PC8 - Corrected

Exercise #1

1) $i_{€} = 0.75\%$, $i_{\$} = 0.30\%$ and S = 1.50. UIP: $\left(1+i_{\$}\right)=\left(1+i_{€}\right)\frac{S^e}{S}$ = 0.3, where S^e is the expected exchange rate. Thus $S^e = S\frac{1+i_{\$}}{1+i_{€}} = 1.5\times\frac{1.003}{1.0075} = 1.4933$: the euro is expected to depreciate by 0.4% against the dollar, which compensates for higher remuneration on bonds denominated in euro.
2) $\pi_F = 1\%$ and $\pi_{US} = 2\%$,

   Thus $r_F = i_{€} - \pi_F = 0.75\% - 1\% = -0.25\%$ and $r_{US} = i_{\$} - \pi_{US} = 0.3\% - 2\% = -1.7\%$, where r_i denotes the real interest rate of country i (i=F, US). Denoting by Q the bilateral real exchange rate, we have $\left(1+r_{US}\right)=\left(1+r_F\right)\frac{Q^e}{Q}$, hence $\frac{Q^e}{Q}=\frac{1+r_{US}}{1+r_F}=\frac{0.983}{0.9975}=0.9855$. The euro is expected to depreciate by 1.45% in real terms against the dollar.

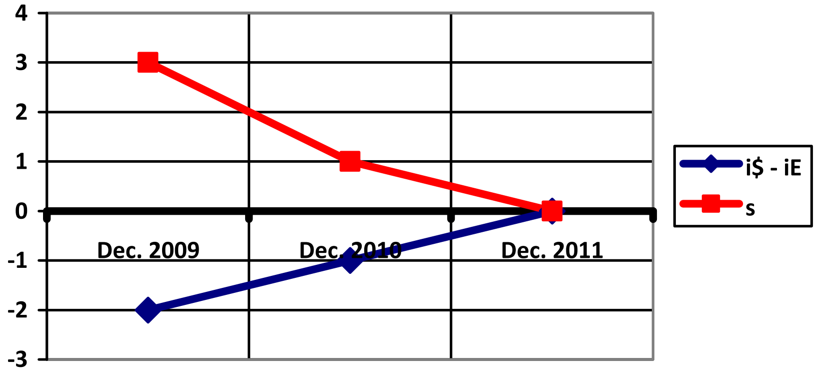
3) Dec. 2009: $i_{\$} - i_{€} = -2\% = s_{10} - s_{09}$, where $s = Log\left(\frac{S}{\bar{S}}\right)$ with $\bar{S}$ the exchange rate before the shock.

   Dec. 2010: $i_{\$} - i_{€} = -1\% = s_{11} - s_{10}$

   Summing up: $-3\% = s_{11} - s_{09}$: the euro is expected to depreciate by 3% between Dec. 2009 and Dec. 2011.

   If the terminal condition is unchanged ($S_{11} = \bar{S}$, hence $s_{11} = 0$) then: $s_{09} = -3\%$: the euro depreciates immediately by 3%. It then appreciates by 2% the first year and by 1% the second year, as shown on the graph.

Interest-rate differential and exchange rate



Exercise #2

1) In the long run, all variables are constant:

Money market $m = \bar{p} + \alpha y - \beta \bar{i}$ (1LR)

UIP $\bar{i} = i^*$ (2LR)

Prices $\bar{d} = y$ (3LR)

Aggregate demand $\bar{e} + \bar{p} = -\frac{1}{\delta}\left((1-\gamma)y + \sigma \bar{i}\right)$ (4LR)

From Equations (1LR) and (2LR), we get the long-run price level: $\bar{p} = m - \alpha y + \beta i^*$

From this expression and from Equation (4LR) we get the long-run value of the exchange rate:

$$\bar{e} = -\frac{1}{\delta}((1-\gamma)y + \sigma i^*) - \bar{p}$$

In the long run, the nominal exchange rate is 100% indexed on the price level (no nominal illusion). The real exchange rate $\bar{e} + \bar{p}$ is lower if output or the world interest rate increases. In both cases, the shock generates a lack of demand (d) compared to supply (y). The real exchange-rate depreciation allows aggregate demand to increase.

