PC1 corrected

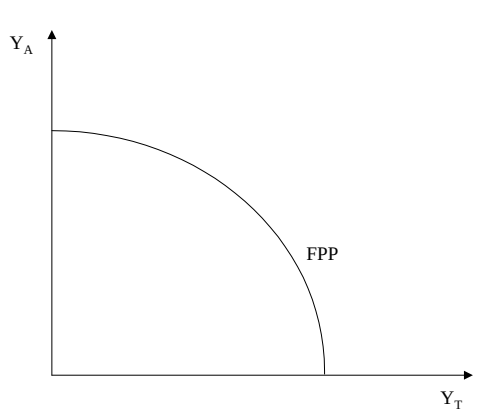
Exercise #1: Gains from trade

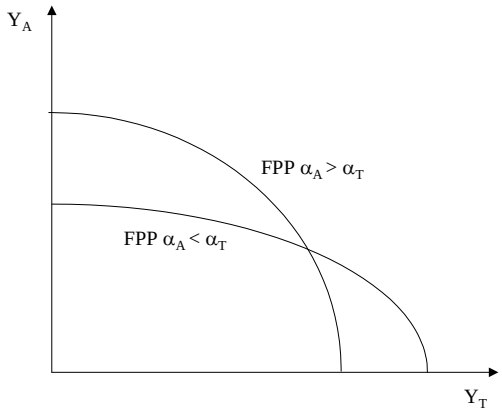
1) Autarky

a. We have: $Y_A = L_A^{\alpha_A} = (L - L_T)^{\alpha_A} = \left(L - Y_Y^{1/\alpha_T}\right)^{\alpha_A}$, which implies:

$$MRT = -\frac{\partial Y_A}{\partial Y_T} = \frac{\alpha_A}{\alpha_T}\frac{L_A^{\alpha_A - 1}}{L_T^{\alpha_T - 1}} \text{ and } \frac{\partial^2 Y_A}{\partial Y_T^2} < 0$$

PPF is the locus of all outcomes (Y_T,Y_A) such that $L_A+L_T=L$ (full employment). The tangent of this curve is the -MRT. The lower α_A/α_T, the lower the MRT for a given factor allocation, hence the flatter the curve (PPF tilted to the right).





The marginal productivity of labor is: $\frac{\partial Y_i}{\partial L_i} = \alpha_i L_i^{\alpha_i - 1}$, i=A,T. For a given volume of labor, $\alpha_A<\alpha_T$ implies that the marginal productivity is higher in textile than in aircraft: textile is more labor-intensive than aircraft.

b. In a perfectly competitive economy, the FOC of profit maximization implies that marginal cost equals marginal productivity:

$$w/P_A = \partial Y_A/\partial L_A = \alpha_A L_T^{\alpha_A - 1} \text{ and } w/P_T = \partial Y_T/\partial L_T = \alpha_T L_T^{\alpha_T - 1}$$

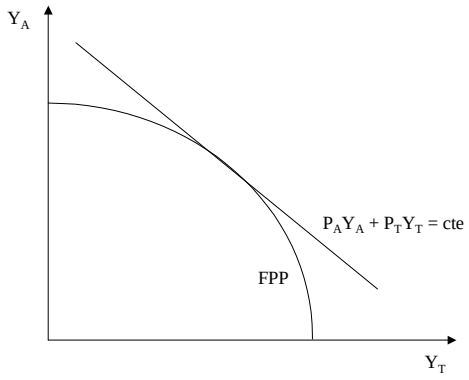
Since nominal wages are equal, the ratio of real wages yields the relative price *p*:

$$p = \frac{P_T}{P_A} = \frac{\alpha_A}{\alpha_T}\frac{L_A^{\alpha_A - 1}}{L_T^{\alpha_T - 1}} = -\frac{\partial Y_A}{\partial Y_T} = MRT$$

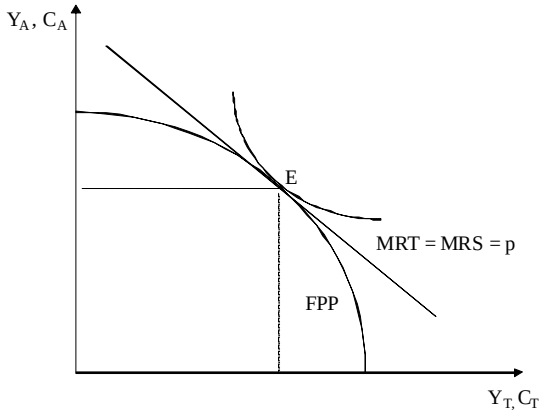
The MRT between the two goods is thus equal to their relative price. MTR is key to the comparative-advantage approach to international trade. When it comes to opening up the economy, the key issue is how much good A the economy would produce out of labor currently employed to produce good T.

