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Highlights

- We incorporate a small and time-varying "disaster risk" in a New Keynesian model.
- A higher probability of disaster is sufficient to generate a recession without effective occurrence of the disaster.
- The dynamic responses of macroeconomic quantities are discussed relatively to a flexible price benchmark.

Abstract

This paper incorporates a small and time-varying "disaster risk" à la Gourio (2012) in a New Keynesian model. A change in the probability of disaster may affect macroeconomic quantities and asset prices. In particular, a higher risk is sufficient to generate a recession without effective occurrence of the disaster. By accounting for monopolistic competition, price stickiness, and a Taylor-type rule, this paper provides a baseline framework of the dynamic interactions between the macroeconomic effects of rare events and nominal rigidity, particularly suitable for further analysis of monetary policy. We also set up our next research agenda aimed at assessing the desirability of several policy measures in case of a variation in the probability of rare events.

JEL Classification: E17, E20, E32, G12

Keywords: disaster risk, rare events, DSGE models, business cycles.

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Disaster Risk in a New Keynesian Model

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Points clefs

- Nous incorporons un risque faible et variable dans le temps de "désastre" économique dans un modèle néo-keynésien.
- Un accroissement de la probabilité de désastre s'avère suffisant pour générer une récession sans occurrence effective de ce désastre.
- Nous comparons les dynamiques des quantités macroéconomiques ainsi obtenues en présence et en l'absence de rigidités nominales.

Résumé

Ce document introduit dans un modèle néo-keynésien un risque faible et variable dans le temps de "désastre" économique à la Gourio (2012). Une légère variation de la probabilité de désastre affecte les quantités macroéconomiques et les prix des actifs. En particulier, un accroissement du risque s'avère suffisant pour générer une récession sans occurrence effective du désastre. En étudiant cet effet dans un cadre néo-keynésien standard, en concurrence monopolistique avec rigidité des prix et en présence d'une règle de politique monétaire, nous fournissons un cadre de référence pour l'étude des interactions dynamiques entre les effets macroéconomiques des événements rares et les rigidités nominales. Nous proposons enfin un programme de recherche visant à évaluer l'impact de différentes politiques économiques, notamment de mesures monétaires non conventionnelles, en cas d'augmentation de la probabilité de désastre.

Classification JEL : E17, E20, E32, G12

Mots clés : Risque de désastre, événements rares, modèles DSGE, business cycles.

DISASTER RISK IN A NEW KEYNESIAN MODEL[1]

Marlène Isoré* and Urszula Szczerbowicz†

1. INTRODUCTION

A recent but growing literature studies how the risk of rare events — sometimes called economic "disasters" — affects the dynamic interactions between macroeconomic quantities and asset prices — risk premia in particular. However, disaster risk is still rarely accounted for in general equilibrium models, especially in the models used to conduct monetary policy where variations in the expected returns are generally entirely driven by variations in the risk-free interest rate. Yet understanding the efficiency and the desirability of monetary policy facing — realized or potential — rare events is of main interest. In order to design an appropriate intervention, studying the effects of a time-varying disaster risk in this class of models is a prerequisite.

Early papers on disaster risk were restricted to endowment economies (Rietz, 1988, Barro, 2006, Gabaix, 2012) such that policy implications could have hardly been derived. Gourio (2012) has gone a step further by introducing a small and stochastically time-varying risk premium into a real business cycle model. His model has thus provided a tractable way to analyze the feedback effects between changes in aggregate risk and the macroeconomic variables, as well as to reproduce some important empirical facts in terms of asset pricing including the countercyclicality of the risk premia. In particular, an increase in the probability of disaster leads investment and output to fall as capital becomes riskier. Meanwhile precautionary savings lower the yield on risk-free assets, such that the spread rises in distressed times.

This paper builds on Gourio's approach and introduces a time-varying risk of disaster in an otherwise standard New Keynesian DSGE model, providing a baseline framework that will allow to evaluate the role of monetary policy facing changes in the probability of rare events. The occurrence of a disaster is associated with the destruction of a share of capital, but the appealing feature of the model is that business cycles are significantly affected by the disaster risk even when disasters do not effectively arrive. We especially focus on the responses of macroeconomic quantities to a sudden rise in the probability of disaster, and get some interesting preliminary results.

[1] We thank Pierpaolo Benigno, Julio Carrillo, Benjamin Carton, Rich Clarida, Marco Del Negro, François Gourio, Salvatore Nisticò, Henri Sterdyniak, and Philippe Weil for many discussions from the early stages of this work. All remaining errors are ours.

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First, we are able to relax one essential assumption in Gourio's work which consists in imposing a reduction in total factor productivity by exactly the same amount than the capital stock to replicate the data. We show that the output fall may be large enough by introducing investment adjustment costs and monopolistic competition in intermediate goods instead. The response of output is much more important under time-dependent price stickiness, however, since firms may be more inclined to adjust their prices when the aggregate risk rises (Caplin and Leahy, 1991), we also allow for some state-dependent price adjustment.

Second, we find that consumption falls on impact in case of a rise in disaster risk while Gourio found the opposite response with a more stylized model. Similarly, we get a drop in wages which is not observed in the pure flexible-price but otherwise similar version of the model, that seems more reminiscent of distressed economic times, whether under time-dependent or state-dependent price stickiness. Finally, we compare the responses of the model to standard monetary, fiscal, and productivity shocks, with and without the presence of a disaster risk.

This version of the model does not study the feedback effects between these macroeconomic quantities and the impact of disaster risk on asset pricing yet. However, the set-up is such that we will be able to do so quite easily by already incorporating a stochastic discount factor from which the term premium will be derived and some features that proved effective in replicating the variations of equity premia, including habit formation.[2] Gourio (2012) shows that the presence of a time-varying disaster risk allows to replicate well the first- and second moments of asset returns, as well as their correlation with the macroeconomic quantities. This suggests that the degree of risk aversion or the amount of risk in the economy has a significant impact on macroeconomic dynamics while Tallarini (2000)'s "observational equivalence" only holds when the probability of disaster is constant over time. In our model, solved under certainty-equivalence so far, linear and nonlinear approximations give almost identical results since we study the responses of macroeconomic quantities to a (small) change in the probability of disaster instead of the responses to a (large) disaster shock. Asset pricing in the presence of a time-varying disaster risk would however require the combination of nonlinear methods and aggregate uncertainty.[3]

The remainder of the paper is as follows. Section 2 develops the model, Section 3 discusses how the steady state is affected by the presence of a disaster risk and presents the calibration, Section 4 describes the response functions to a shock to the probability of disaster as well as to standard shocks. Section 5 gives our further research agenda, and Section 6 concludes.

[2] See Campbell and Cochrane (1999) and Uhlig (2007).

[3] See Bloom (2009) for a model with uncertainty shocks for instance.

2. MODEL

2.1. Households

Households consume goods, supply labor, and save through risk-free bonds and capital accumulation so as to maximize the expected discounted sum of utility flows given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - hC_{t-1})^{1-\gamma}}{1-\gamma} - \chi \frac{L_t^{1+\phi}}{1+\phi} \right) \tag{1}$$

where β is the subjective discount factor, E_0 the expectation operator, C and L consumption and labor flows respectively, h a habit formation parameter, γ the coefficient of relative risk aversion or the inverse of the intertemporal elasticity of substitution, and ϕ the inverse of the elasticity of work effort with respect to the real wage. Households own the capital stock K_t and lease a fraction u_t of it to the firms. Thus their budget constraint is

$$C_t + I_t + \frac{B_{t+1}}{p_t} \leq W_t L_t + (1+i_{t-1}) \frac{B_t}{p_t} + R_t^k u_t K_t + \Pi_t - T_t \tag{2}$$

where I_t is investment, B_t are one-period bonds, w_t is the real wage, Π_t are profits from firms, and R_t^k is the real rental rate of capital, at time t.

Capital is considered as a risky asset here in the sense that it may be hit by a “disaster”. In Barro (2006) and Gourio (2012)’s spirit, a disaster occurrence may be either a war which physically destroys a part of the capital stock, the expropriation of capital holders, a technological revolution that make it worthless, or the loss of intangible capital due to a prolonged recession. We assume that the disaster destroys a share b_k of the capital stock if realized.[4] Therefore the law of capital accumulation is given by

$$K_{t+1} = \left\{ (1-\delta_t) K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t \right\} (1 - x_{t+1} b_k) \tag{3}$$

where $\delta_t = \delta u_t^{\eta}$ is the depreciation rate increasing with capital utilization (Burnside and Eichenbaum, 1996), and $S = \frac{\tau}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2$ is a capital adjustment cost function which verifies the usual properties ($S(0) = 0$, $S'(0) = 0$, and $S''(.) > 0$). The disaster is captured by the indicator x_{t+1} which is equal to 1 with probability θ_t and equal to 0 otherwise. Gourio (2012) argues that this probability can be considered as strict rational expectations or more generally account for

[4] As a disaster lowers the return on capital because investing in capital is riskier one can equally consider ex ante that this is the price or the quantity of capital which is affected by the disaster.

time-varying beliefs which may differ from the objective probability.[5] We consider that the log of the probability of disaster follows a first-order autoregressive process as

$$\log \theta_t = (1-\rho_\theta)\log\bar{\theta} + \rho_\theta \log\theta_{t-1} + \sigma_\theta \varepsilon_{\theta_t} \tag{4}$$

and assume that the shocks θ_{t+1} and x_{t+1} are independent, conditional on θ_t, in line with the evidence that a disaster occurrence tomorrow is not likely if there is a disaster today (Gourio, 2008).

We relax Gourio (2012)'s assumption that total factor productivity is reduced by exactly the same amount than the capital (b_k) in case of a disaster here. This assumption has been made for two reasons. First, detrending the capital by the (stochastic) technology level gives a stationary variable and reduces the dimension of the state space, so as to obtain analytical results and simplify the numerical analysis. Second, it delivers an empirically relevant magnitude for the recession. However, the combination of adjustment costs and monopolistic competition allows us to replicate a large enough fall in output following a rise in disaster risk without having to maintain this assumption here. Moreover, while Gourio argues that some disasters were associated with a fall in TFP (South America since 1945, Russia in 1917), some papers find, on the contrary, that TFP may rise in recessions as the least productive firms are shut down (for instance Petrosky-Nadeau, 2010).

Maximizing (1) subject to (2), (3), and (4) gives standard first-order conditions for consumption, labor, and the riskfree bonds, respectively as

$$\lambda_t = (C_t - hC_{t-1})^{-\gamma} - \beta h E_t (C_{t+1} - hC_t)^{-\gamma} \tag{5}$$

$$\chi L_t^{\phi} = w_t \lambda_t \tag{6}$$

$$\lambda_t = \beta E_t \lambda_{t+1} (1+i_t)(1+\pi_{t+1})^{-1} \tag{7}$$

in which $1+\pi_{t+1} \equiv \frac{p_{t+1}}{p_t}$ where π is the (net) inflation rate, whereas the first-order conditions for capital and capital utilization are both affected by the disaster probability and the disaster size effect, $\theta_t b_k$, as follows[6]

$$\mu_t = \beta E_t \left[\lambda_{t+1} R^k_{t+1} u_{t+1} + \mu_{t+1}\left(1-\delta u^{\eta}_{t+1}\right)(1-\theta_{t+1} b_k) \right] \tag{8}$$

[5] Building on the behavioral macroeconomics literature would help to disentangle whether this probability is objective or stemming from agents' sentiments or "animal spirits" (waves of optimism or pessimism) but this is out of the scope of our paper for now (see Section 5).

[6] These expressions hold under certainty-equivalence, such that disaster risk is not an *uncertainty* shock in this version of the paper. See Section 5 and Appendix.

$$\lambda_t R_t^k = \mu_t \delta \eta u_t^{\eta-1}(1-\theta_t b_k) \tag{9}$$

Finally the first-order condition on investment, also affected by the disaster risk, is

$$\begin{aligned}\lambda_t = \mu_t \left(1-\theta_t b_k\right) & \left[1-\frac{\tau}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2 - \tau\left(\frac{I_t}{I_{t-1}}-1\right)\frac{I_t}{I_{t-1}}\right] \\ & +\beta E_t \mu_{t+1}\left(1-\theta_{t+1} b_k\right)\tau\left(\frac{I_{t+1}}{I_t}-1\right)\left(\frac{I_{t+1}}{I_t}\right)^2\end{aligned} \tag{10}$$

Without investment adjustment cost ($\tau = 0$), the Euler equation would be

$$\beta E_t \frac{\lambda_{t+1}}{\lambda_t} = E_t \left\{\left[R_{t+1}^k u_{t+1} + (1-\delta_{t+1})\right](1-\theta_t b_k)\right\}^{-1}$$

and would further reduce to the standard Euler equation if the probability of disaster was equal to zero.[7] This states that the marginal utility from consumption tomorrow λ_{t+1} will be greater than the marginal utility from consumption today λ_t if the probability θ_t drawn today that a disaster arrives tomorrow increases given that the disaster would destroy a share of capital tomorrow. Consumption may fall or rise on impact following a shock to the disaster risk depending on the value of the elasticity of intertemporal substitution (see Sections 3 and 4).