2) We aim to reduce the system to a linear dynamics in (p_t, e_t). To obtain the first equation, we substract (1LR) from (1), with $\bar{i} = i^*$:

$$p_t - \bar{p} = -\beta\left(i_t - i^*\right) = -\beta\left(e_{t+1} - e_t\right)$$

We get: $\boxed{e_{t+1} - e_t = \frac{1}{\beta}\left(\bar{p} - p_t\right)}$ $e_{t+1} > e_t \Leftrightarrow p_t < \bar{p}$

If prices are below their long-term value, the domestic currency appreciates. This is because of the money market balance (the ‘LM’ curve): money supply in volume $m - p_t$ is higher than its long-term level and i_t is therefore lower than its long-term level i^*. This is consistent with UIP only if the domestic currency is expected to appreciate.

3) The second equation results from (3) and (4). Substituting in (3) aggregate demand d_t with its value implied by (4) yields:

$$p_{t+1} - p_t = \theta\left(-(1-\gamma)y - \delta\left(e_t + p_t\right) - \sigma i_t\right)$$

In the long-run: $0 = \theta\left(-(1-\gamma)y - \delta\left(\bar{e} + \bar{p}\right) - \sigma i^*\right)$

Substracting the two:

$$p_{t+1} - p_t = -\theta\left(\delta\left(e_t - \bar{e}\right) + \left(p_t - \bar{p}\right) + \sigma\left(i_t - i^*\right)\right) = -\theta\left(\delta\left(e_t - \bar{e}\right) + \left(p_t - \bar{p}\right) - \sigma\left(e_{t+1} - e_t\right)\right)$$

We get: $p_{t+1} - p_t = \theta\delta\left(\bar{e} - e_t\right) + \theta\delta\left(\bar{p} - p_t\right) + \theta\sigma\left(e_{t+1} - e_t\right)$

Knowing that $e_{t+1} - e_t = \frac{1}{\beta}\left(\bar{p} - p_t\right)$, we finally have: $\boxed{p_{t+1} - p_t = \delta\theta\left(\bar{e} - e_t\right) + \theta(\delta + \frac{\sigma}{\beta})(\bar{p} - p_t)}$

$$p_{t+1} > p_t \Leftrightarrow e_t < \bar{e} + \left(1 + \frac{\sigma}{\delta\beta}\right)(\bar{p} - p_t)$$

If the exchange rate is too low, then aggregate demand exceeds supply and prices increase.

4) How to draw a phase diagram

First draw the two lines $\{(e,p) \mid e_{t+1} = e_t\}$ and $\{(e,p) \mid p_{t+1} = p_t\}$ in the (*e*,*p*) space:

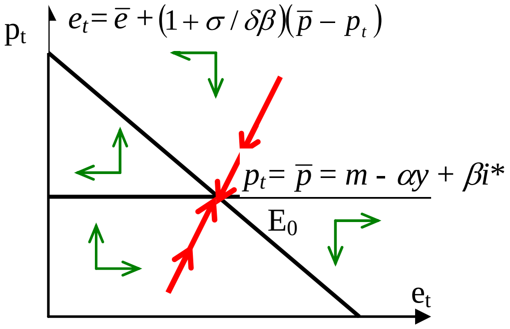
$$e_{t+1} = e_t \Leftrightarrow p_t = \overline{p} = m - \alpha y + \beta i^*$$

$$p_{t+1} = p_t \Leftrightarrow e_t = \overline{e} + \left(1 + \frac{\sigma}{\delta\beta}\right)(\overline{p} - p_t)$$

The first line is parallel to the *e*-axis and moves upright in case of an exogenous increase of money supply *m*, un exogenous decrease of output *y*, or an exogenous increase of the world interest rate *i**. The second line is decreasing with slope -1/(1+σ/δβ).

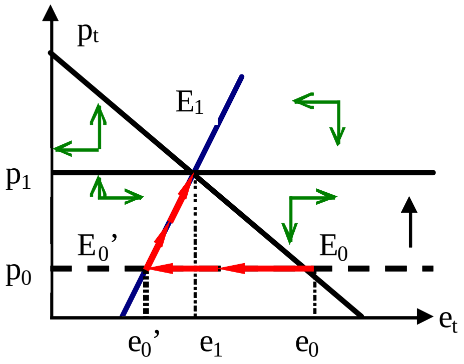
At the intersection of the two black lines lies the long-term steady-state, E_0. Outside of E_0, the price and the exchange-rate are not constant:

- below the horizontal line, the exchange-rate appreciates. We denote this dynamics with green *rightward arrows* below the line and *leftward arrows* above the line;
- left of the decreasing line, the price increases. We denote this dynamics with green *upward arrows* left of the line and *downward arrows* right of the line.



