PC6 - Corrected

Exercise #1. Trade deficit and the exchange rate in a two-good model

Static equilibrium

1. Goods markets equilibria :

$$Y_N = C_N \quad \text{and} \quad Y_T = C_T + X$$

In order to aggregate the two goods, one of them needs be used as the numeraire. Taking the tradable good as the numeraire, we have:

$Y_T + QY_N = C_T + QC_N + X$

$Y = QY_N + Y_T$ is domestic income

2. The consumer's programme is: Max $U = 0.25 \log C_T + 0.75 \log C_N$
   s.c. $QC_N + C_T + X \leq Y$

The Lagrangian writes : $L = 0.25 Log C_T + 0.75 Log C_N + \lambda(Y - QC_N - C_T - X)$

The FOCs yield: $\lambda = \frac{1}{4C_T}$, $\frac{3}{4C_N} = \lambda Q$

Eliminating λ, we get: $\frac{C_T}{C_N} = Q/3$

The share of each good in consumption depends on their relative price: a fall in Q (fall in the relative price of the non-traded good) rises the consumption of N at the expense of T.

3. $Y_T = C_T + X = \frac{1}{3}QC_N + X = \frac{1}{3}QY_N + X$. Given that $Y_N = 3Y_T$, we get:

$$\frac{X}{Y_T} = 1 - Q$$

A rise in the relative price of the tradable good means a fall in Q, hence a rise in X/Y_T: consumption shifts away from the tradable good to the non-tradable one. A higher share of the production of the tradable good must then be exported in order for the market to clear.

4. Since $Y = Y_T + QY_N = (1+3Q)Y_T$, we have: $\frac{Y_T}{Y} = \frac{1}{1+3Q}$. With Q = 1, the tradable good represents 25% of domestic income and trade is balanced (*X*/*Y* = 0). With Q = 1.3, the tradable good represents only 20% of domestic income. Domestic consumption of the non-tradable good is now satisfied thanks to net imports:

$$\frac{X}{Y} = \frac{X}{Y_T} \times \frac{Y_T}{Y} = \frac{1-Q}{1+3Q} = 6.1\%$$

Assuming fixed production levels, the trade balance ratio only depends on the relative price of the non-tradable good. In order to bring the US current account from–6% of GDP to zero, the relative price of the non-tradable good must fall from 1.3 to 1, which amounts to a 24% depreciation. This relative price adjustment allows the volume of consumption to be redirected to the non-tradable good.

The subsequent fall in the consumption of the tradable good means that domestic producers export a larger share of their production.

5. In the medium term, prices can be considered flexible. With $P = P_T^{1/4} P_N^{3/4}$ the consumer price index, and assuming that the central bank manages to keep this price index constant, we have: $P_T^{1/4}(QP_T)^{3/4} = \overline{P}$. In logs, and using the law of one price ($P^T = P^*/S$), we get :

$$s = p^* - \overline{p} + \frac{3}{4}q$$

Both p* and $\overline{p}$ are constant. The nominal exchange-rate variation thus is: $ds=0.75\ dq$. The 30% fall in Q found in the previous question (to erase the US deficit) translates into a 22.5% nominal exchange-rate depreciation (fall in *S*). This calculation however assumes complete pass-through of nominal exchange-rate variations on the tradable-good price and a constant consumer price index.

- With incomplete pass-through, P_T will react less than proportionally to S, so a larger variation in S is needed to move Q in a given proportion;
- With CPI inflation, $\overline{p}$ increases, which reduces the necessary adjustment of S.

In this model, the exchange-rate depreciation affects the trade balance not through higher price competitiveness (there is only one traded good whose price is given in foreign currency), but through shifting domestic consumption from the tradable (imported) good to the non-tradable one. In reality, there is a combination of competitiveness and demand-shifting effects.