Moreover, complete markets imply that there is a unique stochastic discount factor, denoted $Q_{t,t+1}$ such that

$$1 + i_t = (E_t Q_{t,t+1})^{-1} \tag{11}$$

If $\tau = 0$, we can easily derive, from the first-order condition on bonds and the Euler equation above, that

$$E_t Q_{t,t+1} = E_t \left\{(1+\pi_{t+1})\left[R_{t+1}^k u_{t+1} + (1-\delta_{t+1})\right](1-\theta_t b_k)\right\}^{-1}$$

such that the stochastic discount factor also accounts for the disaster risk, while remains standard if $\theta_t = 0$.[8]

The existence of a risk of disaster on capital also affects the level of the Tobin's q. Defined as the ratio of the market value of one additional unit of investment to the marginal replacement

[7] For the purpose of the quantitative exercise, we keep adjustment costs positive ($\tau > 0$) though, in order to get a more gradual response of investment to changes in the probability of disaster, without qualitative impact on the Euler equation.

[8] Our time-varying stochastic discount factor however differs from Gourio (2012)'s because we do not assume that total factor productivity is reduced by the same amount than the capital stock in case of a disaster.

cost of installed capital,[9] it is given by the ratio of the Lagrange multipliers on (3) and (4), that is,

$$q_t = \frac{\mu_t}{\lambda_t} \tag{12}$$

Without disaster risk, the first-order condition on investment would imply that, in steady-state, $\bar{\lambda} = \bar{\mu}$, and thus $\bar{q} = 1$. Therefore whenever $q_{t+s} > 1$ in any period $t+s$ more investment would then add to the value of the firm, whereas with $q_{t+s} < 1$ it would be optimal for firms to disinvest. Here the disaster risk implies that $\bar{\lambda} = \bar{\mu}(1-\bar{\theta}b_k)$, and thus

$$\bar{q} = \frac{1}{1-\bar{\theta}b_k} \quad > 1 \quad \text{if} \quad \bar{\theta} > 0$$

The higher the disaster risk in steady-state, the higher the Tobin's q: the threshold value for (dis-)investment incentives is higher in the presence of a disaster risk. This is because a rise in disaster risk today leads to a higher marginal replacement cost of capital tomorrow, associated with a rise in the level of investment that is required to increase firms' net market value.

2.2. Firms

The production block is roughly similar to the New Keynesian literature,[10] except that we will allow the price adjustment to depend on the disaster risk. Production is split into a monopolistic competition market producing intermediate goods and a competitive sector producing the final consumption good as a CES composite of the intermediate goods.

2.2.1. Final goods producers

With intermediate goods indexed by j over a continuum of unit interval, the aggregate is given by

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\nu-1}{\nu}} dj\right)^{\frac{\nu}{\nu-1}}$$

which corresponds to a downward sloping demand curve for each good j as

$$Y_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} y_t$$

[9] In microeconomic terms, the ratio of the marginal benefit in terms of utility of an extra unit of investment over the marginal benefit in terms of utility of sacrificing a unit of current consumption in order to have an extra unit of investment.

[10] See for instance FernÃ¡ndez-Villaverde and Rubio-RamÃrez (2006)

and to an aggregate price index given by

$$p_t = \left(\int_0^1 p_{j,t}^{1-\nu} dj \right)^{\frac{1}{1-\nu}}$$

2.2.2. Intermediate goods producers

Intermediate goods are produced with capital and labor, according to a standard Cobb-Douglas production function

$$Y_{j,t} = A_t \tilde{K}_{j,t}^{\alpha} L_{j,t}^{1-\alpha}$$

in which the capital leased to the firms is

$$\tilde{K}_t = u_t K_t \tag{13}$$

where u_t is the variable utilization rate of capital, and in which total factor productivity, denoted A_t, is driven by

$$\log A_t = (1-\rho_A)\log \bar{A} + \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A_t} \tag{14}$$

where the shocks are small and normally distributed (ε_t is i.i.d. $N(0,1)$).

There is a two-step problem for firms producing the intermediate goods. First, each firm j minimizes capital and labor costs at each date, independently of price adjustment, subject to the restriction of producing at least as much as the intermediate good is demanded at the selling price, that is,

$$\min_{L_{j,t},\tilde{K}_{j,t}} p_t (w_t L_{j,t} + R_t^k \tilde{K}_{j,t})$$

$$\text{s.t.} \quad A_t \tilde{K}_{j,t}^{\alpha} L_{j,t}^{1-\alpha} \geq \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} Y_t$$

The first-order conditions for this problem give a capital-labor ratio which holds at the aggregate level since it is the same across all firms

$$\left(\frac{\tilde{K}_{j,t}}{L_{j,t}} \right)^* = \frac{w_t}{R_t^k} \frac{\alpha}{(1-\alpha)}$$

and allows to write the optimal marginal input costs as

$$mc_t^* = w_t^{1-\alpha} \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} \frac{R_t^{\alpha}}{A_t}$$

from which the aggregate first-order conditions are expressed as

$$w_t = mc^* (1-\alpha) A_t \left(\frac{\tilde{K}_t}{L_t}\right)^{\alpha} \tag{15}$$

$$R_t^k = mc^* \alpha A_t \left(\frac{\tilde{K}_t}{L_t}\right)^{\alpha-1} \tag{16}$$

Then, given the optimal input mix, some firms maximize their profits by choosing their selling price $p_{j,t}$. We consider two alternative ways to introduce nominal stickiness. One is standard Calvo time-dependent pricing so that firms in the intermediate sector face a constant probability ζ_0 of being unable to change their price at each time t despite the disaster risk. The other one is to assume that firms' price adjustment increases in the aggregate risk, *i.e.* the gap between the current value of the probability ζ_t of being unable to change one's price and the Calvo probability ζ_0 is given by

$$\zeta_t - \zeta_0 = -\theta_t^{\iota}$$

where ι is the elasticity of the gap to the probability of disaster.[11] [12]

Writing ζ as standing either for ζ_0 in the first case or for ζ_t in the second, the profit-maximizing problem in both cases is

$$\max_{p_{j,t}} E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\left(\frac{p_{j,t}}{p_{t+s}}\right)^{1-\nu} y_{t+s} - mc_{t+s}^* \left(\frac{p_{j,t}}{p_{t+s}}\right)^{-\nu} y_{t+s} \right)$$

The solution to this problem holds at the aggregate level ($p_t^* = p_{j,t}^*$). The gap between this optimal price p_t^* and the consumer price index p_t is

$$\frac{p_t^*}{p_t} = \frac{\nu}{\nu-1} E_t \frac{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\frac{p_{t+s}}{p_t}\right)^{\nu} Y_{t+s} mc_{t+s}^*}{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\frac{p_{t+s}}{p_t}\right)^{\nu-1} Y_{t+s}}$$

[11] Note that this function requires to impose a parameter restriction so that ζ remains positive. With $\bar{\theta} = 0.01$ in particular, ι cannot be lower than 0.05.

[12] This price setting reminds the 'SS pricing' literature (Caplin and Leahy, 1991) although the firms do not react to the effective realization of aggregate shocks but to the expected risk. This is because the probability of disaster is incorporated in the forward-looking agents' optimization problem and the size of an effective disaster is constant here.

This expression can finally be rewritten recursively in order to stress out the price adjustment dynamics and in terms of an inflation gap to allow for a non-zero inflation steady-state, such that

$$\frac{1+\pi_t^*}{1+\pi_t} = \frac{\nu}{\nu-1} E_t \frac{\Xi_{1_t}/p_t^{\nu}}{\Xi_{2_t}/p_t^{\nu-1}} \tag{17}$$

with $\pi_t = \frac{p_t}{p_{t-1}} - 1$ the net inflation rate, π_t^* the net reset inflation rate, and

$$\frac{\Xi_{1_t}}{p_t^{\nu}} = \frac{Q_{t+s}}{\beta} Y_t mc_t^* + \zeta\beta E_t \frac{\Xi_{1_{t+1}}}{p_{t+1}^{\nu}} (1+\pi_{t+1})^{\nu}, \quad \text{and} \tag{18}$$

$$\frac{\Xi_{2_t}}{p_t^{\nu-1}} = \frac{Q_{t+s}}{\beta} Y_t + \zeta\beta E_t \frac{\Xi_{2_{t+1}}}{p_{t+1}^{\nu-1}} (1+\pi_{t+1})^{\nu-1} \tag{19}$$

All the computational details are given in Appendix.

2.3. Public authority

The public authority consumes some output G_t, charges lump sum taxes T_t to households, and issues debt D_t which pays interest i_t set up according to a standard Taylor-type rule that depends on the deviation of inflation from steady-state and on an output growth gap as

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[\psi_\pi(\pi_t - \bar{\pi}) + \psi_Y(y_t - \bar{y}) + \bar{i}\right] + \sigma_i \varepsilon_{i_t} \tag{20}$$

in which y is the growth rate of output and where an overbar indicates the steady-state value of a variable. The public authority's budget constraint equates spending plus payment on existing debt to collected taxes plus new debt issuance[13], that is,

$$G_t + (1+i_t)\frac{D_t}{p_t} = T_t + \frac{D_{t+1}}{p_t}$$

in which G_t follows a first-order autoregressive process in the logs

$$\log G_t = (1-\rho_G)\log(\omega\bar{Y}) + \rho_G \log G_{t-1} + \sigma_G \varepsilon_{G_t} \tag{21}$$

where ω is the steady-state share of output devoted to public expenditures.

[13] We assume that there is no money, hence no seignorage revenue in the model.

3. EQUILIBRIUM

3.1. Market clearing

Market-clearing in the bond market implies that the total amount of debt is equal to the total amount of bounds in period t

$$D_t = B_t$$

and market-clearing in output implies that

$$Y_t = C_t + I_t + G_t \tag{22}$$

Moreover, knowing the demand for individual intermediate goods firms, we are able to derive the aggregate production function as a function of the individual firms' production function and a measure of the inefficiency introduced by the dispersion in relative prices, $\Omega_t = \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj$, such that

$$Y_t = \frac{A_t \tilde{K}_t^{\alpha} L_t^{1-\alpha}}{\Omega_t} \tag{23}$$

in which the aggregate price distortion is given by the recursive equation

$$\Omega_t = (1-\zeta)\left(\frac{1+\pi_t}{1+\pi_t^*}\right)^{\nu} + \zeta(1+\pi_t)^{\nu}\Omega_{t-1} \tag{24}$$

Finally, given that a fraction ζ of firms do not readjust their prices, the aggregate price index, $p_t^{1-\nu} = \int_0^1 p_{j,t}^{1-\nu} dj$, is given by $p_t^{1-\nu} = (1-\zeta)p_t^{*1-\nu} + \zeta p_{t-1}^{1-\nu}$, further rewritten in inflation terms as

$$(1+\pi_t)^{1-\nu} = (1-\zeta)(1+\pi_t^*)^{1-\nu} + \zeta \tag{25}$$

Equilibrium is characterized by equations (3) to (25) in 23 unknowns: $\{Y, C, I, G, A, L, K, \tilde{K}, u, w, R^k, \Omega, \pi, \pi^*, \tilde{\Xi}$

3.2. Calibration and steady-state analysis

Our calibration, summarized in Table 1, is mostly based on the standard New Keynesian literature (Smets and Wouters, 2003, Rudebusch and Swanson, 2008). In particular the value of the inverse of the elasticity of intertemporal substitution (EIS) ranges from 0.5 to 6 under CRRA preferences with a baseline value of 2. In addition, Barro (2006) found on historical data that the average share of capital that is destroyed in case of disaster is 43%, while Gourio (2012) estimates that the average probability of a such a disaster is 1.7% annually, backing it out from evidence on asset prices under the assumption that the fall in total factor productivity is also exactly equal to 43%. Since we use the quarterly calibration of standard New Keynesian models and are not able to replicate the estimation so far, we test for several values of $\bar{\theta}$ around a 1% benchmark, as well as for several values of b_k and of the persistence in the shock to θ, without significant changes in our results.

In our steady-state, the capital stock, output, and consumption are lower in the presence of a disaster risk as compared to the same economy without disaster for all values of risk aversion/EIS. Steady-state investment and labor may be larger in the presence of disasters if the EIS is very high ($\gamma = 0.5$), but are generally weaker, such that wages are generally lower. The firms can substitute labor to capital such that their steady-state marginal costs are unchanged even though the cost of capital is higher in case of disaster. Therefore the non-zero steady-state inflation rate is unaffected by disaster risk and equal to the public authority's target that we set at 2% annually. The main ratios, C/Y, I/Y, G/Y are in all cases slightly above or below their standard values, 60%, 20% and 20%, respectively. Finally, the steady-state risk premium in case of disaster corresponds to the wedge between the higher steady-state return on capital and the unchanged riskfree rate.