If $0< \alpha_A < \alpha_T <1$, textile is more labor-intensive than aircraft and.

c. The isovalue line is defined by: $P_A Y_A + P_T Y_T$ = constant. It is thus tangent to the PPF.



d. The consumer's FOC equals the marginal rate of substitution rate (MRS) and relative price, and therefore MRS and MRT: MRS = $\frac{\partial U / \partial C_T}{\partial U / \partial C_A} = \frac{1-\beta}{\beta}\frac{C_A}{C_T} = \frac{P_T}{P_A} = p$



Moreover, each good's consumption equals its production since the economy is closed.

2) The open economy

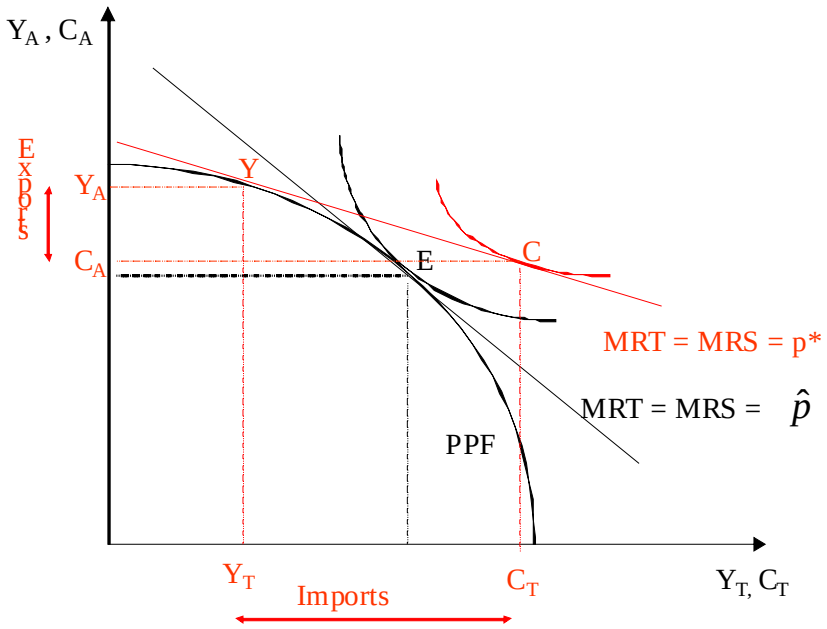
a)

Production is optimal if MTR=p*: $p^* = \frac{\alpha_A}{\alpha_T}\frac{L_A^{\alpha_A - 1}}{L_T^{\alpha_T - 1}}$

Consumption is optimal if MSR=p*: $p^* = \frac{1-\beta}{\beta}\frac{C_A}{C_T}$

Trade is balanced if: $p^*(Y_T - C_T) + (Y_A - C_A) = 0$

b) Equilibrium with trade flows is represented below in the case where $p^* < \hat{p}$. The country specializes into aircraft: its exports airplanes and imports clothes. Specialization makes it possible for producers to produce less textile and more airplanes, and for consumers to consumer more of each product.



Exercise #2: Comparative advantages

Question "1

- **North**

PPF: $4000 = 2\ y_{1N} + 4\ y_{2N}$, or else $y_{1N} = 2000 - 2\ y_{2N}$.

$p_N = p_{2N}/p_{1N} = MRT = 2$.

$c_{1N} = 0{,}5y_N = y_{1N}$ and $c_{2N} = 0{,}5y_N/2 = y_{2N}$. Full employment implies:

$4000 = 2y_{1N} + 4y_{2N} = y_N + y_N = 2y_N$, hence: $y_N = 2000$, $y_{1N} = 1000$, $y_{2N} = 500$.