The dynamics *diverges* in the South-Eastern and the North-Western areas and *converges* in the North-Eastern and South-Western areas, as divided by the two lines. We establish in question 5 the existence of a saddle path, which is drawn in red.

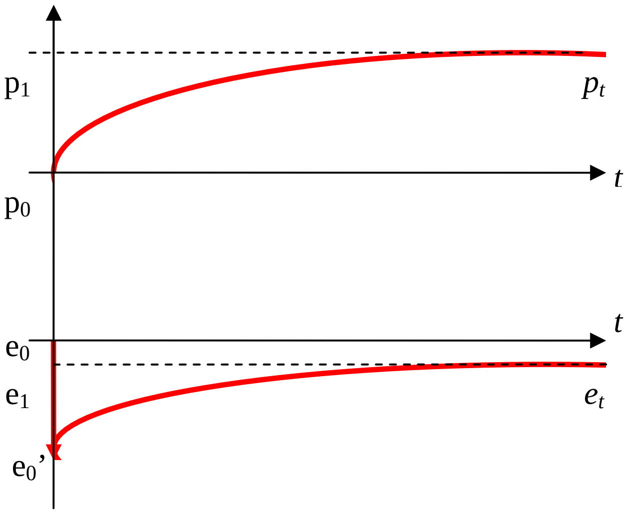
We study the impact of a permanent shock $\Delta\overline{m} > 0$. We have shown that a permanent increase in $\overline{m}$ moves the { $p = \overline{p}$ } locus upwards and does not move the { $e = \overline{e}$ } locus. The long-term equilibrium therefore moves from E_0 to E_1. The price level increases from p_0 to p_1 and the exchange rate depreciates from e_0 to e_1.



Knowing the joint dynamics of *e* and *p* depending on their position with respect to the two lines, we can now describe the full path between E_0 and E_1.

In the short run, prices are rigid but the exchange rate is fully flexible. All trajectories outside the saddle path are diverging. The economy therefore 'jumps' onto the saddle path, adjusting *e* downwards but keeping $p = p_0$ constant. From this new short-term equilibrium E_0', the economy converges along the saddle path to the new long-term equilibrium E_1. This implies a gradual exchange-rate appreciation from e_0' to e_1 and a gradual increase in prices from p_0 to p_1.

The time profile of the exchange rate is described below. The exchange rate *overshoots* with respect to its long-term level. This is because prices are rigid in the short run; if θ was large, the economy would directly move from E_0 to E_1 and there would be no overshooting.



5) Equations (5) and (6) can be written as follows:

$$X_{t+1} - X_t = A\left(X_t - \bar{X}\right) \qquad \text{with: } X_t = \begin{pmatrix} e_t \\ p_t \end{pmatrix} \text{ and } A = \begin{pmatrix} 0 & -\dfrac{1}{\beta} \\ -\theta\delta & -\theta\left(\delta + \dfrac{\sigma}{\beta}\right) \end{pmatrix}$$

The dynamics of such a linear system is described by the two eigenvalues (in French, ‘valeurs propres’) λ_1 and λ_2 of matrix A. These are the roots of the characteristic polynomial:

$P(\lambda) = det\ (\lambda\, I - A) = \lambda^2 - tr(A)\ \lambda\ + det(A)$

where $tr(A) = \lambda_1 + \lambda_2$ is the matrix trace of A and $det(A) = \lambda_1\lambda_2$ is the determinant of A.

Here, $tr(A) = -\theta\left(\delta + \dfrac{\sigma}{\beta}\right) < 0$ and $\det(A) = -\theta\delta/\beta < 0$. The discriminant of the second-order polynomial is positive: $\Delta = tr(A)^2 - 4d\acute{e}t(A) = \theta^2\left(\delta + \dfrac{\sigma}{\beta}\right)^2 + 4\dfrac{\theta\delta}{\beta} > 0$

The eigenvalues of A are real numbers of opposite signs, $\lambda_1 < 0$ and $\lambda_2 > 0$. Let V_1 and V_2 be the eigenvectors associated with λ_1 and λ_2. Since $AV_1 = \lambda_1 V_1$ and $AV_2 = \lambda_2 V_2$, we have:

- If $X_0 = \bar{X} + xV_1$, then $X_t = \bar{X} + (1+\lambda_1)^t xV_1$
- If $X_0 = \bar{X} + xV_2$, then $X_t = \bar{X} + (1+\lambda_2)^t xV_2$

There are two linear trajectories associated with the eigenvectors of A. One converges with speed λ_1 towards the long-term equilibrium and is called the *saddle path*. The other one diverges with speed λ_2.