Gourio (2012) found that the model quantities shift to a lower steady-state in the economy with disaster risk (as compared to an economy without disaster) if and only if the EIS is larger than unity. Therefore, it is noteworthy to clarify at this point why we do get a lower steady-state for all values of the EIS here, on the one hand, and its further implications on the model dynamics, on the other hand. First, with Epstein-Zin preferences, *i.e.* dissociating the risk aversion coefficient from the inverse of the EIS, it would be possible to show that, when investment on capital becomes riskier, the *risk-adjusted* return on capital goes down for risk averse agents, while the effect of this change on the consumption-savings decisions depends on the value of the EIS (Weil, 1990, Angeletos, 2007). In particular, when the EIS is larger than unity ($\gamma < 1$), the substitution effect of a higher risk-adjusted return is larger than the income effect and savings fall. Therefore the steady-state capital stock and output are lower. However, when the EIS is equal to 1, both effects cancel each other out and savings are unaffected by changes in the risk-adjusted return, that is, are unaffected by changes in the return on capital even if agents are risk-averse.[14] Our specification, where risk aversion is only the inverse of the EIS, does not allow to disentangle the two effects, yet remains preferable in order to solve the equity premium puzzle by incorporating habit formation (Weil, 1989, Uhlig, 2007, Angeletos, 2007).

More importantly, the reason why we get lower steady-state macro quantities even when the EIS is unity is because we solve the model such that the disaster risk is treated as a small but

[14]The EIS determines the *sign* of the effect of increased uncertainty on savings while the risk aversion only affects its *magnitude* (Weil, 1990).

certain probability of disaster instead of being a large uncertain shock. This allows to solve the model quite easily without having to maintain Gourio's assumption that the disaster is a strict combination of a depreciation shock to capital and a negative shock to the total factor productivity by the same amount. Meanwhile, this does not substantially restrict our business cycle analysis for two reasons. First, we capture the main first-moment effect of disaster risk by the fact that depreciation of capital will be higher in the future, even though we do not have the second-moment effect associated with higher uncertainty about future depreciation.[15] Second, Gourio shows that Tallarini (2000)'s *observational equivalence* in the dynamics of the macroeconomic variables in case there is an aggregate risk or not does not hold when the probability of disaster is not constant. When the disaster risk is time-varying, Gourio finds that risk aversion matters for the macroeconomic dynamics, and this is captured here.

4. Impulse responses of the macroeconomic variables

Analyzing the effects of a time-varying risk on asset pricing would require to treat the disaster risk as an uncertainty shock and to use nonlinear methods to solve the model. However, since there is a consensus about the irrelevance of approximation beyond the first-order for the macroeconomic quantities, on the one hand, and given that we do not consider the case of a large shock, on the other hand, we maintain certainty-equivalence and first-order methods in this version of the paper, although we keep track of some second-order corrections in the Appendix.[16][17] For each (small) shock below, we compare the responses obtained in our model (solid line) to their counterpart in a flexible-price but otherwise similar model[18] (dashed line) and in a standard sticky-price New Keynesian model without disaster risk (dotted line).

4.1. A rise in the probability of disaster

Figure 1 depicts the responses of the main variables to a rise in the probability of disaster, θ. Investment and capital fall on impact as households foresee the upcoming depreciation of capital when the probability of disaster, θ, rises. These effects are much more important under Calvo price stickiness ($\zeta = 0.8$) than under flexible prices ($\zeta = 0$) as all firms do not adjust their prices downwards as much as they would optimally do to match the fall in aggregate demand. The capital stock still goes down next periods because of the depressed investment even though the probability of disaster gradually returns to its initial level (from the autoregressive process).

[15] Gourio admits that the two effects are present but cannot be disentangled in his article. In every case, both effects push the variables in the same direction, and the first-moment effect is far more important for macroeconomic quantities.

[16] Since certainty-equivalence holds, these correction terms are naturally very small.

[17] The effective occurrence of a disaster would be a large shock, whereas the rise in the probability of disaster considered here is a small one.

[18] The flexible-price model is different from Gourio's RBC with disaster risk since we have CRRA preferences with habit formation, a public authority, and variable utilization rate of capital, on the one hand, and because we do not assume a fall to TFP by the same amount as simultaneous to the rise in the probability of disaster, on the other hand.

Labor supply decreases when prices are flexible because it is less attractive for workers to work today when the return on savings is low (intertemporal effect), despite a negative wealth effect that tends to push employment up.[19] Wages thus slightly rise. However, when prices are sticky, the firms that cannot readjust their prices downwards as much as they want face an even lower demand for their own intermediate goods, and thus in turn lower their demand of labor, leading wages to fall. Because capital and labor decrease more under sticky prices, combined with the fact that decrease in aggregate demand is more severe, the slump in output is far larger with nominal rigidity.

In the flexible case, consumption increases on impact as households substitute consumption for investment in the first period, while lower output leads consumption to fall in the next periods, for standard values of the EIS and/or risk aversion.[20] With sticky prices however, consumption falls on impact for the baseline calibration ($\gamma = 2$), or lower values of the EIS (higher risk aversion). For very low risk aversion, consumption moves up on impact similarly to the flexible-price case but a quantitative difference due to price stickiness remains, as shown in Figure 5.

As investment in capital is riskier, households' demand for safer government bonds rises, so that the short-term nominal interest rate falls ("flight to quality" effect). However, because of the inertia in the Taylor-type reaction, the interest rate — and therefore inflation — falls less under price stickiness Finally, actual inflation decreases less than reset inflation, so that the price dispersion falls, but still falls more than the nominal interest rate, so that the real rate rises.

Figures 6 to 10 present some robustness checks and alternative specifications. Figure 6 considers different values of the steady-state probability of disaster ($\bar{\theta}$). While the magnitude of the effects increases in the steady-state disaster risk, the qualitative responses are all identical. Figure 7 gives some alternative values for the persistence of the shock (ρ_θ). Figure 8 tests for different values for the share of capital which is destroyed in case of disaster (b_k), including a possible negative value.[21]

More importantly, Figure 9 gives the responses under state-dependent price stickiness for different values of the parameter $\iota < 1$.[22] The responses still differ significantly from the pure flexible-price version of the model ($\zeta = 0$) and our main results hold, notably the drop in wages, including for an extreme $\iota = 0.1$.

We finally consider a fall in the probability of disaster in Figure 10. Table 2 gives the second-order correction terms associated with this shock, naturally found to be very small under the

[19]The relative importance of the two effects would depend on the EIS with Epstein-Zin preferences. However this result is familiar with standard calibration of CRRA preferences.
[20]Gourio (2012) found a similar effect with a slightly different flexible-price model and a simultaneous shock to the TFP.
[21]A negative value of b_k verifies that the model works symmetrically such that the rare event could be a "miracle" instead of a "disaster".
[22]When $\iota \geq 1$, the responses are almost identical to the time-dependent pricing case.

certainty-equivalence assumption.

To sum up, a rise in the probability of disaster creates a recession, a fall in inflation, a flight to quality in terms of asset demand, depressed investment and labor, as well as lower consumption for standard risk aversion. The fact that the probability of a disaster is higher suffices to generate this recession, without effective occurrence of the disaster.

4.2. Standard shocks

The responses to standard shocks in the model with disaster risk are very close to the responses in a standard New Keynesian model.

For a TFP shock (Figure 2), output and investment rise because the marginal returns on labor and capital rise. However this is slightly less important in the presence of a disaster risk which depreciates capital. Consumption rise more however from the substitution effect between investment and consumption for households. The response of labor is discussed extensively in the literature: in opposition to a RBC where labor increases because the marginal return on labor is higher, sticky prices prevent some firms from lowering their prices leading them to lower their labor demand because of the contraction in demand for their own intermediate goods (Galí, 1999). In addition, higher incomes for households make leisure more desirable so that the supply of labor does not substantially rise neither. As reset inflation is higher than actual inflation, price dispersion falls and the real interest rate goes up despite the fall in the nominal rate.

A positive shock to public expenditures (Figure 3) also replicates the very well-known reactions. In all cases, there is a temporary rise in output from the rise in aggregate demand, an eviction effect on private consumption and investment, hence a fall in capital. Thus firms rely more on labor and wages go up. High reset inflation creates more price dispersion, and the nominal rate is increased.

Finally, a monetary contraction (Figure 4) generates the standard decrease in all macro quantities, as well as in inflation and price dispersion.

5. FURTHER RESEARCH

This paper provides a baseline framework that could be used to develop a number of innovative research ideas, including the role of monetary policy to prevent self-fulfilling recessions in case of misperceptions about the disaster risk. This Section presents our research agenda, which broadly consists in three steps.

First, we would like to account for a *perceived* risk of disaster along with the *real* disaster risk. Gourio (2012) considers that the probability of disaster introduced in his model (and in ours) may result from the economic agents' perception, probably because considering that the probability taken as given by the agents is the real risk would be associated with perfect individual rationality and knowledge about disasters while one could be more agnostic by considering it as merely perceived, especially for *rare* events. We think that it would be helpful to build on the behavioral macroeconomics literature (Gabaix and Laibson, 2002, De Grauwe, 2010, Fuster, Laibson and Mendel, 2010, Angeletos and La'O, 2012, Barsky and Sims, 2012) in order to disentangle a *perceived* from a *real* disaster risk. Another mean would be the use of computational methods in order to keep the disaster variable (x_{t+1}) as an indicator in the Euler equation instead of substituting the time-varying probability θ_t of an effective future occurrence. This would allow to simulate a rise in the probability of disaster while preventing the real occurrence of a disaster by accounting for uncertainty in the model.

As a second step, we will evaluate the model predictions in terms of asset pricing, especially the countercyclicality of the risk premium. Some interactions between price rigidity and the risk of disaster may affect equity returns. The asset price volatility may in turn have important consequences on consumption volatility. In particular the perception of disaster risk may be one of the psychological mechanisms that alter the reactivity of consumption changes to asset price movements (see Lynch, 1996, or Gabaix and Laibson, 2002, for instance), in addition to habit formation (Campbell and Cochrane, 1999, Uhlig, 2007), or adjustment costs (Grossman and Laroque, 1990). On practical grounds, pricing assets requires a few more sophistications in our setup. One is to go beyond the first-order approximation in the Taylor expansion. The consensus in the literature is that these higher-order terms do not matter for the responses of macroeconomic quantities we have focused on so far but have an important role in the asset pricing in the presence of a time-varying risk. Another key element will be to add corporate bonds in the model since leverage is a standard way to make equity returns more volatile and procyclical — in line with the data — in the literature, which may be even more relevant in a model in which firms' prices are sticky.

Finally, we would like to assess the desirability of monetary policy to prevent a (self-fulfilling) recession from a sudden rise in the (perceived) probability of disaster. Several conventional and unconventional interventions could be compared with one another by incorporating a welfare function measuring their effectiveness. In particular we think of adding an extra term in the Taylor-type rule which would represent a direct response of the monetary authority in the face of a wave of pessimism. This would be a quasi-conventional intervention, making changes in the nominal interest rate more reactive but still limited by the zero lower bound. A more unconventional measure could consist in purchasing corporate bonds (which may encompass bank debt), directly affected by the disaster risk, by selling riskfree government bonds (as far as sovereign default is excluded).

6. Conclusion

This paper provides a baseline framework to analyze the business cycle responses of macroeconomic quantities in the presence of a small time-varying disaster risk in an otherwise standard New Keynesian model. While following Gourio (2012) on the description of an economic disaster, we relax the assumption that total factor productivity needs to fall by the same amount than the capital stock in case of a disaster. By incorporating investment adjustment costs and monopolistic competition, we show that the magnitude of the recession following a shock to the probability of disaster may be far increased. As compared with the early papers on rare events, we also account for the fact that consumption and wages do not rise in distressed economic times, whether nominal rigidity is time-dependent or state-dependent. More generally, this paper is a first step towards the introduction of rare events into the models used to conduct monetary policy, and will be used to compare the effectiveness of several interventions in the presence of such a risk.