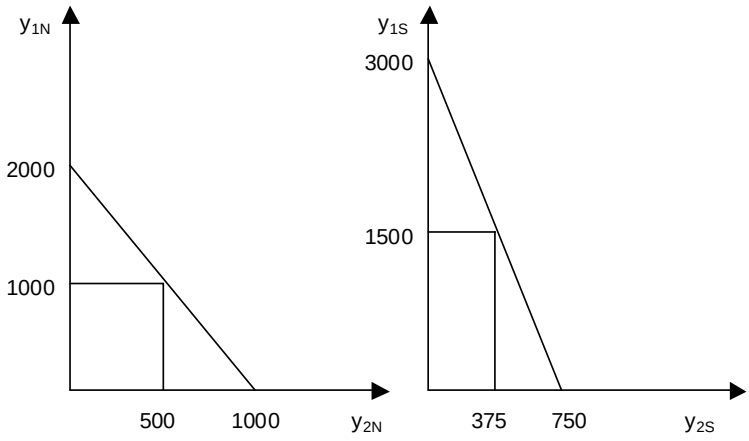
- **South**

PPF: $9000 = 3\ y_{1S} + 12\ y_{2S}$, or else $y_{1S} = 3000 - 4\ y_{2S}$.

$p_S = p_{2S}/p_{1S} = MRT = 4$.

$c_{1S} = 0{,}5y_S = y_{1S}$ and $c_{2S} = 0{,}5y_S/4 = y_{2S}$. Full employment implies:

$9000 = 3y_{1S} + 12y_{2S} = 1{,}5y_S + 1{,}5y_S = 3y_S$, hence $y_S = 3000$, $y_{1S} = 1500$, $y_{2S} = 375$.



Question #2

North: comparative advantage for good 2 (lower MTR). South: good 1.

Both countries specialize fully since returns to scale are constant.

North: $y_{1N} = 0$; $y_{2N} = 4000/4 = 1000$; $y_N = py_{2N} = 1000p$; $c_{1N} = 500p$; $c_{2N} = 500$.

South: $y_{1S} = 9000/3 = 3000$; $y_{2S} = 0$; $y_S = y_{1S} = 3000$; $c_{1S} = 1500$; $c_{2S} = 1500/p$.

Market clearing:

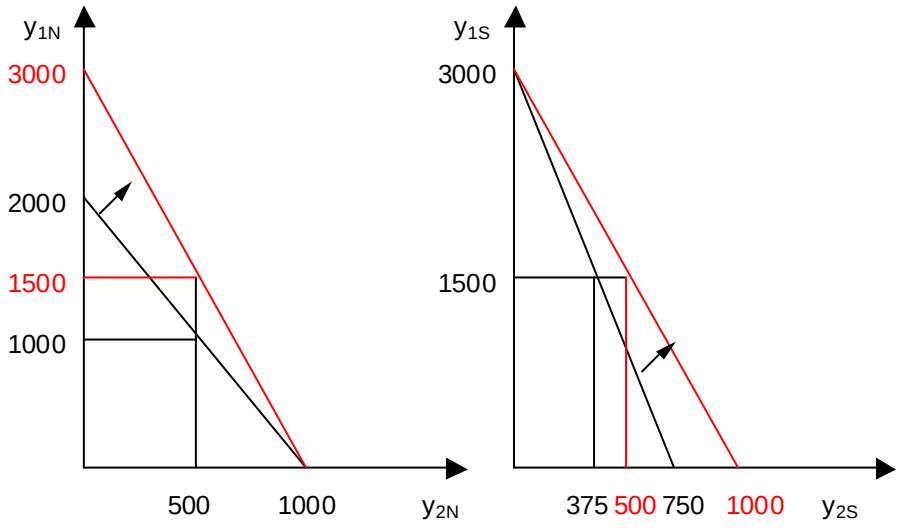
- Good 1: $3000 = 500p + 1500$, hence $p = 3$.
- One can check that the other market is balanced: $1000 = 500 + 1500/3$

Question #3

North: produces $y_{2N} = 1000$; consumes $c_{1N} = 1500$ and $c_{2N} = 500$, as compared to 1000 and 500 in autarky. Consumption therefore increases. North imports good 1 in quantity $m_{1N} = 1500$ and exports good 2 in quantity $x_{2N} = 500$. Northern trade is balanced: $m_{1N} = p\, x_{2N}$.

South: produces $y_{1S} = 3000$; consumes $c_{1S} = 1500$ and $c_{2S} = 500$, as compared to 1500 and 375 in autarky. Consumption therefore increases. South exports good 1 and imports good 2.

	North					South				
	Y_{1N}	Y_{2N}	C_{1N}	C_{2N}	X_{2N}	Y_{1S}	Y_{2S}	C_{1S}	C_{2S}	X_{2S}
Autarky	1000	500	1000	500	0	375	1500	375	1500	0
Openness	0	1000	1500	500	500	3000	0	1500	1500	1500



Gains to trade: there is more consumption in each country. It can be seen on the figure that for each utility function strictly concave in (y_{1j}, y_{2j}), a higher utility level is reached (the equilibrium moves in the North-East direction).