

## PC6: Balance of payments dynamics

### Exercise #1. Trade deficit and the exchange rate in a two-good model

Consider an open economy with two goods, a tradable good (T) and a non-tradable one (N). Production and consumption of each good are denoted  $Y_i$  and  $C_i$ , respectively ( $i=T,N$ ). The price of each good is denoted  $P_i$  and  $Q$  is the relative price of the non-tradable good in terms of the tradable one ( $Q = P_N/P_T$ ). Net exports (*i.e.* exports minus imports) are denoted by  $X$ . The imported good is assumed to be identical to the domestic traded good, and there are no restrictions to trade, so that the law of one price applies:  $P_T = P^*/S$ , where  $P^*$  is the (exogenous) price of the imported good, expressed in the foreign currency, and  $S$  is the nominal exchange rate (units of the foreign currency in one unit of the domestic currency).

#### The static equilibrium

1. Write the market equilibrium for each good and express domestic income  $Y$  in terms of the traded good.
2. Assume that the consumer's utility function is:  $U = \frac{1}{4} \log C_T + \frac{3}{4} \log C_N$ . Show that constrained utility maximization yields:

$$(1) \quad \frac{C_T}{C_N} = \frac{1}{3} Q$$

3. Assuming that the production of each good is such as  $Y_N = 3 Y_T$ , show that:

$$(2) \quad \frac{X}{Y_T} = 1 - Q$$

What is the impact on net exports of an increase in the relative price of the tradable good? Explain.

4. Express the  $X/Y$  ratio as a function of  $Q$ . What is the value of this ratio when  $Q = 1$ ?  $Q = 1.3$ ? Conclude on the way a current account deficit amounting to 6 percent of GDP (the US level in 2007) can be brought to balance.
5. In the medium term, prices can be assumed to be perfectly flexible. Denoting  $P = P_T^{1/4} P_N^{3/4}$  the consumer price index and assuming that the central bank keeps this price index constant, show that the elasticity of the nominal exchange rate to the relative price of the non-tradable good (in terms of the tradable one) is  $3/4$ . Calculate the nominal exchange-rate variation that is consistent with bringing the current-account deficit from 6% of GDP to zero.

#### The inter-temporal equilibrium

6. The model is now extended to two periods. The consumer's inter-temporal utility function is:

$$(3) \quad V = U^1 + \beta U^2 \quad \beta > 0$$

The subscript refers to the period (1, 2). Like in the static case, we have  $U^t = \frac{1}{4} \log C_T^t + \frac{3}{4} \log C_N^t$ .

Consumers are assumed to access freely capital markets at the exogenous world interest rate  $r$ . What is the interpretation of coefficient  $\beta$ ? Show that the inter-temporal budget constraint writes:

$$(4) \quad Q^1 C_N^1 + C_T^1 + \frac{Q^2 C_N^2 + C_T^2}{1+r} = \Omega$$

$\Omega$  is the inter-temporal income. Show that inter-temporal optimization yields:

$$(5) \quad \frac{C_N^2}{C_N^1} = \beta(1+r) \frac{Q^1}{Q^2} \quad \text{and} \quad \frac{C_T^2}{C_T^1} = \beta(1+r)$$

Explain the meaning of these two relationships. Show that, if  $Y_N$  and  $Y_T$  are constant, there is a current account deficit in period 1 if  $\beta < 1/(1+r)$ . How does the relative price of the non-tradable evolve between periods 1 and 2? Explain.

## Exercise #2: Dynamics of the net foreign asset position

Find the equation that gives the net foreign asset position of a country at the end of period  $t$ ,  $B_t$ , as a function of period  $t$ 's trade balance  $b_t$ , gross assets held by residents at the end of period  $t-1$ ,  $A_{t-1}$ , gross liabilities at the end of period  $t-1$ ,  $L_{t-1}$ , the return on gross assets ( $r^A$ ) and on gross liabilities ( $r^L$ ) and the exchange-rate variation between end of  $t-1$  and end of  $t$ ,  $S_t/S_{t-1}$  (where  $S_t$  denotes the number of foreign currency units in one domestic currency unit). Two cases will be successively studied:

- Assets are denominated in foreign currency and liabilities are denominated in domestic currency;
- Assets are denominated in domestic currency and liabilities are denominated in foreign currency.

In all cases,  $B_t$ ,  $A_t$ ,  $L_t$  and  $b_t$  are expressed in the domestic currency. Explain the impact of exchange-rate variations on the net foreign asset position. Comment on the following table.

Decomposition of net foreign asset position dynamics, 1996–2004

	Initial NFA	Change in NFA	Trade balance	Net returns			Other factors
				Total	Invest. income	Capital gains	
United States	-5.5	-16.3	-29.6	10.6	1.9	8.7	2.7
Japan	15.5	23.3	9.8	11.8	12.6	-0.7	1.7
United Kingdom	-2.4	-7.6	-17.8	6.2	8.2	-2.0	4.1
Germany	5.1	5.5	5.7	-3.3	-2.3	-1.0	3.1
Switzerland	92.5	32.9	18.0	20.5	60.3	-39.9	-5.6
Finland	-41.5	31.1	49.1	-30.1	-10.6	-19.4	12.0
Norway	5.9	59.3	67.3	15.7	-0.9	16.5	-21.2
Spain	-22.0	-25.0	-9.1	-31.1	-8.6	-22.6	15.2
Argentina	-15.0	-16.5	18.6	-18.1	-43.5	25.4	-17.0
Mexico	-56.8	13.3	1.9	-20.8	-17.4	-3.4	32.2
Korea	-7.0	-2.6	17.8	-19.9	-3.2	-16.6	-0.5
Malaysia	-55.0	47.2	96.1	-59.4	-43.0	-16.5	10.5

Note: All variables expressed as ratios to GDP. Trade balance is balance on goods, services, and transfers. Net returns are the sum of investment income and capital gains. the term 'other factors' is the sum of the growth term, capital account and errors and omissions.

Source: Ph. Lane and G.M. Milesi-Ferretti, 2007, "The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970-2004," *Journal of International Economics*, 73(2), pp. 223-250.