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APPENDIX

8. MATHEMATICAL APPENDIX

A. Households

Given that next period disaster x_{t+1} is equal to 1 with probability θ_t and equal to 0 with probability $1-\theta_t$, the law of accumulation of capital can be rewritten as

$$
\begin{aligned}
K_{t+1} &= [\theta_t(1-b_k)+(1-\theta_t)]\{(1-\delta_t)K_t+[1-S(I_t/I_{t-1})]I_t\} \\
&= (1-\theta_t b_k)\{(1-\delta_t)K_t+[1-S(I_t/I_{t-1})]I_t\}
\end{aligned}
$$

Therefore the Lagrangian for the households' problem is

$$
\begin{aligned}
\mathscr{L} =& E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{(C_t-hC_{t-1})^{1-\gamma}}{1-\gamma} - \chi \frac{L_t^{1+\phi}}{1+\phi} \right) \right. \\
&+\lambda_t \left(W_t L_t + (1+i_{t-1})\frac{B_t}{p_t} + \frac{M_t}{p_t} - \frac{B_{t+1}}{p_t} - \frac{M_{t+1}}{p_t} + R_t^k u_t K_t + \Pi_t - T_t - I_t - C_t \right) \\
&\left. + \mu_t \left[\left((1-\delta u_t^{\eta})K_t + \left(1 - \frac{\tau}{2}\left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t \right) (1-\theta_t b_k) - K_{t+1} \right] \right\}
\end{aligned}
$$

and the first-order conditions are

- Consumption: $\lambda_t = (C_t-hC_{t-1})^{-\gamma} - \beta h E_t (C_{t+1}-hC_t)^{-\gamma}$
- Labor: $\chi L_t^{\phi} = w_t \lambda_t$
- Bonds: $\lambda_t = \beta E_t \lambda_{t+1}(1+i_t)(1+\pi_{t+1})^{-1}$, with$1+\pi_{t+1} \equiv \frac{p_{t+1}}{p_t}$
- Capital: $\mu_t = \beta E_t \left[\lambda_{t+1} R_{t+1}^k u_{t+1} + \mu_{t+1}\left(1-\delta u_{t+1}^{\eta}\right)(1-\theta_{t+1}b_k) \right]$
- Capital utilization rate: $\lambda_t R_t^k = \mu_t \delta \eta u_t^{\eta-1}(1-\theta_t b_k)$
- Investment: $\lambda_t = \mu_t(1-\theta_t b_k)\left[1 - \frac{\tau}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2 - \tau\left(\frac{I_t}{I_{t-1}}-1\right)\frac{I_t}{I_{t-1}} \right]$

  $+\beta E_t \mu_{t+1}(1-\theta_{t+1}b_k)\tau\left(\frac{I_{t+1}}{I_t}-1\right)\left(\frac{I_{t+1}}{I_t}\right)^2$

With no investment adjustment cost ($\tau = 0$), the FOC on investment becomes $\lambda_t = \mu_t(1-\theta_t b_k)$, which in turn implies from the FOC on the capital utilization rate that $R_t^k = \delta_t'$. Substituting into the FOC on capital gives the Euler equation (11) in case $\tau = 0$.

B. Firms

- Production aggregation

The aggregate of intermediate goods is given by

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}$$

so that the profit maximization problem of the representative firm in the final sector is

$$\max_{Y_{t,j}} p_t \left(\int_0^1 Y_{j,t}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} - \int_0^1 p_{j,t} Y_{j,t} dj$$

The first-order condition with respect to $Y_{t,j}$ yields a downward sloping demand curve for each intermediate good j as

$$Y_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} Y_t$$

The nominal value of the final good is the sum of prices times quantities of intermediates

$$p_t Y_t = \int_0^1 p_{j,t} Y_{j,t} dj$$

in which Y_t is substituted to give the aggregate price index as

$$p_t = \left(\int_0^1 p_{j,t}^{1-\nu} dj \right)^{\frac{1}{1-\nu}}$$

- Cost minimization

Firms are price-takers in the input markets, facing a nominal wage $w_t p_t$ and a nominal rental rate $R_t^k p_t$ (w_t and R_t^k are in real terms). Therefore, they choose the optimal quantities of labor and capital given the input prices and subject to the restriction of producing at least as much as the intermediate good is demanded at the given price. The intratemporal problem is

$$\min_{L_{j,t}, \tilde{K}_{j,t}} w_t p_t L_{j,t} + R_t^k p_t \tilde{K}_{j,t}$$

$$\text{s.t.} \quad a_t \tilde{K}_{j,t}^{\alpha} L_{j,t}^{1-\alpha} \geq \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} Y_t$$

The first-order conditions are

$$(L_{j,t}:) \quad w_t = \frac{\varphi_{j,t}}{p_t}(1-\alpha)A_t\left(\frac{\tilde{K}_{j,t}}{L_{j,t}}\right)^{\alpha}$$

$$(\tilde{K}_{j,t}:) \quad R_t^k = \frac{\varphi_{j,t}}{p_t}\alpha A_t\left(\frac{\tilde{K}_{j,t}}{L_{j,t}}\right)^{\alpha-1}$$

in which the Lagrange multiplier $\varphi_{j,t}$ can be interpreted as the (nominal) marginal cost associated with an additional unit of capital or labor. Rearranging gives the optimal capital over labor ratio as

$$\left(\frac{\tilde{K}_{j,t}}{L_{j,t}}\right)^{*} = \frac{w_t}{R_t^k}\frac{\alpha}{(1-\alpha)}$$

in which none of the terms on the right hand side depends on j, and thus holds for all firms in equilibrium, *i.e.*, $\frac{\tilde{K}_t}{L_t} = \frac{\tilde{K}_{j,t}}{L_{j,t}}$. Replacing in the first-order conditions further gives $mc_t^* = \frac{\varphi_t}{p_t}$ as

$$mc_t^* = w_t^{1-\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{\left(R_t^k\right)^{\alpha}}{A_t}$$

- Profit maximization

Let us now consider the pricing problem of a firm that gets to update its price in period t and wants to maximize the present discounted value of future profits. First, the (nominal) profit flow, $p_{j,t}Y_{j,t} - w_t p_t L_{j,t} - R_{j,t}^k p_t \tilde{K}_{j,t}$, can be rewritten as $\Pi_{j,t} = (p_{j,t} - \varphi_t)Y_{j,t}$, that is, in real terms, $\frac{\Pi_{j,t}}{p_t} = \frac{p_{j,t}}{p_t}Y_{j,t} - mc_t^* Y_{j,t}$. Firms will discount future profit flows by both the stochastic discount factor, $Q_t = \beta^s \lambda_{t+s}$, and by the probability ζ^s that a price chosen at time t is still in effect at time s. Replacing $Y_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} Y_t$, the profit maximization problem is

$$\max_{p_{j,t}} E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s}\left(\left(\frac{p_{j,t}}{p_{t+s}}\right)^{1-\nu} Y_{t+s} - mc_{t+s}^*\left(\frac{p_{j,t}}{p_{t+s}}\right)^{-\nu} Y_{t+s}\right)$$

Given that $mc_t^* = \frac{\varphi_t}{p_t}$ and factorizing, we can rewrite it as

$$\max_{p_{j,t}} E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p_{t+s}^{\nu-1} Y_{t+s}\left(p_{j,t}^{1-\nu} - \varphi_t p_{j,t}^{-\nu}\right)$$

The first-order condition is

$$E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p_{t+s}^{\nu-1} Y_{t+s} \left((1-\nu) p_{j,t}^{-\nu} + \nu \varphi_t p_{j,t}^{-\nu-1} \right) = 0$$

which simplifies as

$$p_{j,t}^* = \frac{\nu}{\nu-1} E_t \frac{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p_{t+s}^{\nu} Y_{t+s} mc_{t+s}^*}{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p_{t+s}^{\nu-1} Y_{t+s}}$$

Note that this optimal price depends on aggregate variables only, so that $p_t^* = p_{j,t}^*$. The gap between the current price and the optimal aggregate price is thus given by

$$\frac{p_t^*}{p_t} = \frac{\nu}{\nu-1} E_t \frac{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\frac{p_{t+s}}{p_t}\right)^{\nu} Y_{t+s} mc_{t+s}^*}{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\frac{p_{t+s}}{p_t}\right)^{\nu-1} Y_{t+s}}$$

In order to stress out the recursive price adjustment, let define p_t^* as

$$p_t^* = \frac{\nu}{\nu-1} E_t \frac{\Xi_{1_t}}{\Xi_{2_t}}$$

in which Ξ_{1_t} and Ξ_{2_t} can be expressed recursively as

$$\Xi_{1_t} = Q_{t+s} p_t^{\nu} Y_t mc_t^* + \zeta E_t \Xi_{1_{t+1}}$$

$$\Xi_{2_t} = Q_{t+s} p_t^{\nu-1} Y_t + \zeta E_t \Xi_{2_{t+1}}$$

and rewritten as

$$\beta \frac{\Xi_{1_t}}{p_t^{\nu}} = Q_{t+s} Y_t mc_t^* + \zeta \beta^2 E_t \frac{\Xi_{1_{t+1}}}{p_{t+1}^{\nu}} \left(\frac{p_{t+1}}{p_t}\right)^{\nu}$$

$$\beta \frac{\Xi_{2_t}}{p_t^{\nu-1}} = Q_{t+s} Y_t + \zeta \beta^2 E_t \frac{\Xi_{2_{t+1}}}{p_{t+1}^{\nu-1}} \left(\frac{p_{t+1}}{p_t}\right)^{\nu-1}$$

Therefore, we have

$$\frac{p_t^*}{p_t} = \frac{\nu}{\nu-1} E_t \frac{\frac{\Xi_{1_t}}{p_t^{\nu}}}{\frac{\Xi_{2_t}}{p_t^{\nu-1}}}$$

C. Aggregation

C.1. Bonds market

Market-clearing requires that:

$$D_t = B_t$$

C.2. Aggregate demand

First replace $D_t = B_t$ into the public authority's budget constraint, and express T_t as

$$T_t = G_t + (1+i_t)\frac{B_t}{p_t} - \frac{B_{t+1}}{p_t}$$

which can be plugged into the household budget constraint as

$$C_t + I_t + \frac{B_{t+1}}{p_t} = w_t L_t + (1+i_t)\frac{B_t}{p_t} + R_t^k \tilde{K}_t + \Pi_t - \left(G_t + (1+i_t)\frac{B_t}{p_t} - \frac{B_{t+1}}{p_t}\right)$$

This further simplifies to:

$$C_t + I_t + G_t = w_t L_t + R_t^k \tilde{K}_t + \Pi_t$$

where we have to verify that the RHS is equal to Y_t. Total profits Π_t must be equal to the sum of profits earned by intermediate good firms, that is

$$\Pi_t = \int_0^1 \Pi_{j,t} dj$$

Real profits earned by intermediate good firms j are given by

$$\Pi_{j,t(real)} = \frac{p_{j,t}}{p_t} Y_{j,t} - w_t L_{j,t} - R_t^k \tilde{K}_{j,t}$$

Substituting $Y_{j,t}$, we have

$$\Pi_{j,t(real)} = \left(\frac{p_{j,t}}{p_t}\right)^{1-\nu} Y_t - w_t L_{j,t} - R_t^k \tilde{K}_{j,t}$$

Therefore,

$$\Pi_{t(real)} = \int_0^1 \left(\left(\frac{p_{j,t}}{p_t}\right)^{1-\nu} Y_t - w_t L_{j,t} - R_t^k \tilde{K}_{j,t} \right) dj = \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{1-\nu} Y_t dj \\ - \int_0^1 w_t L_{j,t} dj - \int_0^1 R_t^k \tilde{K}_{j,t} dj$$

$$\Pi_{t(real)} = \int_0^1 \left(\left(\frac{p_{j,t}}{p_t}\right)^{1-\nu} Y_t - w_t L_{j,t} - R_t^k \tilde{K}_{j,t} \right) dj = Y_t \frac{1}{p_t^{1-\nu}} \int_0^1 (p_{j,t})^{1-\nu} dj \\ - w_t \int_0^1 L_{j,t} dj - R_t^k \int_0^1 \tilde{K}_{j,t} dj$$

Given that

- the aggregate price level is $p_t^{1-\nu} = \int_0^1 p_{j,t}^{1-\nu} dj$,
- aggregate labor demand must equal supply, $\int_0^1 L_{j,t} dj = L_t$, and
- aggregate supply of capital services must equal demand $\int_0^1 \tilde{K}_{j,t} dj = \tilde{K}_t$,

the aggregate profit is

$$\Pi_{t(real)} = Y_t - w_t L_t - R_t^k \tilde{K}_t$$

Plugging this expression into the household budget constraint finally gives the aggregate accounting identity as

$$Y_t = C_t + I_t + G_t$$

C.3. Inflation

Firms have a probability $1-\zeta$ of getting to update their price each period. Since there are an infinite number of firms, there is also the exact fraction $1-\zeta$ of total firms who adjust their prices and the fraction ζ who stay with the previous period price. Moreover, since there is a random sampling from the entire distribution of firm prices, the distribution of any subset of firm prices is similar to the entire distribution. Therefore, the aggregate price index, $p_t^{1-\nu} = \int_0^1 p_{j,t}^{1-\nu} dj$, is rewritten as

$$p_t^{1-\nu} = \int_0^{1-\zeta} p_t^{*1-\nu} dj + \int_{1-\zeta}^1 p_{j,t-1}^{1-\nu} dj$$

which simplifies to

$$p_t^{1-\nu} = (1-\zeta) p_t^{*1-\nu} + \zeta p_{t-1}^{1-\nu}$$

Dividing both sides of the equation by $p_{t-1}^{1-\nu}$

$$\left(\frac{p_t}{p_{t-1}}\right)^{1-\nu} = (1-\zeta)\left(\frac{p_t^*}{p_{t-1}}\right)^{1-\nu} + \zeta\left(\frac{p_{t-1}}{p_{t-1}}\right)^{1-\nu}$$

and defining gross inflation as $1+\pi_t = \frac{p_t}{p_{t-1}}$ and gross reset inflation as $1+\pi_t^* = \frac{p_t^*}{p_{t-1}}$, we get

$$(1+\pi_t)^{1-\nu} = (1-\zeta)(1+\pi_t^*)^{1-\nu} + \zeta$$

Finally, from $p_t^* = \frac{\nu}{\nu-1} E_t \frac{\Xi_{1_t}}{\Xi_{2_t}}$, we have

$$\frac{p_t^*}{p_t} = \frac{\nu}{\nu-1} E_t \frac{\Xi_{1_t}/p_t^{\nu}}{\Xi_{2_t}/p_t^{\nu-1}}$$

