ECO572 - International Economics
Agnès Bénassy-Quéré – Benoît Cœuré 30 September 2009

PC2: Specialization and Factor Endowment

Consider a small open economy producing two goods (i = 1,2) out of two production factors (labor and capital). Capital and labor are mobile across industries *within* a country, but they are not mobile *internationally*.

The country is endowed with a combination (K, L) of capital and labor and technology is embedded in the following production functions (for goods 1 and 2, respectively):

$$Y_1 = F_1(K_1, L_1) = K_1^{\alpha} L_1^{1-\alpha} \qquad Y_2 = F_2(K_2, L_2) = K_2^{\beta} L_2^{1-\beta} \qquad 0< \alpha < \beta <1$$

In the following, we will denote $k_i=K_i/L_i$ the capital-labor combination used in the production of good i (i=1,2), and $k=K/L$ the capital-labor combination available in the country.

Let w and r be the unit costs of labor and of capital, $p = p_1/p_2$ the relative price of goods 1 and 2 and $\omega = w/r$ the relative compensation of production factors. Firms are perfectly competitive on the two good markets and on the markets for production factors.

1. Derive the optimal levels of k_1 and k_2 as a function of ω. Explain the relationship between k_i and ω. Show that, at the producer's equilibrium, $\kappa = k_2/k_1$ depends on α and β only and that $\kappa >1$. Plot $k_1^{-1}(\omega)$ and $k_2^{-1}(\omega)$ in the (k,ω) space. Show that it is only when ω lies within a segment [ω_{min}, ω_{max}] that the economy does not specialize in the production of a single good. In what follows, we assume that: $\omega_{\min} < \omega < \omega_{\max}$.

2. The Stolper-Samuelson theorem. Noting that production functions can be rewritten as $Y_1 = k_1^{\alpha} L_1$ and $Y_2 = k_2^{\beta} L_2$, compute the marginal rate of transformation - $dY_2/dY_{1|\omega=cst}$ as a function of ω. What is the relationship between p and ω in the competitive equilibrium?

   Infer from this relationship that *a rise of the relative price of the labor-intensive good goes together with a rise in the relative compensation of labor*.

3. The Rybczynski theorem. Noting that:

   $k_1 L_1 = K - k_2(L-L_1)$ and $k_2 L_2 = K - k_1(L-L_2)$,

   write L_1/L and L_2/L as functions of k, k_1 and k_2. Compute the impact of a variation in L for employment, and then for output in both sectors, when ω and K are held constant.

   Infer from this relationship that *when total labor force increases, production increases in the labor-intensive industry and it is reduced in the capital-intensive industry*.

4. Show that the relative production of the two goods can be written: $\dfrac{Y_1}{Y_2} = -\dfrac{k_1^{\alpha}}{k_2^{\beta}}\left(\dfrac{k-k_2}{k-k_1}\right)$.

   Suppose that consumers' utility functions are such that consumptions of the two goods have equal shares in their budgets ($C_2 = pC_1$). What would be the equilibrium price p in autarky, depending on ω?

5. Assume that on the world market the relative price of good 1 in terms of good 2 is lower than the one that would be obtained in the small country in autarky. What is the implication of opening up the economy for the relative compensation ω?