Inter-temporal equilibrium

6. Q^i is the relative price of the non-tradable good in terms of the tradable one in period i. Hence $C^i = C_T^i + Q^i C_N^i$ is total consumption of period i expressed in terms of the tradable good. The intertemporal budget constraint writes:

   $C^1 + C^2/(1+r) \leq \Omega$

   With $\Omega = Y^1 + Y^2/(1+r)$ (note that $X^1 + X^2/(1+r) = 0$ due to the inter-temporal external constraint). Hence, the consumer's programme is:

   Max $V = ¼ \log C^1_T + ¾ \log C^1_N + ¼\ \beta \log C^2_T + ¾\ \beta \log C^2_N$

   s.c. $Q^1C^1_N + C^1_T + (Q^2C^2_N + C^2_T)/(1 + r) \leq \ \Omega = Y^1 + Y^2/(1 + r)$

β is the discount factor, which is a decreasing function of preference for present. The consumer compares it to 1/(1 + r). Writing the Lagrangian and the FOCs, we get:

$$\frac{C_T^2}{C_T^1} = \beta(1+r) \qquad \frac{C_N^2}{C_N^1} = \beta(1+r)\frac{Q^1}{Q^2} \qquad \frac{C_T^1}{C_N^1} = \frac{1}{3}Q^1 \qquad \frac{C_T^2}{C_N^2} = \frac{1}{3}Q^2$$

The ratios of consumption levels in the two periods depend on the $\beta(1+r)$: for a given discount factor, a higher interest rate makes consumption shift from the first to the second period; for a given interest rate, a higher preference for present (lower *β*) makes consumption shift from the second to the first period.

The C_N^2/C_N^1 also depends on Q^1/Q^2: if the relative price of the non-tradable is higher in the first than in the second period, consumption of the non-tradable good shifts from the first to the second period.

Finally, the intra-temporal allocation of consumption depends on the relative price of the two goods in the same period.

If $\beta(1+r) < 1$, then $\frac{C_T^2}{C_T^1} = \beta(1+r) < 1$: consumption is lower in period 2 than in period 1. If the production of the tradable good is constant, this means a deficit in the first period and a surplus in the second one.

If Y_N and Y_T are constant (no growth), then $\frac{Q^2}{Q^1} = \beta(1+r)\frac{C_N^1}{C_N^2} = \beta(1+r) < 0$. The relative price of the non-tradable good falls from period 1 to period 2 in order to make the consumption of this good lower in period 1, higher in period 2 and thus counteract the high preference for present (this good cannot be imported).

Exercise #2: Dynamics of the net foreign asset position

By definition: $B_t = A_t - L_t$.

a) Assets in the foreign currency, liabilities in the domestic currency.

$$B_t = A_{t-1}\frac{S_{t-1}}{S_t}(1+r^A) - L_{t-1}(1+r^L) + b_t$$

A depreciation of the domestic currency (fall in S_t) raises the net foreign asset position through two channels : a rise in the trade balance (if the Marshall-Lerner condition is met) and re-valuation of foreign assets when expressed in the domestic currency.

This situation applies to advanced economies, which have more foreign assets than debts in foreign currencies because they can issue debts in the domestic currency.

b) Assets in the domestic currency, liabilities in the foreign currency.

$$B_t = A_{t-1}(1+r^A) - L_{t-1}\frac{S_{t-1}}{S_t}(1+r^L) + b_t$$

A depreciation of the domestic currency (fall in S_t) raises the net foreign asset position through the trade balance channel but it lowers the NFA position through the valuation channel. This is because the value of liabilities increases when expressed in the domestic currency.

This situation covers the case of some emerging and developing economies, which have more liabilities than assets in foreign currencies because they are unable to issue debts in their own currencies (« original sin »). However this situation has become less well-spread during the 2000s because (i) most emerging countries have become net creditors, and (ii) many have been able to issue debts denominated in their own currencies.

The table shows that the change in NFA between 1996 and 2004 is only partly due to cumulated trade disequilibria. Without capital returns, for instance, the US NFA position would have fallen by almost 30 percent of GDP, instead of 'only' 16 percent in reality. Capital gains (including exchange-rate variations) account for the bulk of the difference.

In Japan, the NFA increased by 23% of GDP between 1996 and 2004, but the cumulated trade balance only accounts for 10% of GDP. The difference owes to net investment income, which was possible thanks to very low interest rates in Japan (yen carry trade). The situation is opposite in some emerging countries (Argentina, Mexico, Malaysia).