Rewritting the left-hand side as $\frac{p_t^*}{p_t}\frac{p_{t-1}}{p_{t-1}}$, and rearranging, we get

$$\pi_t^* = \pi_t \frac{\nu}{\nu-1} E_t \frac{\Xi_{1_t}/p_t^{\nu}}{\Xi_{2_t}/p_t^{\nu-1}}$$

Therefore we have

$$\frac{\Xi_{1_t}}{p_t^{\nu}} = \frac{Q_{t+s}}{\beta} Y_t mc_t^* + \zeta\beta E_t \frac{\Xi_{1_{t+1}}}{p_{t+1}^{\nu}} (1+\pi_{t+1})^{\nu}$$

$$\frac{\Xi_{2_t}}{p_t^{\nu-1}} = \frac{Q_{t+s}}{\beta} Y_t + \zeta\beta E_t \frac{\Xi_{2_{t+1}}}{p_{t+1}^{\nu-1}} (1+\pi_{t+1})^{\nu-1}$$

C.4. Aggregate supply

We know that the demand to individual firm j is given by

$$Y_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} Y_t$$

and that firm j hires labor and capital in the same proportion than the aggregate capital to labor ratio (common factor markets). Hence, substituting in the production function for the intermediate good j we get

$$A_t \left(\frac{\tilde{K}_t}{L_t}\right)^{\alpha} L_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} Y_t$$

Then, summing up across the intermediate firms gives

$$A_t \left(\frac{\tilde{K}_t}{L_t}\right)^{\alpha} \int_0^1 L_{j,t} dj = Y_t \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj$$

Given that aggregate labor demand equals aggregate labor supply $\int_0^1 L_{j,t} dj = L_t$, we have

$$\int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} djY_t = A_t \tilde{K}_t^{\alpha} L_t^{1-\alpha}$$

Thus, the aggregate production function can be written as

$$Y_t = \frac{A_t \tilde{K}_t^{\alpha} L_t^{1-\alpha}}{\Omega_t}$$

where $\Omega_t = \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj$ measures a distortion introduced by the dispersion in relative prices.[23] In order to express Ω_t in aggregate terms, let decompose it according to the Calvo pricing assumption again, so that

$$\Omega_t = \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj = p_t^{\nu} \int_0^1 p_{j,t}^{-\nu}$$

[23] This distortion is not the one associated with the monopoly power of firms but an additional one that arises from the relative price fluctuations due to prie stickiness.

$$p_t^{\nu}\int_0^1 p_{j,t}^{-\nu} = p_t^{\nu}\left(\int_0^{1-\zeta} p_t^{*-\nu}dj + \int_{1-\zeta}^1 p_{j,t-1}^{-\nu}dj\right)$$

$$p_t^{\nu}\int_0^1 p_{j,t}^{-\nu} = p_t^{\nu}(1-\zeta)p_t^{*-\nu} + p_t^{\nu}\int_{1-\zeta}^1 p_{j,t-1}^{-\nu}dj$$

$$p_t^{\nu}\int_0^1 p_{j,t}^{-\nu} = (1-\zeta)\left(\frac{p_t^*}{p_t}\right)^{-\nu} + p_t^{\nu}\int_{1-\zeta}^1 p_{j,t-1}^{-\nu}dj$$

$$p_t^{\nu}\int_0^1 p_{j,t}^{-\nu} = (1-\zeta)\left(\frac{p_t^*}{p_{t-1}}\right)^{-\nu}\left(\frac{p_{t-1}}{p_t}\right)^{-\nu} + p_t^{\nu}\int_{1-\zeta}^1 p_{j,t-1}^{-\nu}dj$$

$$p_t^{\nu}\int_0^1 p_{j,t}^{-\nu} = (1-\zeta)(1+\pi_t^*)^{-\nu}(1+\pi_t)^{\nu} + p_{t-1}^{-\nu}p_t^{\nu}\int_{1-\zeta}^1 \left(\frac{p_{j,t-1}}{p_{t-1}}\right)^{-\nu}dj$$

Given random sampling and the fact that there is a continuum of firms

$$\Omega_t = (1-\zeta)(1+\pi_t^*)^{-\nu}(1+\pi_t)^{\nu} + \zeta(1+\pi_t)^{\nu}\Omega_{t-1}$$

D. Full set of equilibrium conditions

$$K_{t+1} = \left\{(1-\delta u_t^{\eta})K_t + \left[1-\frac{\tau}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2\right]I_t\right\}(1-\theta_t b_k) \tag{26}$$

$$\log\theta_t = (1-\rho_\theta)\log\bar{\theta} + \rho_\theta\log\theta_{t-1} + \sigma_\theta\varepsilon_{\theta_t} \tag{27}$$

$$\tilde{K}_t = u_t K_t \tag{28}$$

$$\lambda_t = (C_t - hC_{t-1})^{-\gamma} - \beta h E_t(C_{t+1} - hC_t)^{-\gamma} \tag{29}$$

$$\chi L_t^{\phi} = w_t\lambda_t \tag{30}$$

$$\lambda_t = \beta E_t\lambda_{t+1}(1+i_{t+1})(1+\pi_{t+1})^{-1} \tag{31}$$

$$\mu_t = \beta E_t \left[\lambda_{t+1} R^k_{t+1} u_{t+1} + \mu_{t+1} \left(1 - \delta u^{\eta}_{t+1}\right) \left(1 - \theta_{t+1} b_k\right) \right] \tag{32}$$

$$\lambda_t R^k_t = \mu_t \delta \eta u_t^{\eta - 1} (1 - \theta_t b_k) \tag{33}$$

$$\begin{aligned} \lambda_t = \mu_t \left(1 - \theta_t b_k\right) & \left[1 - \frac{\tau}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \tau \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] \\ & + \beta E_t \mu_{t+1} \left(1 - \theta_{t+1} b_k\right) \tau \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \end{aligned} \tag{34}$$

$$1 + i_t = \left(E_t Q_{t,t+1}\right)^{-1} \tag{35}$$

$$q_t = \frac{\mu_t}{\lambda_t} \tag{36}$$

$$\log A_t = (1 - \rho_A) \log \bar{A} + \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A_t} \tag{37}$$

$$w_t = mc^* (1 - \alpha) A_t \left(\frac{\tilde{K}_t}{L_t} \right)^{\alpha} \tag{38}$$

$$R^k_t = mc^* \alpha A_t \left(\frac{\tilde{K}_t}{L_t} \right)^{\alpha - 1} \tag{39}$$

$$(1 + \pi^*_t) = (1 + \pi_t) \frac{\nu}{\nu - 1} E_t \frac{\tilde{\Xi}_{1_t}}{\tilde{\Xi}_{2_t}} \tag{40}$$

where $\tilde{\Xi}_{1_t} = \frac{\Xi_{1_t}}{p_t^{\nu}}$ and $\tilde{\Xi}_{2_t} = \frac{\Xi_{2_t}}{p_t^{\nu-1}}$.

$$\tilde{\Xi}_{1_t} = \lambda_t Y_t mc^*_t + \zeta \beta E_t \tilde{\Xi}_{1_{t+1}} \left(1 + \pi_{t+1}\right)^{\nu} \tag{41}$$

$$\tilde{\Xi}_{2_t} = \lambda_t Y_t + \zeta \beta E_t \tilde{\Xi}_{2_{t+1}} \left(1 + \pi_{t+1}\right)^{\nu - 1} \tag{42}$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[\psi_\pi(\pi_t - \bar{\pi}) + \psi_Y\left(y_t - \bar{y}\right) + \bar{i}\right] + \sigma_i \varepsilon_{i_t} \tag{43}$$

$$\log G_t = (1-\rho_G)\log(\omega \bar{Y}) + \rho_G \log G_{t-1} + \sigma_G \varepsilon_{G_t} \tag{44}$$

$$Y_t = C_t + I_t + G_t \tag{45}$$

$$Y_t = \frac{A_t \tilde{K}^\alpha L_t^{1-\alpha}}{\Omega_t} \tag{46}$$

$$(1+\pi_t)^{1-\nu} = (1-\zeta)(1+\pi_t^*)^{1-\nu} + \zeta \tag{47}$$

$$\Omega_t = (1-\zeta)(1+\pi_t^*)^{-\nu}(1+\pi_t)^\nu + \zeta(1+\pi_t)^\nu \Omega_{t-1} \tag{48}$$

This is a system of 23 equations in 23 unknowns: $\{Y, C, I, G, A, L, K, \tilde{K}, u, w, R^k, \Omega, \pi, \pi^*, \tilde{\bar{\Xi}}_1, \tilde{\bar{\Xi}}_2, mc^*, \lambda, \mu, i, q,$

E. Steady-state

From the FOC on investment (34), we have

$$\bar{\lambda} = \bar{\mu}(1 - \bar{\theta} b_k) \tag{49}$$

which implies by (36) that

$$\bar{q} = \frac{\bar{\mu}}{\bar{\lambda}} = \frac{1}{1-\bar{\theta} b_k} \tag{50}$$

Without disaster risk, we would have $\bar{q} = 1$ determining the threshold under which firms invest or disinvest to raise their market value. Here disaster risk implies that this threshold is greater than unity since, for a given replacement cost in terms of utility, firms find it less profitable to invest as the probability that a part of their capital turns out to be destroyed rises.

Normalizing $\bar{u} = 1$, we have $\bar{\tilde{K}} = \bar{K}$ from (28), and from (33)

$$\bar{R}^k = \delta \eta \tag{51}$$

Moreover (32) implies that

$$\bar{R}^k = \frac{1}{\beta(1-\bar{\theta}b_k)} - (1-\delta) \tag{52}$$

The last two equations imply a parameter restriction of η as

$$\eta = 1 + \frac{\frac{1}{\beta(1-\bar{\theta}b_k)} - 1}{\delta} \tag{53}$$

Therefore, with parameter values $\beta = .99$, $\delta = .025$, $\bar{\theta} = .017$, and $b_k = .43$, we have $\eta = 1.7$ (and $\eta = 1.404$ in a world without disasters).

Then from (47), and given the target inflation rate $\bar{\pi}$, we have the steady-state reset inflation rate as

$$(1+\bar{\pi}^*) = \left(\frac{(1+\bar{\pi})^{1-\nu} - \zeta}{1-\zeta}\right)^{\frac{1}{1-\nu}} \tag{54}$$

and, since from (40) we have,

$$(1+\bar{\pi}^*) = (1+\bar{\pi})\frac{\nu}{\nu-1}\frac{\bar{\bar{\Xi}}_1}{\bar{\bar{\Xi}}_1} \tag{55}$$

where, from (41) and (42),

$$\bar{\bar{\Xi}}_1 = \frac{\bar{\lambda}\bar{Y}\bar{mc}^*}{1-\zeta\beta(1+\bar{\pi})^{\nu}} \tag{56}$$

$$\bar{\bar{\Xi}}_2 = \frac{\bar{\lambda}\bar{Y}}{1-\zeta\beta(1+\bar{\pi})^{\nu-1}} \tag{57}$$

we get

$$(1+\bar{\pi}^*) = (1+\bar{\pi})\frac{\nu}{\nu-1}\bar{mc}^*\frac{1-\zeta\beta(1+\bar{\pi})^{\nu-1}}{1-\zeta\beta(1+\bar{\pi})^{\nu}} \tag{58}$$

which gives the steady-state marginal cost $\bar{mc}^*$ as

$$\bar{mc}^* = \frac{\nu-1}{\nu}\frac{1}{(1+\bar{\pi})}\frac{1-\zeta\beta(1+\bar{\pi})^{\nu}}{1-\zeta\beta(1+\bar{\pi})^{\nu-1}}\left(\frac{(1+\bar{\pi})^{1-\nu}-\zeta}{1-\zeta}\right)^{\frac{1}{1-\nu}} \tag{59}$$

Note that we must therefore restrict parameter values so that $\zeta\beta(1+\bar{\pi})^{\nu} < 1$.

