PC 9 corrected

Exercise #1

1. In a fixed exchange-rate regime, PPP involves a constant price level (Eq. (3)) and UIP involves a constant interest rate (Eq. (4)). To keep the price level constant while having a constant interest rate, money supply must remain constant (Eq. (1)). Hence, official reserves must decrease if domestic credit increases, and vice versa (Eq. (2)).

   In a free-floating regime, official reserves remain constant. Hence, money supply, the price level and the nominal exchange rate move in parallel to $(1-\gamma)d_t$, for a given interest rate. Eq. (4) will then lead to a differential equation between the exchange rate and its variation.

2. In a fixed exchange-rate regime, we have $i_t = i^*$ and $p_t = p^* + \bar{e}$: i_t and p_t are constant. Money supply must then remain constant: $(1-\gamma)(d_0 + \mu t) + \gamma r_t = m_t = \text{cst} = (1-\gamma)d_0 + \gamma r_0$. Hence :

   $$r_t = r_0 - \frac{1-\gamma}{\gamma}\mu t$$

   The (fixed) exchange rate is $\bar{e}$ such as:

   $$\bar{e} = p_t = m_t + \beta\, i^* = (1-\gamma)d_0 + \gamma\, r_0 + \beta\, i^*$$

   $$\boxed{\bar{e} = \gamma\, r_0 + \beta\, i^*}$$

   The regime is sustainable as long as $r_t > 0$. Official reserves are depleted in:

   $$\boxed{T = \frac{\gamma}{1-\gamma}\frac{r_0}{\mu}}$$

   Reserve depletion comes sooner the higher credit growth μ. If the central bank can borrow reserves from other central banks (cf. ERM, Chiang Mai) or on international capital markets, the regime can last longer than T because official reserves can be negative.

3. When reserves have been depleted ($r_t = 0$), money supply shrinks to: $m_t = (1-\gamma)\mu t$. From Eq. (1) and PPP, we get: $m_t = e_t - \beta\, i_t$. Using UIP (Eq. (4)), we get the differential equation:

   $$e_t - \beta\,(i^* + \hat{e}_t) = (1 - \gamma)\mu\, t$$

   $$e_t - \beta\,\hat{e}_t = \beta i^* + (1 - \gamma)\mu\, t$$

The solution is of the form:

$$e_t = a + bt + ce^{t/\beta}$$

The exponential, bubble term is eliminated by assuming c = 0. Then, we have $\hat{e}_t = b$. Thus, the solution is:

$$\tilde{e}_t = \beta\hat{e}_t + \beta i^* + (1-\gamma)\mu t = \beta(1-\gamma)\mu + \beta i^* + (1-\gamma)\mu t$$

We can check that $\hat{\tilde{e}}_t = (1-\gamma)\mu$, hence that $\tilde{e}_t - \beta\hat{\tilde{e}}_t = \beta i^* + (1-\gamma)\mu t$

Another way of finding the solution is to derivate the following expression with respect to t :

$$e_t - \beta\,\hat{e}_t = \beta i^* + (1 - \gamma)\mu\, t$$

We get: $\hat{e}_t - \beta\, d\hat{e}_t/dt = (1-\gamma)\mu$

An obvious solution is $\hat{e}_t = (1-\gamma)\mu$. A specific solution then is:

$$\boxed{\tilde{e}_t = \beta(1-\gamma)\mu + \beta i^* + (1-\gamma)\mu t}$$

The solution without second term is $\tilde{e}_t = e^{t/\beta}$. This solution can be discarded because if is explosive when t goes to infinity (speculative bubble). Hence, there is a single solution (the specific solution written above).

The shadow exchange rate $\tilde{e}_t$ can be written as a function of the fixed exchange rate $\bar{e}$:

$$\boxed{\tilde{e}_t = \bar{e} + \beta(1-\gamma)\mu - \gamma r_0 + d_t}$$

The shadow exchange rate $\tilde{e}_t$ crosses the fixed exchange rate $\bar{e}$ at time T' such as:

$$d_{T'} = \gamma r_0 - \beta(1-\gamma)\mu$$

Hence: $(1-\gamma)\mu T' = \gamma r_0 - \beta(1-\gamma)\mu$

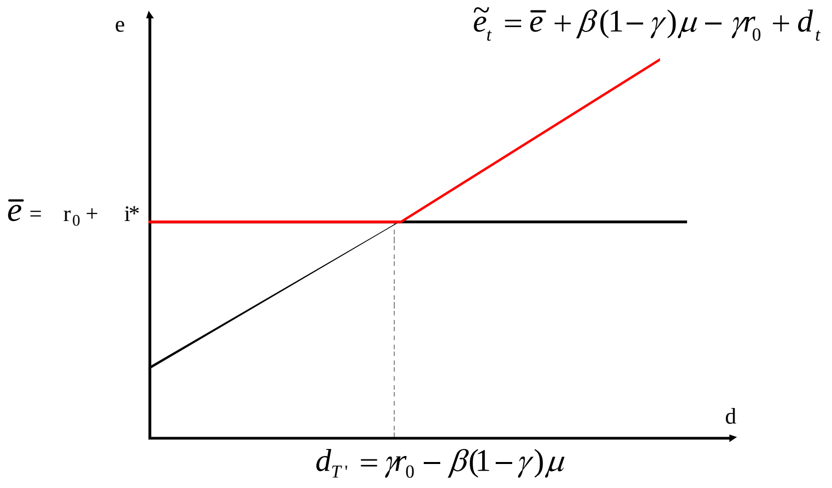
We get: $$\boxed{T' = \frac{\gamma r_0}{(1-\gamma)\mu} - \beta = T - \beta}$$

T' < T. At time T', a speculator could make riskless profit by selling the domestic currency. Remaining reserves would be instantaneously depleted. The amount of reserves at time T' is:

$$r_t = r_0 - \frac{1-\gamma}{\gamma}\mu T' = \beta\frac{1-\gamma}{\gamma}\mu > 0$$

At time T', reserves jump from $r_{T'}$ to zero and the exchange rate starts a new path where it depreciates at rate μ.

Relationship between domestic credit and the nominal exchange rate



Exercise #2

1. The government has an incentive to devalue to curb the unemployment rate, but this will entail a fixed cost *c*. It decides to devalue if the gain in terms of lower unemployment exceeds the cost in terms of foregone reputation. It is a policy decision, not an obligation triggered by reserve depletion. However, a speculative attack makes the defense of the peg more costly, hence if is an incentive to devalue.

2. No expected devaluation: $\pi^e = 0$.

- If the government does not devalue, its loss is: $L_0^0 = (\rho u_{t-1})^2$
- If the government devalues, its loss is: $L_0^d = (\rho u_{t-1} - \lambda\delta)^2 + c$

It devalues if $L_0^d < L_0^0$, *i.e.* if $\Phi < -\lambda\delta$, with $\Phi = \frac{c}{\lambda d} - 2\rho u_{t-1}$

This condition is more easily satisfied if the devaluation cost *c* is small and if inherited unemployment is high. Φ summarizes economic 'fundamentals'.

3. Expected devaluation: $\pi^e = \delta$.

- If the government does not devalue, its loss is: $L_d^0 = (\rho u_{-1} + \lambda\delta)^2$
- If the government devalues, its loss is: $L_d^d = (\rho u_{-1})^2 + c$

It devalues if $L_d^d < L_d^0$, *i.e.* if $\Phi < \lambda d$.

4.