our finding of increased pension fund investment in safe assets in period T-1 can obviously not be generalized to generations beyond T. Indeed, from the analyses of the previous sections, it is likely that individual savings will decrease, hence pushing down pension fund investment in stocks. Therefore, the projections of tables 2 and 3 are not inconsistent with our model.

4. Conclusion

Our paper set out to study the impact of a demographic shock in a monetary union. While it is widely believed that a fully funded system provides insurance against such shocks, we show that this result does not hold true in a general setting. Indeed, for the case of a monetary union with countries similar in their demographic structure but leading different and independent policies with respect to the pensions system, we find that the FFS country will bear part of the burden of the demographic transition. This result hinges on the availability of free capital flows between countries, which allow capital owners to optimally react to variations in factor returns. Our model is designed to approximate European Union reality. Hence, we find some rationale for empirical observations often made. First, the first generation affected by the demographic changes displays a pattern of increased savings when young that clearly helps understanding the higher observed savings rates of PAYG countries in the European Union. Second, the role of pension funds is also highlighted. While individuals do not have the facility of access to the stock market, pension funds do. Hence, they act as market regulators by investing more heavily on the stock market. Doing so, they implicitly decrease the demand for bond emanating from the FFS country, and hence relax the implicit price pressure on the stock market. Hence, pension funds of the FFS country increase their holdings abroad, again a result empirically observed across the European Union.

Appendix

Table 1
Main characteristics of pensions and savings for European selected countries

Countries	Pay-as-you-go			Expenditures (% of the GDP, 1998)	Pension funds liabilities (% of the GDP, 1999)	Saving rates Average rates 1990-2001
	Replacement rates (% , 1998)					
	Half the average wage	Average wage	Twice the average wage			
France	84	84	73	12.50	6.00	15.15
Germany	76	72	75	12.80	8.40	10.00
Netherlands	73	43	25	5.20	110	10.00
United Kingdom	72	50	35	4.40	81	5.00

Table 2
The Netherlands : Change in the asset allocation.

Age group	2000			2050		
	Pension assets	Asset allocation per age group		Pension assets	Asset allocation per age group	
		Equities	Non equities		Equities	Non equities
25-29	21 518	100%	0%	33 180	100%	0%
30-34	159 516	100%	0%	203 399	100%	0%
35-39	310 876	100%	0%	380 847	100%	0%
40-44	442 292	96%	4%	597 863	96%	4%
45-49	570 012	80%	20%	902 020	80%	20%
50-54	761 294	60%	40%	1 309 177	60%	40%
55-59	713 362	40%	60%	1 792 411	40%	60%
60-64	737 922	20%	80%	2 095 929	20%	80%
65-69	715 441	0%	100%	2 123 877	0%	100%
70-74	434 973	0%	100%	1 523 035	0%	100%
75-79	192 510	0%	100%	1 023 279	0%	100%
80+	0	0%	100%	741 178	0%	100%
Total	5 059 716	2 262 226	2 797 490	12 726 194	3 834 647	8 891 548
Weighted average asset allocation		44.7%	55.3%		30.1%	69.9%

Source: Merrill Lynch. The assumptions made in order to have the pension assets per age group are detailed in Jan Mantel (2000).

Table 3
Changes in Asset allocation

	U.K.		U.S.		Netherlands		Japan	
	2000	2050	2000	2050	2000	2050	2000	2050
Equities	72%	60%	62%	54%	45%	30%	40%	28%
Non equities	28%	40%	38%	46%	55%	70%	60%	72%

Source: Merrill Lynch

References

Artus P. (2001), « Quelles politiques budgétaires dans une économie vieillissante, le cas du Japon et un peu de théorie, *Épargne et retraite, Revue d'Economie Politique*, n° spécial, pp.153-176.

Artus P. (2000), « Le placement optimal de l'épargne lorsque les pays les plus avancées connaissent un cycle démographique », *Revue d'Economie Politique*, n°2 , mars-avril, pp.243-265.

Artus P., Legros F. (1999), *Le choix du système de retraite*, Economica

Blanchet D. (1990) « Retraite par capitalisation et par répartition selon le contexte démographique : quelques résultats comparatifs », *Annales d'Economie et de Statistique*, n°18, pp.63-90

Börsch-Supan A., Ludwig A., Winter J. (2001), « Ageing and international capital flows », W.P. NBER n° 8553, octobre.

Ingénue (2001) *Épargne et retraite, Revue d'Economie Politique*, n° spécial, pp.

Mantel J.(2000): "Demographics and the funded pension system, Ageing populations, Mature pension funds and negative cash flow", Merrill Lynch, 30 octobre 2000.