With the expressions for $\bar{R}^k$ and $\bar{mc}^*$, we can express the steady-state capital-labor ratio as a function of the steady-state characteristics of disaster from (39)

$$\frac{\bar{K}}{\bar{L}} = \left(\frac{\bar{mc}^*\alpha\bar{a}}{\bar{R}^k}\right)^{\frac{1}{1-\alpha}} \tag{60}$$

Therefore the steady-state wage is given by (38)

$$\bar{w} = \bar{mc}^*(1-\alpha)\bar{a}\left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha} \tag{61}$$

From (48), we have

$$\bar{\Omega} = \frac{(1-\zeta)(1+\bar{\pi}^*)^{-\nu}(1+\bar{\pi})^{\nu}}{1-\zeta(1+\bar{\pi})^{\nu}} \tag{62}$$

From the law of capital accumulation (26) in steady-state, we have

$$\bar{I} = \bar{K}\left(\frac{1}{1-\bar{\theta}b_k} - (1-\delta)\right) \tag{63}$$

and given that from (44),

$$\bar{G} = \omega\bar{Y} \tag{64}$$

the accounting identity (45) becomes in steady-state

$$\bar{Y} = \frac{1}{1-\omega}\left\{\bar{C} + \bar{K}\left[\frac{1}{1-\bar{\theta}b_k} - (1-\delta)\right]\right\} \tag{65}$$

in which $\frac{1}{1-\omega}$ is the keynesian multiplier of public expenditures. Further dividing each side by $\bar{L}$ gives

$$\frac{\bar{Y}}{\bar{L}} = \frac{1}{1-\omega}\left\{\frac{\bar{C}}{\bar{L}} + \frac{\bar{K}}{\bar{L}}\left[\frac{1}{1-\bar{\theta}b_k} - (1-\delta)\right]\right\} \tag{66}$$

Replacing the left-hand side by the output-labor ratio obtained from the aggregate production function (46), we have

$$\frac{\bar{A}}{\bar{\Omega}}\left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha}=\frac{1}{1-\omega}\left\{\frac{\bar{C}}{\bar{L}}+\frac{\bar{K}}{\bar{L}}\left[\frac{1}{1-\bar{\theta}b_k}-(1-\delta)\right]\right\} \tag{67}$$

which can be solved for the steady-state consumption-labor ratio as

$$\frac{\bar{C}}{\bar{L}}=\frac{\bar{A}(1-\omega)}{\bar{\Omega}}\left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha}-\frac{\bar{K}}{\bar{L}}\left[\frac{1}{1-\bar{\theta}b_k}-(1-\delta)\right] \tag{68}$$

Combining the FOC on consumption (29) in steady-state

$$\bar{\lambda}=[(1-h)\bar{C}]^{-\gamma}(1-\beta h) \tag{69}$$

with the FOC on labor (30) in steady-state

$$\bar{L}=\left(\frac{\bar{w}\bar{\lambda}}{\chi}\right)^{1/\phi} \tag{70}$$

we can express $\bar{L}$ as a function of the steady state consumption-labor ratio

$$\bar{L}=\left[\frac{\bar{w}(1-h)^{-\gamma}\left(\frac{\bar{C}}{\bar{L}}\right)^{-\gamma}(1-\beta h)}{\chi_L}\right]^{\frac{1}{\phi_L+\gamma}} \tag{71}$$

which gives $\bar{\lambda}$ by (69) and therefore $\bar{\mu}$. $\bar{L}$ also gives $\bar{Y}$ by (66) and $\bar{K}$ by (60). Then $\bar{G}$ is obtained by (64) and $\bar{I}$ by the accounting identity or by (63). Then we get $\bar{\bar{\Xi}}_1$ and $\bar{\bar{\Xi}}_2$ by (56) and (57).

Finally, from the FOC on bonds (31) we have the standard Fisher relation between the subjective discount factor, the nominal interest rate and the inflation rate, $1/\beta=(1+\bar{i})/(1+\bar{\pi})$, such that, by (35), the one-period stochastic discount factor is

$$\bar{Q}=\frac{1}{1+\bar{i}}=\frac{\beta}{1+\bar{\pi}} \tag{72}$$

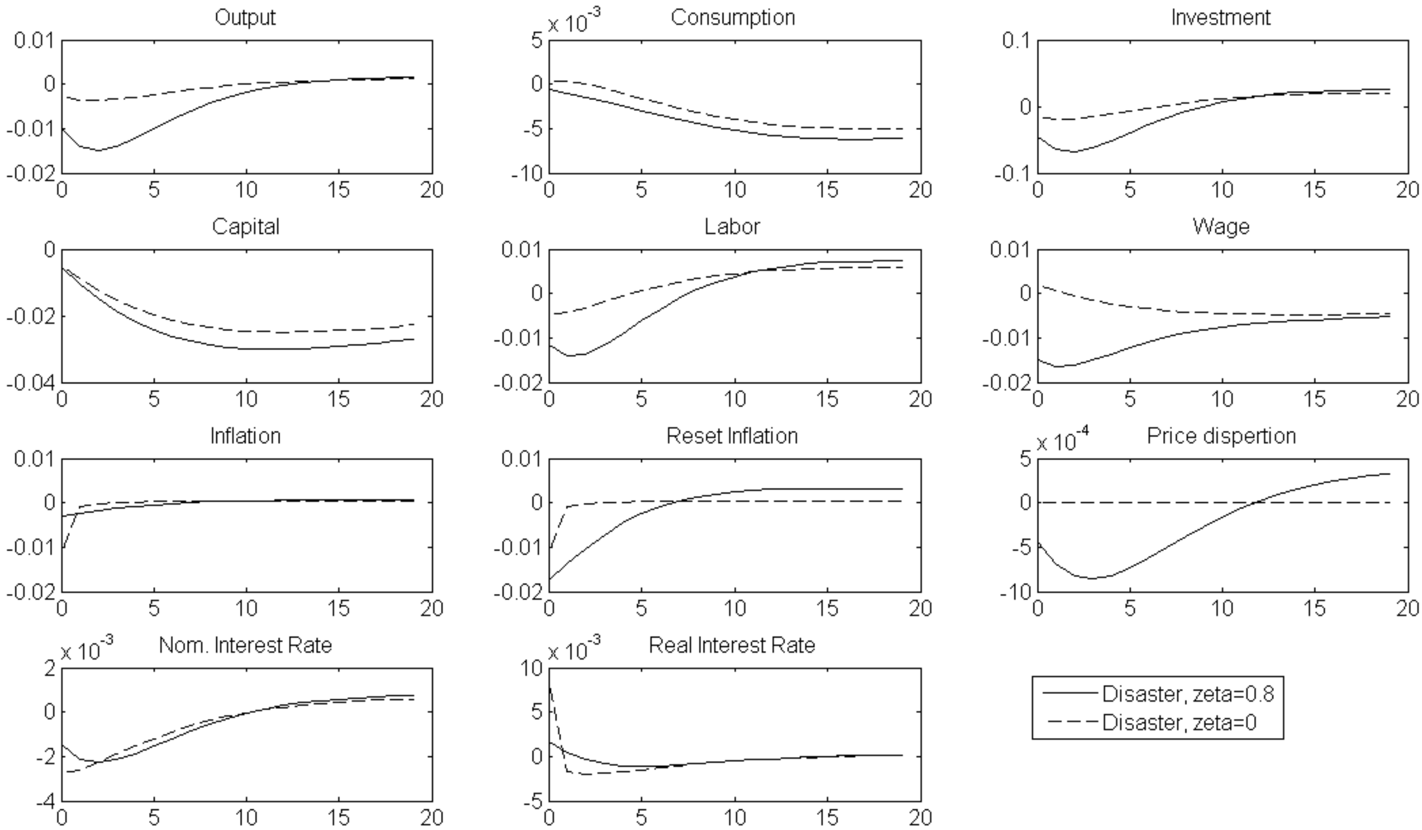

Figure E.1 – Standard-deviation responses to a shock to the probability of disaster (increase in θ). Solid line: model with disaster risk and sticky prices (ζ = 0.8). Dashed line: model with disaster risk and flexible prices (ζ = 0).

Table E.1 – Baseline calibration parameters (quarterly values)

Utility function		
β	discount factor	0.99
γ	inverse of EIS / risk aversion coefficient	2
h	habit in consumption	0.7
ϕ	inverse of the elasticity of work effort to the real wage	1
χ	labor disutility weight	4.74
Investment		
δ	capital depreciation rate	0.025
τ	investment adjustment costs	0.5
$\bar{u}$	utilization rate of capital	1
Production		
α	capital share of production	0.33
ζ_0	Calvo probability	0.8
ν	elasticity of substitution among intermediate goods	6
Public authority		
ω	steady-state G/Y ratio	0.2
ψ_π	Taylor rule inflation weight	1.5
ψ_Y	Taylor rule output weight	0.5
$\bar{\pi}$	target inflation rate	0.005
ρ_A	TFP smoothing parameter	0.9
ρ_G	government expenditures smoothing parameter	0.85
ρ_i	interest rate smoothing parameter	0.85
Disaster risk		
$\bar{\theta}$	disaster risk	0.01
b_k	share of capital destroyed if disaster	0.43
ρ_θ	disaster risk smoothing parameter	0.85
σ	standard deviation of shocks	0.01

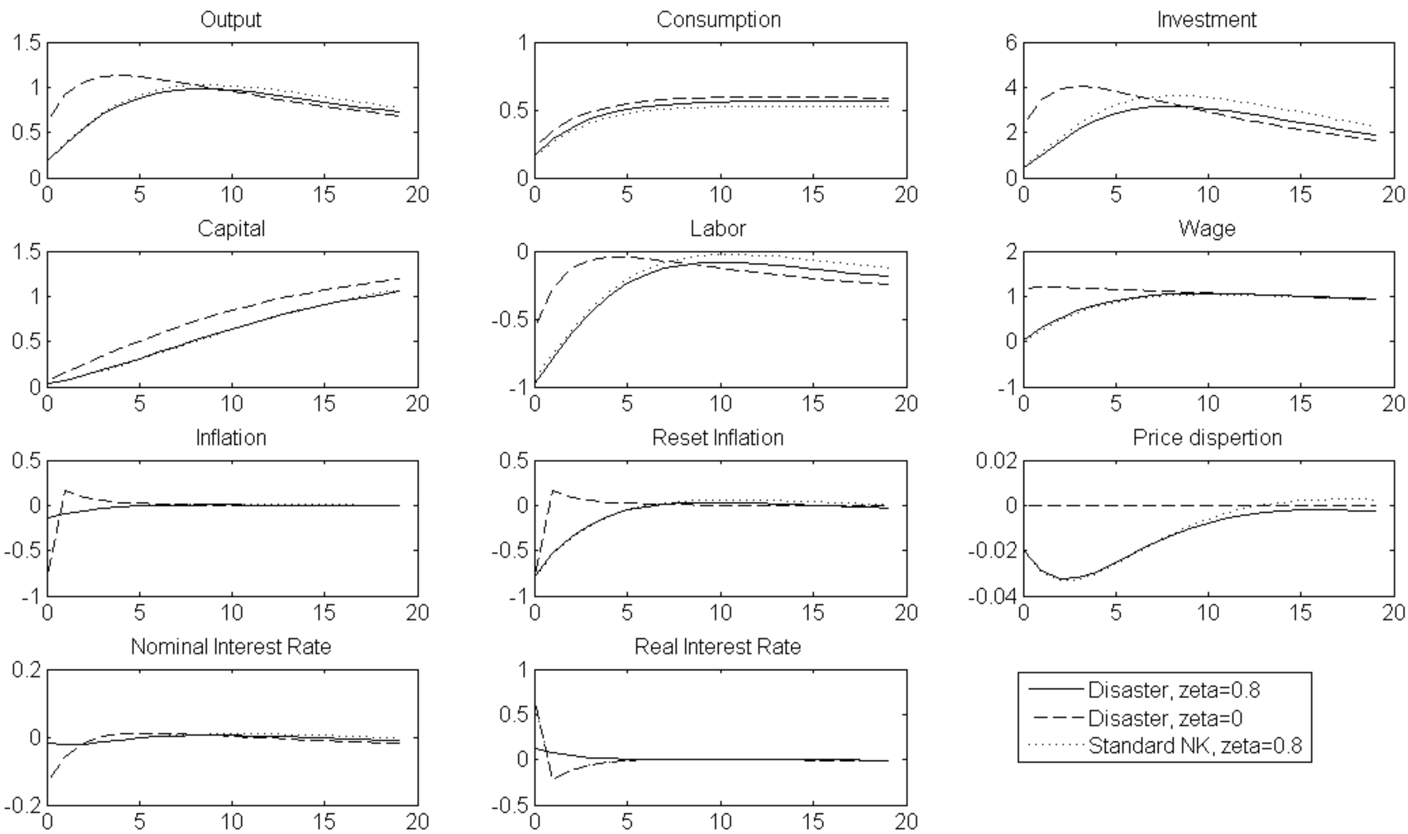

Figure E.2 – Standard-deviation responses to a productivity shock. Solid line: model with disaster risk and sticky prices (ζ = 0.8). Dashed line: model with disaster risk and flexible prices (ζ = 0). Dotted line: model without disasters, with sticky prices.

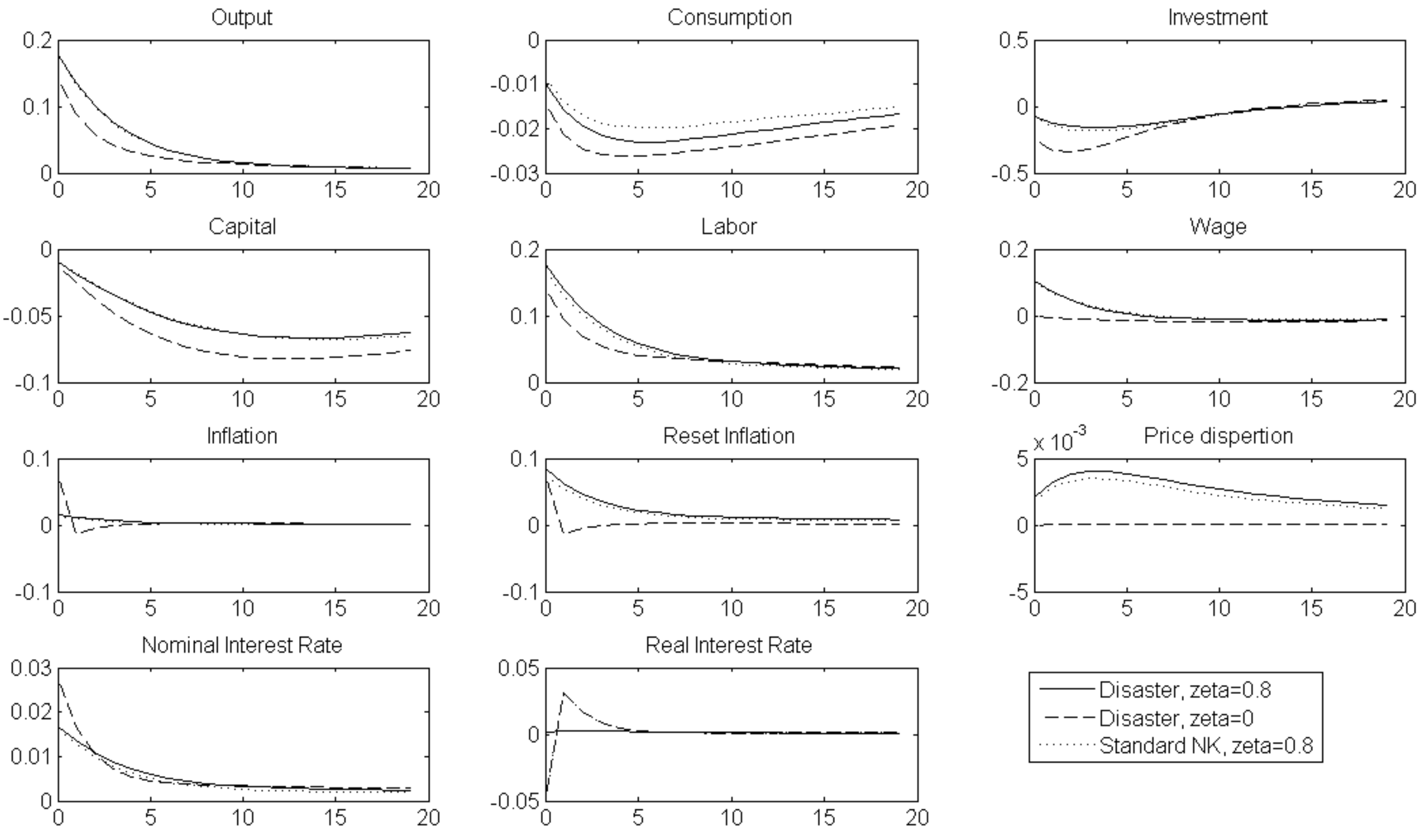

Figure E.3 – Standard-deviation responses to a public spending shock. Solid line: model with disaster risk and sticky prices (ζ = 0.8). Dashed line: model with disaster risk and flexible prices (ζ = 0). Dotted line: model without disasters, with sticky prices.

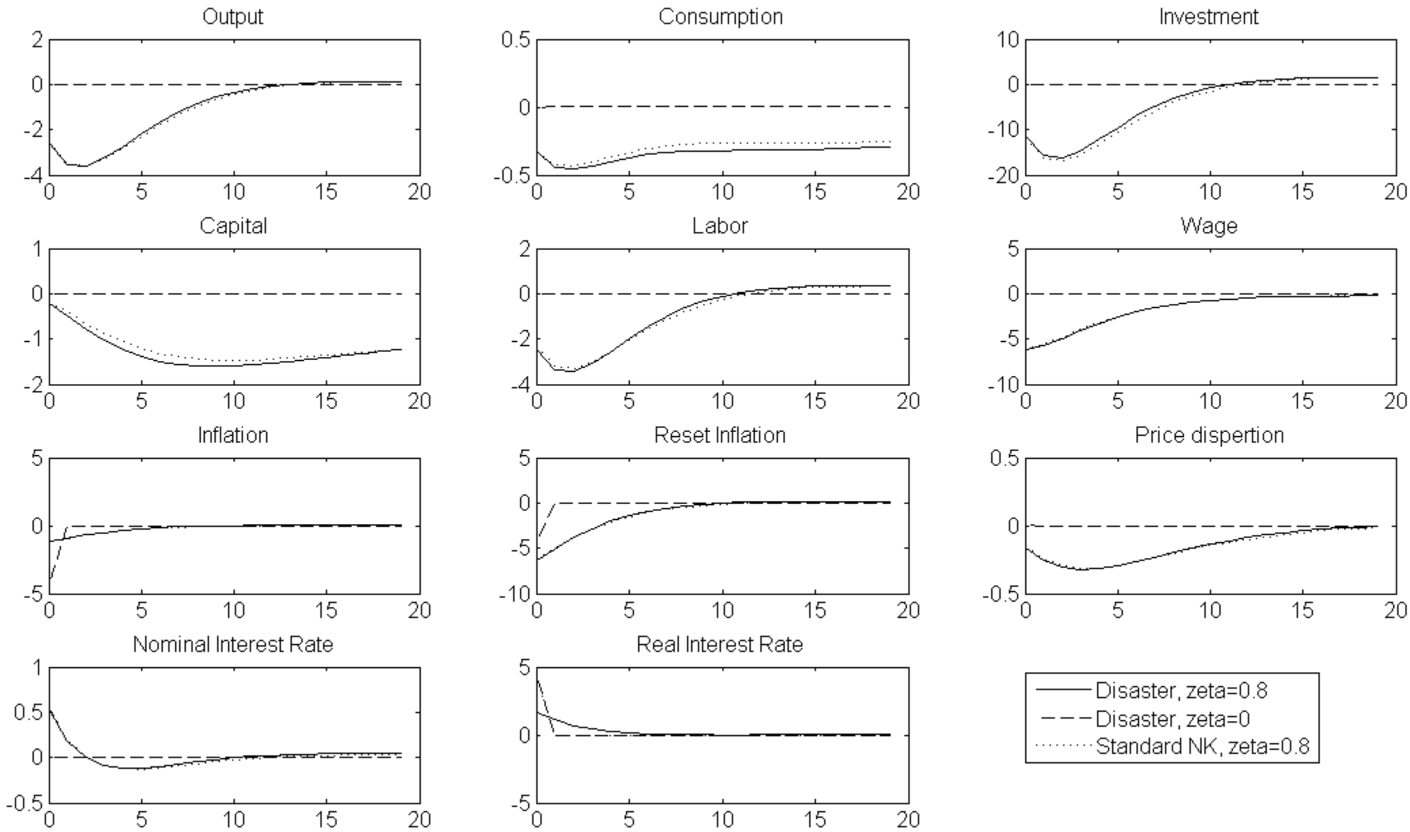

Figure E.4 – Standard-deviation responses to a monetary shock. Solid line: model with disaster risk and sticky prices (ζ = 0.8). Dashed line: model with disaster risk and flexible prices (ζ = 0). Dotted line: model without disasters, with sticky prices.

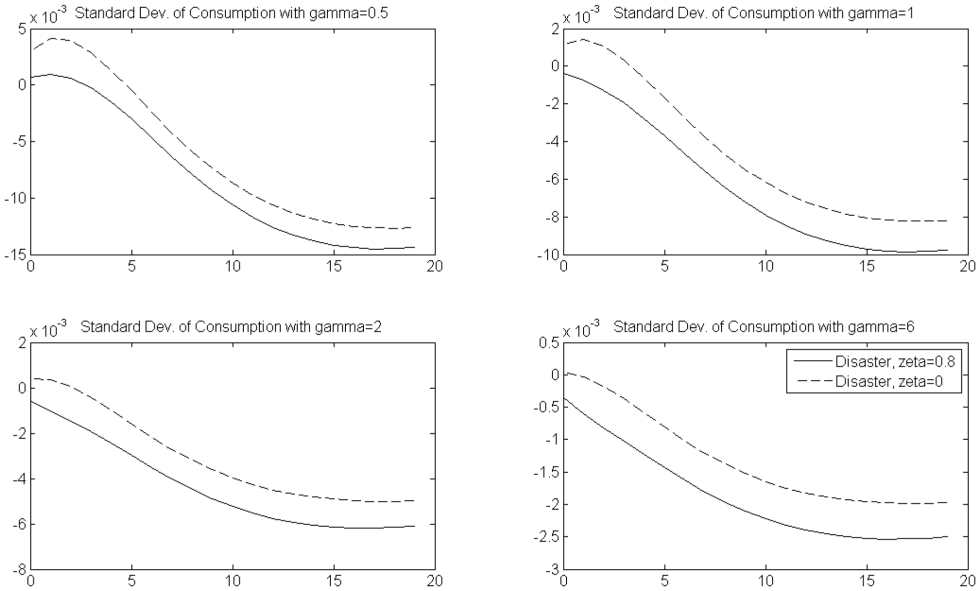

Figure E.5 – Standard-deviation of consumption to a shock to the probability of disaster, for different values of the risk aversion coefficient γ.

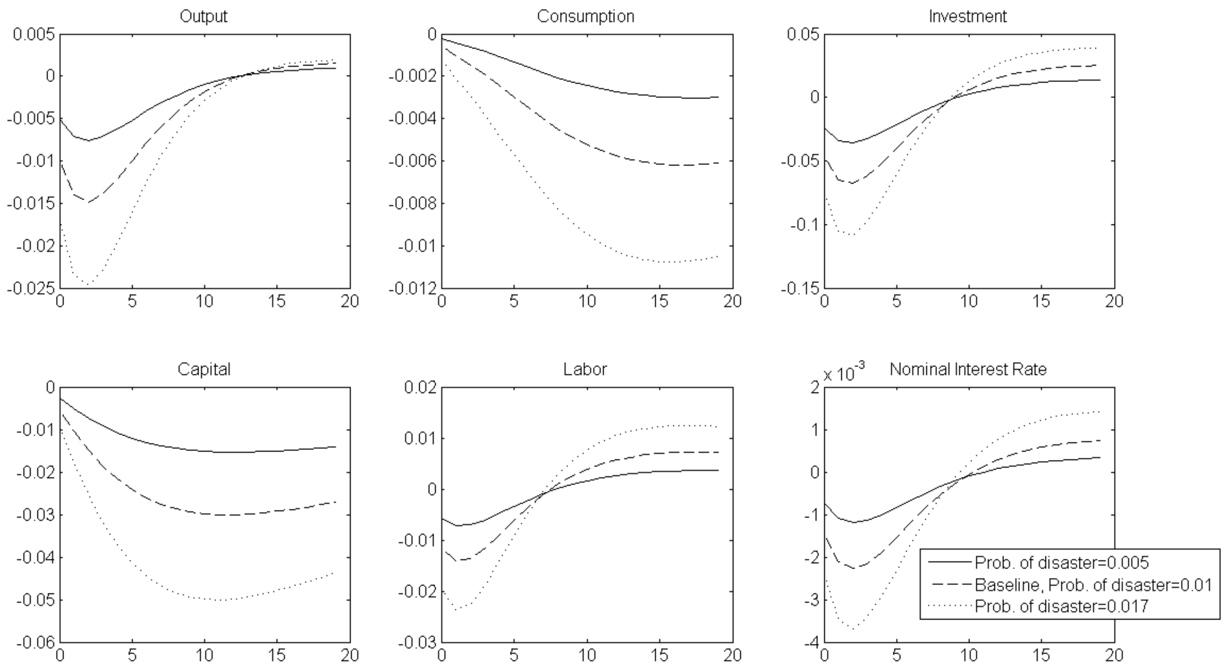

Figure E.6 – Standard-deviation responses to a shock to the probability of disaster, for different values of the steady-state probability of disaster, $\bar{\theta}$.

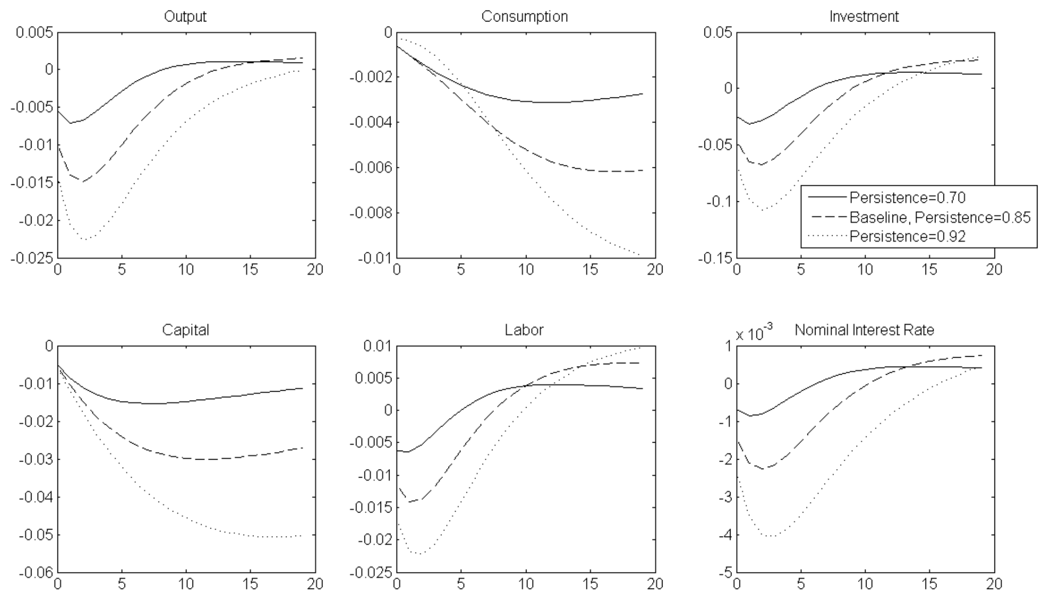

Figure E.7 – Standard-deviation responses to a shock to the probability of disaster, for different values of the persistence of the shock ρ_θ.

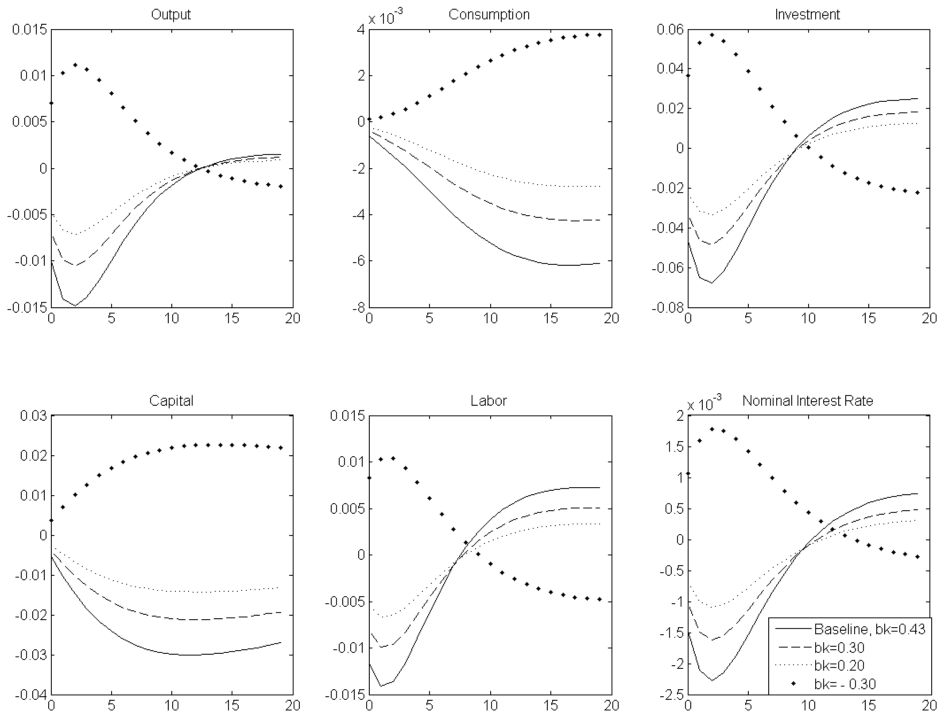

Figure E.8 – Standard-deviation responses to a shock to the probability of disaster, for different values of the destroyed share of capital b_k.

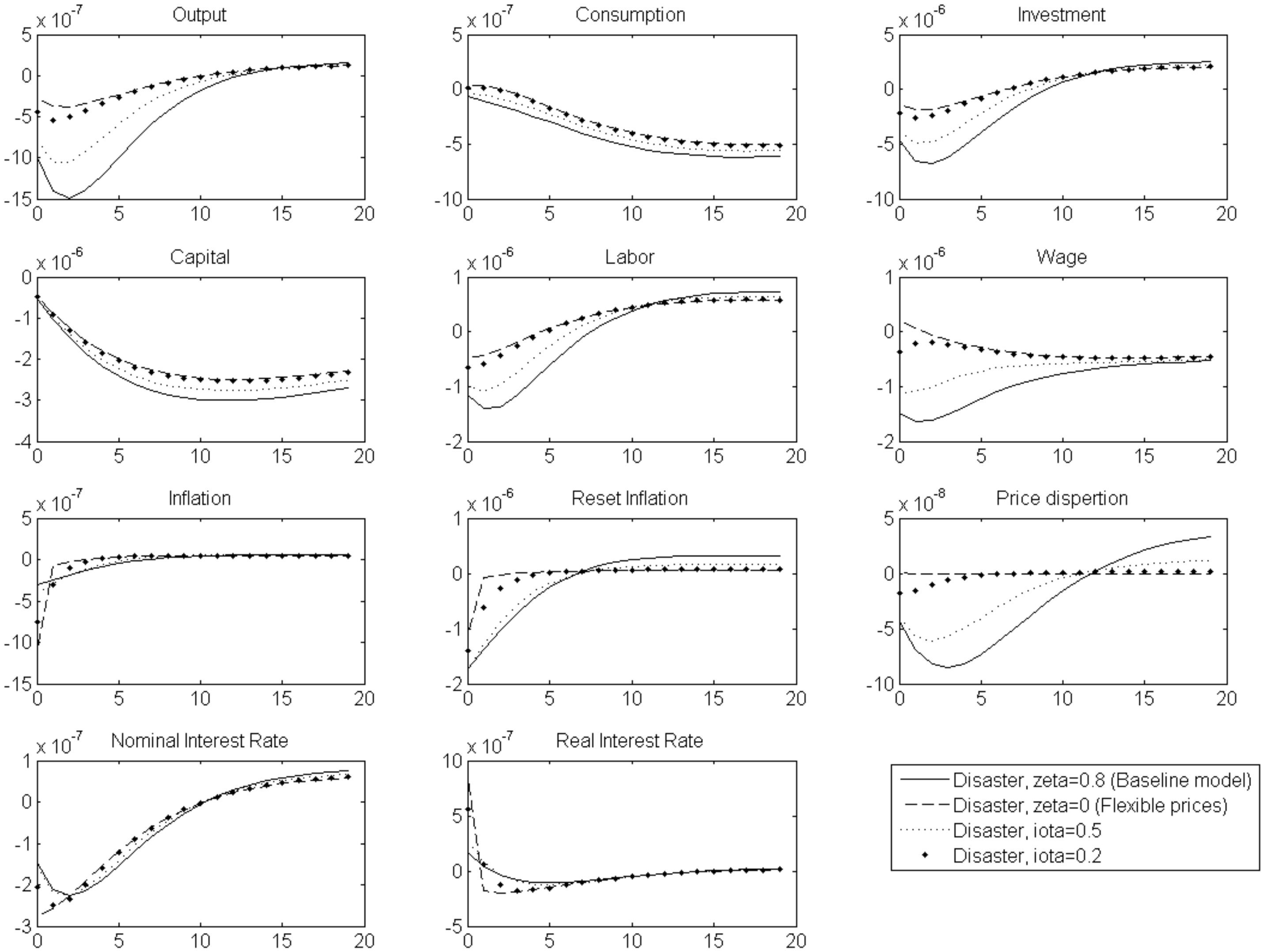

Figure E.9 – Standard-deviation responses to a shock to the probability of disaster, with state-dependent price stickiness. We assume that $\zeta_t = \zeta_0 - \theta_t^{\iota}$. With $\iota \geq 1$, the responses are very close to the Calvo pricing case ($\zeta = 0.8$), thus not included.

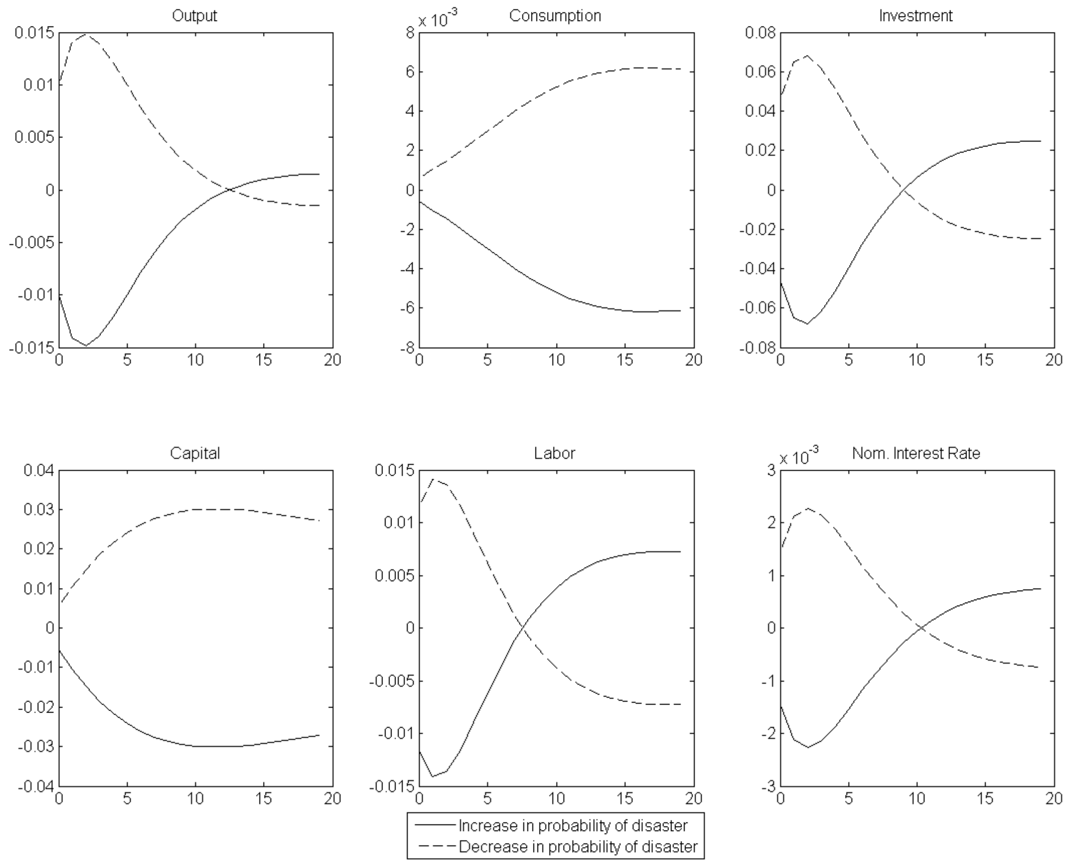

Figure E.10 – Standard-deviation responses to a shock to the probability of disaster. Negative and positive shocks.

Table E.2 – Correction terms for the second-order approximation, shock to θ

	$\hat{Y}$	$\hat{C}$	$\hat{I}$	$\hat{G}$	$\hat{K}$	$\hat{L}$	$\hat{\Omega}$
Constant	0.691892	0.169961	-0.884952	-0.917545	2.644587	-0.281788	0.001777
2nd-order correction	-0.000001	0	-0.000004	0	0	0	0
	$\hat{\pi}$	$\hat{\pi}^*$	$\hat{mc}^*$	$\hat{w}$	$\hat{R}^k$	$\hat{q}$	$\hat{Q}$
Constant	0.004987	0.026281	-0.182864	0.387128	-3.232391	0.004307	-0.015038
2nd-order correction	0	-0.000003	-0.000006	-0.000007	-0.000004	-0.000002	0